

# Package ‘BayesNSGP’

May 6, 2026

**Title** Bayesian Analysis of Non-Stationary Gaussian Process Models

**Description** Enables off-the-shelf functionality for fully Bayesian, nonstationary Gaussian process modeling. The approach to nonstationary modeling involves a closed-form, convolution-based covariance function with spatially-varying parameters; these parameter processes can be specified either deterministically (using covariates or basis functions) or stochastically (using approximate Gaussian processes). Stationary Gaussian processes are a special case of our methodology, and we furthermore implement approximate Gaussian process inference to account for very large spatial data sets (Finley, et al (2017) <[doi:10.48550/arXiv.1702.00434](https://doi.org/10.48550/arXiv.1702.00434)>). Bayesian inference is carried out using Markov chain Monte Carlo methods via the “nimble” package, and posterior prediction for the Gaussian process at unobserved locations is provided as a post-processing step.

**Version** 0.2.0

**Date** 2025-12-11

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**Depends** R (>= 3.4.0),nimble

**Imports** FNN,Matrix,methods,StatMatch

**License** GPL-3

**Encoding** UTF-8

**RoxygenNote** 7.3.2

**NeedsCompilation** no

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**Repository** CRAN

**Date/Publication** 2025-12-11 16:20:21 UTC

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calcQF	<i>Calculate the Gaussian quadratic form for the NNGP approximation</i>
--------	---

---

### Description

calcQF calculates the quadratic form in the multivariate Gaussian based on the NNGP approximation, for a specific parameter combination. The quadratic form is  $t(u)C^{-1}v$ .

### Usage

calcQF(u, v, AD, nID)

**Arguments**

u	Vector; left product.
v	Vector; right product
AD	N x (k+1) matrix; the first k columns are the 'A' matrix, and the last column is the 'D' vector. Represents the Cholesky of $C^{-1}$ .
nID	N x k matrix of neighbor indices.

**Value**

A list with two components: (1) an N x 2 array containing the same spatial coordinates, ordered by MMD, and (2) the same thing, but with any NA values removed.

---

calculateAD_ns	<i>Calculate A and D matrices for the NNGP approximation</i>
----------------	--

---

**Description**

calculateAD\_ns calculates A and D matrices (the Cholesky of the precision matrix) needed for the NNGP approximation.

**Usage**

```
calculateAD_ns(
  dist1_3d,
  dist2_3d,
  dist12_3d,
  Sigma11,
  Sigma22,
  Sigma12,
  log_sigma_vec,
  log_tau_vec,
  nID,
  N,
  k,
  nu,
  d
)
```

**Arguments**

dist1_3d	N x (k+1) x (k+1) array of distances in the x-coordinate direction.
dist2_3d	N x (k+1) x (k+1) array of distances in the y-coordinate direction.
dist12_3d	N x (k+1) x (k+1) array of cross-distances.
Sigma11	N-vector; 1-1 element of the Sigma() process.
Sigma22	N-vector; 2-2 element of the Sigma() process.

Sigma12	N-vector; 1-2 element of the Sigma() process.
log_sigma_vec	N-vector; process standard deviation values.
log_tau_vec	N-vector; nugget standard deviation values.
nID	N x k matrix of neighbor indices.
N	Scalar; number of data measurements.
k	Scalar; number of nearest neighbors.
nu	Scalar; Matern smoothness parameter.
d	Scalar; dimension of the spatial domain.

**Value**

A  $N \times (k+1)$  matrix; the first  $k$  columns are the 'A' matrix, and the last column is the 'D' vector.

---

calculateU_ns	<i>Calculate the (sparse) matrix U</i>
---------------	--

---

**Description**

calculateU\_ns calculates the (sparse) matrix  $U$  (i.e., the Cholesky of the inverse covariance matrix) using a nonstationary covariance function. The output only contains non-zero values and is stored as three vectors: (1) the row indices, (2) the column indices, and (3) the non-zero values. NOTE: this code assumes the all inputs correspond to the ORDERED locations.

**Usage**

```
calculateU_ns(
  dist1_3d,
  dist2_3d,
  dist12_3d,
  Sigma11,
  Sigma22,
  Sigma12,
  log_sigma_vec,
  log_tau_vec,
  nu,
  nID,
  cond_on_y,
  N,
  k,
  d,
  M = 0
)
```

**Arguments**

dist1_3d	N x (k+1) x (k+1) array of distances in the x-coordinate direction.
dist2_3d	N x (k+1) x (k+1) array of distances in the y-coordinate direction.
dist12_3d	N x (k+1) x (k+1) array of cross-distances.
Sigma11	N-vector; 1-1 element of the Sigma() process.
Sigma22	N-vector; 2-2 element of the Sigma() process.
Sigma12	N-vector; 1-2 element of the Sigma() process.
log_sigma_vec	N-vector; process standard deviation values.
log_tau_vec	N-vector; nugget standard deviation values.
nu	Scalar; Matern smoothness parameter.
nID	N x k matrix of (ordered) neighbor indices.
cond_on_y	A matrix indicating whether the conditioning set for each (ordered) location is on the latent process (y, 1) or the observed values (z, 0). Calculated in <code>sgvSetup</code> .
N	Scalar; number of data measurements.
k	Scalar; number of nearest neighbors.
d	Scalar; dimension of the spatial domain.
M	Scalar; number of prediction sites.

**Value**

Returns a sparse matrix representation of the Cholesky of the precision matrix for a fixed set of covariance parameters.

---

`conditionLatentObs`      *Assign conditioning sets for the SGV approximation*

---

**Description**

`conditionLatentObs` assigns  $q_y(i)$  vs  $q_z(i)$  following Section 5.1 in Katzfuss and Guinness (2018). This function only needs to be run once per SGV analysis.

**Usage**

```
conditionLatentObs(nID, coords_ord, N)
```

**Arguments**

nID	N x k matrix of neighbor indices.
coords_ord	N x 2 matrix of locations.
N	Scalar; number of locations (observed only!).

**Value**

A matrix indicating whether the conditioning set for each location is on the latent process (y, 1) or the observed values (z, 0).

---

 crossCy\_sm

*Calculate sparse kernel, core kernel, and determine nonzero entries*


---

### Description

Cy\_sm calculates the normalized sparse kernel for a fixed set of bump function hyperparameters and returns the nonzero entries. Note that the matrix is calculated and returned in dense format.

### Usage

```
crossCy_sm(
  Xdists,
  coords,
  Pcoords,
  d,
  n1,
  n2,
  r0,
  s0,
  cstat_opt,
  normalize,
  bumpLocs,
  rads,
  ampls,
  shps,
  Xdist1_sq,
  Xdist2_sq,
  Xdist12,
  Sigma11,
  Sigma22,
  Sigma12,
  PSigma11,
  PSigma22,
  PSigma12,
  nu,
  log_sigma_vec,
  Plog_sigma_vec
)
```

### Arguments

Xdists	N x N matrix of Euclidean distances
coords	N x d matrix of coordinate/input locations
Pcoords	N x d matrix of coordinate/input locations
d	Scalar; dimension of the spatial domain.
n1	Scalar; number of outer products.

n2	Scalar; number of bump functions in each outer product.
r0	Scalar; length-scale of sparse stationary kernel.
s0	Scalar; signal-variance of sparse stationary kernel.
cstat_opt	Scalar; determines the compactly supported kernel. See Details.
normalize	Logical; should C_sparse have 1's along the diagonal (1 = TRUE)
bumpLocs	Array of bump function locations (n2*d x n1)
rads	Matrix of bump function radii (n1 x n2; denoted $r_{ij}$ )
ampls	Matrix of bump function amplitudes (n1 x n2; denoted $a_{ij}$ )
shps	Matrix of bump function shape parameters (n1 x n2; denoted $b_{ij}$ )
Xdist1_sq	N x N matrix; contains values of pairwise squared distances in the x-coordinate.
Xdist2_sq	N x N matrix; contains values of pairwise squared distances in the y-coordinate.
Xdist12	N x N matrix; contains values of pairwise signed cross- distances between the x- and y-coordinates. The sign of each element is important; see nsDist function for the details of this calculation. in the x-coordinate.
Sigma11	Vector of length N; contains the 1-1 element of the anisotropy process for each station.
Sigma22	Vector of length N; contains the 2-2 element of the anisotropy process for each station.
Sigma12	Vector of length N; contains the 1-2 element of the anisotropy process for each station.
PSigma11	Vector of length N; contains the 1-1 element of the anisotropy process for each station.
PSigma22	Vector of length N; contains the 2-2 element of the anisotropy process for each station.
PSigma12	Vector of length N; contains the 1-2 element of the anisotropy process for each station.
nu	Scalar; Matern smoothness parameter. nu = 0.5 corresponds to the Exponential correlation; nu = Inf corresponds to the Gaussian correlation function.
log_sigma_vec	Vector of length N; log of the signal standard deviation.
Plog_sigma_vec	Vector of length N; log of the signal standard deviation.

### Value

Returns a sparse matrix (N x 3) of the nonzero elements of the product between the core and sparse kernel.

---

 Cy\_sm

---

*Calculate sparse kernel, core kernel, and determine nonzero entries*


---

### Description

Cy\_sm calculates the normalized sparse kernel for a fixed set of bump function hyperparameters and returns the nonzero entries. Note that the matrix is calculated and returned in dense format.

### Usage

```
Cy_sm(
  dists,
  coords,
  N,
  d,
  n1,
  n2,
  r0,
  s0,
  cstat_opt,
  normalize,
  bumpLocs,
  rads,
  ampls,
  shps,
  dist1_sq,
  dist2_sq,
  dist12,
  Sigma11,
  Sigma22,
  Sigma12,
  nu,
  log_sigma_vec,
  lognuggetSD
)
```

### Arguments

dists	N x N matrix of Euclidean distances
coords	N x d matrix of coordinate/input locations
N	Scalar; number of data measurements.
d	Scalar; dimension of the spatial domain.
n1	Scalar; number of outer products.
n2	Scalar; number of bump functions in each outer product.
r0	Scalar; length-scale of sparse stationary kernel.

s0	Scalar; signal-variance of sparse stationary kernel.
cstat_opt	Scalar; determines the compactly supported kernel. See Details.
normalize	Logical; should C_sparse have 1's along the diagonal
bumpLocs	Array of bump function locations (n2*d x n1)
rads	Matrix of bump function radii (n1 x n2; denoted $r_{ij}$ )
ampls	Matrix of bump function amplitudes (n1 x n2; denoted $a_{ij}$ )
shps	Matrix of bump function shape parameters (n1 x n2; denoted $b_{ij}$ )
dist1_sq	N x N matrix; contains values of pairwise squared distances in the x-coordinate.
dist2_sq	N x N matrix; contains values of pairwise squared distances in the y-coordinate.
dist12	N x N matrix; contains values of pairwise signed cross- distances between the x- and y-coordinates. The sign of each element is important; see nsDist function for the details of this calculation. in the x-coordinate.
Sigma11	Vector of length N; contains the 1-1 element of the anisotropy process for each station.
Sigma22	Vector of length N; contains the 2-2 element of the anisotropy process for each station.
Sigma12	Vector of length N; contains the 1-2 element of the anisotropy process for each station.
nu	Scalar; Matern smoothness parameter. nu = 0.5 corresponds to the Exponential correlation; nu = Inf corresponds to the Gaussian correlation function.
log_sigma_vec	Vector of length N; log of the signal standard deviation.
lognuggetSD	Vector of length N; log of the error standard deviation.

**Value**

Returns a sparse matrix (N x 3) of the nonzero elements of the product between the core and sparse kernel.

---

determineNeighbors      *Determine the k-nearest neighbors for each spatial coordinate.*

---

**Description**

determineNeighbors returns an N x k matrix of the nearest neighbors for spatial locations coords, with the ith row giving indices of the k nearest neighbors to the ith location, which are selected from among the 1,...(i-1) other spatial locations. The first row is -1's, since the first location has no neighbors. The i=2 through i=(k+1) rows each necessarily contain 1:i.

**Usage**

determineNeighbors(coords, k)

**Arguments**

coords            N x 2 array of N 2-dimensional (x,y) spatial coordinates.  
 k                 Scalar; number of neighbors

**Value**

An N x k matrix of nearest neighbor indices

**Examples**

```
coords <- cbind(runif(100), runif(100))
determineNeighbors(coords, 20)
```

---

dmnorm\_gp2Scale            *Function for the evaluating the Gaussian likelihood with gp2Scale sparse covariance.*

---

**Description**

dmnorm\_gp2Scale (and rmnorm\_gp2Scale) calculate the usual Gaussian likelihood for a fixed set of parameters (but with sparse matrices). Finally, the distributions must be registered within nimble.

**Usage**

```
dmnorm_gp2Scale(x, mean, Cov, N, Nnz, log = 1)
```

**Arguments**

x                 Vector of measurements  
 mean             Vector of mean values  
 Cov              Matrix of size N x N; sparse kernel  
 N                 Number of measurements in x  
 Nnz              Number of measurements in x  
 log              Logical; should the density be evaluated on the log scale.

**Value**

Returns the Gaussian likelihood using the gp2Scale sparse covariance.

---

dmnorm\_nngp

*Function for the evaluating the NNGP approximate density.*


---

### Description

dmnorm\_nngp (and rmnorm\_nngp) calculate the approximate NNGP likelihood for a fixed set of parameters (i.e., A and D matrices). Finally, the distributions must be registered within nimble.

### Usage

```
dmnorm_nngp(x, mean, AD, nID, N, k, log)
```

### Arguments

x	N-vector of data.
mean	N-vector with current values of the mean
AD	N x (k+1) matrix; the first k columns are the 'A' matrix, and the last column is the 'D' vector.
nID	N x k matrix of neighbor indices.
N	Scalar; number of data measurements.
k	Scalar; number of nearest neighbors.
log	Scalar; should the density be on the log scale (1) or not (0).

### Value

The NNGP approximate density.

---

dmnorm\_sgv

*Function for the evaluating the SGV approximate density.*


---

### Description

dmnorm\_sgv (and rmnorm\_sgv) calculate the approximate SGV likelihood for a fixed set of parameters (i.e., the U matrix). Finally, the distributions must be registered within nimble.

### Usage

```
dmnorm_sgv(x, mean, U, N, k, log = 1)
```

**Arguments**

x	Vector of measurements
mean	Vector of mean values
U	Matrix of size $N \times 3$ ; representation of a sparse $N \times N$ Cholesky of the precision matrix. The first two columns contain row and column indices, respectively, and the last column is the nonzero elements of the matrix.
N	Number of measurements in x
k	Number of neighbors for the SGV approximation.
log	Logical; should the density be evaluated on the log scale.

**Value**

Returns the SGV approximation to the Gaussian likelihood.

---

inverseEigen	<i>Calculate covariance elements based on eigendecomposition components</i>
--------------	---

---

**Description**

inverseEigen calculates the inverse eigendecomposition – in other words, the covariance elements based on the eigenvalues and vectors. For a  $2 \times 2$  anisotropy (covariance) matrix, we parameterize the three unique values in terms of the two log eigenvalues and a rotation parameter on the rescaled logit. The function is coded as a nimbleFunction (see the nimble package) but can also be used as a regular R function.

**Usage**

```
inverseEigen(eigen_comp1, eigen_comp2, eigen_comp3, which_Sigma)
```

**Arguments**

eigen_comp1	N-vector; contains values of the log of the first anisotropy eigenvalue for a set of locations.
eigen_comp2	N-vector; contains values of the log of the second anisotropy eigenvalue for a set of locations.
eigen_comp3	N-vector; contains values of the rescaled logit of the anisotropy rotation for a set of locations.
which_Sigma	Scalar; one of (1, 2, 3), corresponding to which covariance component should be calculated (Sigma11, Sigma22, or Sigma12, respectively).

**Value**

A vector of anisotropy values (Sigma11, Sigma22, or Sigma12; depends on which\_Sigma) for the corresponding set of locations.

**Examples**

```
# Generate some eigendecomposition elements (all three are real-valued)
eigen_comp1 <- rnorm(10)
eigen_comp2 <- rnorm(10)
eigen_comp3 <- rnorm(10)
inverseEigen( eigen_comp1, eigen_comp2, eigen_comp3, 2) # Return the Sigma22 values
```

---

matern\_corr

*Calculate a stationary Matern correlation matrix*


---

**Description**

matern\_corr calculates a stationary Matern correlation matrix for a fixed set of locations, based on a range and smoothness parameter. This function is primarily used for the "npGP" and "approxGP" models. The function is coded as a nimbleFunction (see the nimble package) but can also be used as a regular R function.

**Usage**

```
matern_corr(dist, rho, nu)
```

**Arguments**

dist	N x N matrix; contains values of pairwise Euclidean distances in the x-y plane.
rho	Scalar; "range" parameter used to rescale distances
nu	Scalar; Matern smoothness parameter. nu = 0.5 corresponds to the Exponential correlation; nu = Inf corresponds to the Gaussian correlation function.

**Value**

A correlation matrix for a fixed set of stations and fixed parameter values.

**Examples**

```
# Generate some coordinates
coords <- cbind(runif(100),runif(100))
nu <- 2
# Calculate distances -- can use nsDist to calculate Euclidean distances
dist_list <- nsDist(coords, isotropic = TRUE)
# Calculate the correlation matrix
corMat <- matern_corr(sqrt(dist_list$dist1_sq), 1, nu)
```

nimble\_sparse\_chol     *nimble\_sparse\_chol*

---

**Description**

nimble\_sparse\_chol

**Usage**

nimble\_sparse\_chol(i, j, x, n)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
n	Length of the vector

---

nimble\_sparse\_cholesky  
                          *nimble\_sparse\_chol*

---

**Description**

nimble\_sparse\_chol

**Usage**

nimble\_sparse\_cholesky(i, j, x)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.

---

nimble\_sparse\_crossprod  
*nimble\_sparse\_crossprod*

---

**Description**

nimble\_sparse\_crossprod

**Usage**

nimble\_sparse\_crossprod(i, j, x, z, n, subset, transp)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
z	Vector to calculate the cross-product with.
n	Length of the vector
subset	Optional vector of rows to include in the calculation.
transp	Optional indicator of using the transpose

---

nimble\_sparse\_solve    *nimble\_sparse\_solve*

---

**Description**

nimble\_sparse\_solve

**Usage**

nimble\_sparse\_solve(i, j, x, z)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
z	Vector to calculate the cross-product with.

---

nimble\_sparse\_solveMat

*nimble\_sparse\_crossprod*

---

**Description**

nimble\_sparse\_crossprod

**Usage**

nimble\_sparse\_solveMat(i, j, x, z, transp)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
z	Vector to calculate the cross-product with.
transp	Optional indicator of using the transpose

---

nimble\_sparse\_tcrossprod

*nimble\_sparse\_tcrossprod*

---

**Description**

nimble\_sparse\_tcrossprod

**Usage**

nimble\_sparse\_tcrossprod(i, j, x, subset)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
subset	Optional vector of rows to include in the calculation.

nsCorr

*Calculate a nonstationary Matern correlation matrix***Description**

nsCorr calculates a nonstationary correlation matrix for a fixed set of locations, based on vectors of the unique anisotropy parameters for each station. Since the correlation function uses a spatially-varying Mahalanobis distance, this function requires coordinate-specific distance matrices (see below). The function is coded as a nimbleFunction (see the nimble package) but can also be used as a regular R function.

**Usage**

```
nsCorr(dist1_sq, dist2_sq, dist12, Sigma11, Sigma22, Sigma12, nu, d)
```

**Arguments**

dist1_sq	N x N matrix; contains values of pairwise squared distances in the x-coordinate.
dist2_sq	N x N matrix; contains values of pairwise squared distances in the y-coordinate.
dist12	N x N matrix; contains values of pairwise signed cross-distances between the x- and y-coordinates. The sign of each element is important; see nsDist function for the details of this calculation. in the x-coordinate.
Sigma11	Vector of length N; contains the 1-1 element of the anisotropy process for each station.
Sigma22	Vector of length N; contains the 2-2 element of the anisotropy process for each station.
Sigma12	Vector of length N; contains the 1-2 element of the anisotropy process for each station.
nu	Scalar; Matern smoothness parameter. $\nu = 0.5$ corresponds to the Exponential correlation; $\nu = \text{Inf}$ corresponds to the Gaussian correlation function.
d	Scalar; dimension of the spatial coordinates.

**Value**

A correlation matrix for a fixed set of stations and fixed parameter values.

**Examples**

```
# Generate some coordinates and parameters
coords <- cbind(runif(100),runif(100))
Sigma11 <- rep(1, 100) # Identity anisotropy process
Sigma22 <- rep(1, 100)
Sigma12 <- rep(0, 100)
nu <- 2
# Calculate distances
dist_list <- nsDist(coords)
```

```
# Calculate the correlation matrix
corMat <- nsCorr(dist_list$dist1_sq, dist_list$dist2_sq, dist_list$dist12,
                Sigma11, Sigma22, Sigma12, nu, ncol(coords))
```

---

nsCrosscorr

*Calculate a nonstationary Matern cross-correlation matrix*


---

## Description

nsCrosscorr calculates a nonstationary cross-correlation matrix between two fixed sets of locations (a prediction set with M locations, and the observed set with N locations), based on vectors of the unique anisotropy parameters for each station. Since the correlation function uses a spatially-varying Mahalanobis distance, this function requires coordinate-specific distance matrices (see below). The function is coded as a nimbleFunction (see the nimble package) but can also be used as a regular R function.

## Usage

```
nsCrosscorr(
  Xdist1_sq,
  Xdist2_sq,
  Xdist12,
  Sigma11,
  Sigma22,
  Sigma12,
  PSigma11,
  PSigma22,
  PSigma12,
  nu,
  d
)
```

## Arguments

Xdist1_sq	M x N matrix; contains values of pairwise squared cross-distances in the x-coordinate.
Xdist2_sq	M x N matrix; contains values of pairwise squared cross-distances in the y-coordinate.
Xdist12	M x N matrix; contains values of pairwise signed cross/cross- distances between the x- and y-coordinates. The sign of each element is important; see nsDist function for the details of this calculation. in the x-coordinate.
Sigma11	Vector of length N; contains the 1-1 element of the anisotropy process for each observed location.
Sigma22	Vector of length N; contains the 2-2 element of the anisotropy process for each observed location.

Sigma12	Vector of length N; contains the 1-2 element of the anisotropy process for each observed location.
PSigma11	Vector of length N; contains the 1-1 element of the anisotropy process for each prediction location.
PSigma22	Vector of length N; contains the 2-2 element of the anisotropy process for each prediction location.
PSigma12	Vector of length N; contains the 1-2 element of the anisotropy process for each prediction location.
nu	Scalar; Matern smoothness parameter. $\nu = 0.5$ corresponds to the Exponential correlation; $\nu = \text{Inf}$ corresponds to the Gaussian correlation function.
d	Scalar; dimension of the spatial domain.

**Value**

A  $M \times N$  cross-correlation matrix for two fixed sets of stations and fixed parameter values.

**Examples**

```
# Generate some coordinates and parameters
coords <- cbind(runif(100),runif(100))
Sigma11 <- rep(1, 100) # Identity anisotropy process
Sigma22 <- rep(1, 100)
Sigma12 <- rep(0, 100)
Pcoords <- cbind(runif(200),runif(200))
PSigma11 <- rep(1, 200) # Identity anisotropy process
PSigma22 <- rep(1, 200)
PSigma12 <- rep(0, 200)
nu <- 2
# Calculate distances
Xdist_list <- nsCrossdist(coords, Pcoords)
# Calculate the correlation matrix
XcorMat <- nsCrosscorr(Xdist_list$dist1_sq, Xdist_list$dist2_sq, Xdist_list$dist12,
  Sigma11, Sigma22, Sigma12, PSigma11, PSigma22, PSigma12, nu, ncol(coords))
```

---

nsCrossdist

*Calculate coordinate-specific cross-distance matrices*


---

**Description**

nsCrossdist calculates coordinate-specific cross distances in x, y, and x-y for use in the nonstationary cross-correlation calculation. This function is useful for calculating posterior predictions.

**Usage**

```
nsCrossdist(coords, Pcoords, scale_factor = NULL, isotropic = FALSE)
```

**Arguments**

coords	N x 2 matrix; contains x-y coordinates of station (observed) locations.
Pcoords	M x 2 matrix; contains x-y coordinates of prediction locations.
scale_factor	Scalar; optional argument for re-scaling the distances.
isotropic	Logical; indicates whether distances should be calculated using Euclidean distance ( <code>isotropic = TRUE</code> ) or using the anisotropic formulation ( <code>isotropic = FALSE</code> ).

**Value**

A list of distances matrices, with the following components:

dist1_sq	M x N matrix; contains values of pairwise squared cross- distances in the x-coordinate.
dist2_sq	M x N matrix; contains values of pairwise squared cross- distances in the y-coordinate.
dist12	M x N matrix; contains values of pairwise signed cross- distances between the x- and y-coordinates.
scale_factor	Value of the scale factor used to rescale distances.

**Examples**

```
# Generate some coordinates
coords <- cbind(runif(100),runif(100))
Pcoords <- cbind(runif(200),runif(200))
# Calculate distances
Xdist_list <- nsCrossdist(coords, Pcoords)
```

---

nsCrossdist3d	<i>Calculate coordinate-specific cross-distance matrices, only for nearest neighbors and store in an array</i>
---------------	--

---

**Description**

nsCrossdist3d generates and returns new 3-dimensional arrays containing the former dist1\_sq, dist2\_sq, and dist12 matrices, but only as needed for the k nearest-neighbors of each location. these 3D matrices (dist1\_3d, dist2\_3d, and dist12\_3d) are used in the new implementation of calculateAD\_ns().

**Usage**

```
nsCrossdist3d(
  coords,
  predCoords,
  P_nID,
  scale_factor = NULL,
  isotropic = FALSE
)
```

**Arguments**

coords	N x d matrix; contains the x-y coordinates of stations.
predCoords	M x d matrix
P_nID	N x k matrix; contains indices of nearest neighbors.
scale_factor	Scalar; optional argument for re-scaling the distances.
isotropic	Logical; indicates whether distances should be calculated separately for each coordinate dimension (FALSE) or simultaneously for all coordinate dimensions (TRUE). isotropic = FALSE can only be used for two-dimensional coordinate systems.

**Value**

Arrays with nearest neighbor distances in each coordinate direction. When the spatial dimension  $d > 2$ , `dist1_3d` contains squared Euclidean distances, and `dist2_3d` and `dist12_3d` are empty.

**Examples**

```
# Generate some coordinates and neighbors
coords <- cbind(runif(100),runif(100))
predCoords <- cbind(runif(200),runif(200))
P_nID <- FNN::get.knnx(coords, predCoords, k = 10)$nn.index # Prediction NN
# Calculate distances
Pdist <- nsCrossdist3d(coords, predCoords, P_nID)
```

---

nsDist

*Calculate coordinate-specific distance matrices*


---

**Description**

`nsDist` calculates x, y, and x-y distances for use in the nonstationary correlation calculation. The sign of the cross-distance is important. The function contains an optional argument for re-scaling the distances such that the coordinates lie in a square.

**Usage**

```
nsDist(coords, scale_factor = NULL, isotropic = FALSE)
```

**Arguments**

coords	N x 2 matrix; contains the x-y coordinates of stations
scale_factor	Scalar; optional argument for re-scaling the distances.
isotropic	Logical; indicates whether distances should be calculated separately for each coordinate dimension (FALSE) or simultaneously for all coordinate dimensions (TRUE). <code>isotropic = TRUE</code> can only be used for two-dimensional coordinate systems.

**Value**

A list of distances matrices, with the following components:

dist1_sq	N x N matrix; contains values of pairwise squared distances in the x-coordinate.
dist2_sq	N x N matrix; contains values of pairwise squared distances in the y-coordinate.
dist12	N x N matrix; contains values of pairwise signed cross- distances between the x- and y-coordinates.
scale_factor	Value of the scale factor used to rescale distances.

**Examples**

```
# Generate some coordinates
coords <- cbind(runif(100),runif(100))
# Calculate distances
dist_list <- nsDist(coords)
# Use nsDist to calculate Euclidean distances
dist_Euclidean <- sqrt(nsDist(coords, isotropic = TRUE)$dist1_sq)
```

---

nsDist3d	<i>Calculate coordinate-specific distance matrices, only for nearest neighbors and store in an array</i>
----------	--

---

**Description**

nsDist3d generates and returns new 3-dimensional arrays containing the former `dist1_sq`, `dist2_sq`, and `dist12` matrices, but only as needed for the `k` nearest-neighbors of each location. these 3D matrices (`dist1_3d`, `dist2_3d`, and `dist12_3d`) are used in the new implementation of `calculateAD_ns()`.

**Usage**

```
nsDist3d(coords, nID, scale_factor = NULL, isotropic = FALSE)
```

**Arguments**

coords	N x 2 matrix; contains the x-y coordinates of stations.
nID	N x k matrix; contains indices of nearest neighbors.
scale_factor	Scalar; optional argument for re-scaling the distances.
isotropic	Logical; indicates whether distances should be calculated separately for each coordinate dimension (FALSE) or simultaneously for all coordinate dimensions (TRUE). isotropic = TRUE can only be used for two-dimensional coordinate systems.

**Value**

Arrays with nearest neighbor distances in each coordinate direction.

**Examples**

```
# Generate some coordinates and neighbors
coords <- cbind(runif(100),runif(100))
nID <- determineNeighbors(coords, 10)
# Calculate distances
nsDist3d(coords, nID)
```

---

nsgpModel

---

*NIMBLE code for a generic nonstationary GP model*


---

**Description**

This function sets up and compiles a nimble model for a general nonstationary Gaussian process.

**Usage**

```
nsgpModel(
  tau_model = "constant",
  sigma_model = "constant",
  Sigma_model = "constant",
  sparse_model = "none",
  mu_model = "constant",
  likelihood = "fullGP",
  coords,
  data,
  constants = list(),
  monitorAllSampledNodes = TRUE,
  ...
)
```

**Arguments**

<code>tau_model</code>	Character; specifies the model to be used for the $\log(\tau)$ process. Options are "constant" (spatially-constant), "logLinReg" (log-linear regression), and "approxGP" (approximation to a Gaussian process).
<code>sigma_model</code>	Character; specifies the model to be used for the $\log(\sigma)$ process. See <code>tau_model</code> for options.
<code>Sigma_model</code>	Character; specifies the model to be used for the Sigma anisotropy process. Options are "constant" (spatially-constant), "constantIso" (spatially-constant and isotropic), "covReg" (covariance regression), "compReg" (componentwise regression), "compRegIso" (isotropic componentwise regression), "npApproxGP" (nonparameteric regression via an approximation to a stationary Gaussian process), and "npApproxGPiso" (isotropic nonparameteric regression via an approximation to a stationary Gaussian process)
<code>sparse_model</code>	Character; specifies the model to be used for the sparse kernel when using the gp2Scale likelihood. Defaults to "none".
<code>mu_model</code>	Character; specifies the model to be used for the $\mu$ mean process. Options are "constant" (spatially-constant), "linReg" (linear regression), and "zero" (a fixed zero-mean).
<code>likelihood</code>	Character; specifies the likelihood model. Options are "fullGP" (the exact Gaussian process likelihood), "NNGP" (the nearest-neighbor GP for the response approximate likelihood), and "SGV" (the sparse general Vecchia approximate likelihood).
<code>coords</code>	$N \times d$ matrix of spatial coordinates.
<code>data</code>	$N$ -vector; observed vector of the spatial process of interest
<code>constants</code>	A list of constants required to build the model; depends on the specific parameter process models chosen.
<code>monitorAllSampledNodes</code>	Logical; indicates whether all sampled nodes should be stored (TRUE) or not (FALSE).
<code>...</code>	Additional arguments can be passed to the function; for example, as an alternative to the constants list, items can be passed directly via this argument.

**Value**

A nimbleCode object.

**Examples**

```
# Generate some data: stationary/isotropic
N <- 100
coords <- matrix(runif(2*N), ncol = 2)
alpha_vec <- rep(log(sqrt(1)), N) # Log process SD
delta_vec <- rep(log(sqrt(0.05)), N) # Log nugget SD
Sigma11_vec <- rep(0.4, N) # Kernel matrix element 1,1
Sigma22_vec <- rep(0.4, N) # Kernel matrix element 2,2
Sigma12_vec <- rep(0, N) # Kernel matrix element 1,2
```

```

mu_vec <- rep(0, N) # Mean
nu <- 0.5 # Smoothness
dist_list <- nsDist(coords)
Cor_mat <- nsCorr( dist1_sq = dist_list$dist1_sq, dist2_sq = dist_list$dist2_sq,
                  dist12 = dist_list$dist12, Sigma11 = Sigma11_vec,
                  Sigma22 = Sigma22_vec, Sigma12 = Sigma12_vec, nu = nu )
Cov_mat <- diag(exp(alpha_vec)) %*% Cor_mat %*% diag(exp(alpha_vec))
D_mat <- diag(exp(delta_vec)^2)
set.seed(110)
data <- as.numeric(mu_vec + t(chol(Cov_mat + D_mat)) %*% rnorm(N))
# Set up constants
constants <- list( nu = 0.5, Sigma_HP1 = 2 )
# Defaults: tau_model = "constant", sigma_model = "constant", mu_model = "constant",
# and Sigma_model = "constant"
Rmodel <- nsgpModel(likelihood = "fullGP", constants = constants, coords = coords, data = data )

```

---

nsgpPredict

*Posterior prediction for the NSGP*


---

## Description

nsgpPredict conducts posterior prediction for MCMC samples generated using nimble and nsgpModel.

## Usage

```

nsgpPredict(
  model,
  samples,
  coords.predict,
  predict.process = TRUE,
  constants = list(),
  seed = 0,
  ...
)

```

## Arguments

model	A NSGP nimble object; the output of nsgpModel.
samples	A matrix of J rows, each is an MCMC sample of the parameters corresponding to the specification in nsgpModel.
coords.predict	M x d matrix of prediction coordinates.
predict.process	Logical; determines whether the prediction corresponds to the y(·) process (TRUE) or z(·) (FALSE; this would likely only be used for, e.g., cross-validation).
constants	An optional list of constants to use for prediction; alternatively, additional arguments can be passed to the function via the ... argument.

seed                    An optional random seed argument for reproducibility.

...                    Additional arguments can be passed to the function; for example, as an alternative to the constants list, items can be passed directly via this argument.

### Value

The output of the function is a list with two elements: `obs`, a matrix of  $J$  posterior predictive samples for the  $N$  observed locations (only for `likelihood = "SGV"`, which produces predictions for the observed locations by default; this element is `NULL` otherwise); and `pred`, a corresponding matrix of posterior predictive samples for the prediction locations. Ordering and neighbor selection for the prediction coordinates in the `SGV` likelihood are conducted internally, as with `nsgpModel`.

### Examples

```
## Not run:
# Generate some data: stationary/isotropic
N <- 100
coords <- matrix(runif(2*N), ncol = 2)
alpha_vec <- rep(log(sqrt(1)), N) # Log process SD
delta_vec <- rep(log(sqrt(0.05)), N) # Log nugget SD
Sigma11_vec <- rep(0.4, N) # Kernel matrix element 1,1
Sigma22_vec <- rep(0.4, N) # Kernel matrix element 2,2
Sigma12_vec <- rep(0, N) # Kernel matrix element 1,2
mu_vec <- rep(0, N) # Mean
nu <- 0.5 # Smoothness
dist_list <- nsDist(coords)
Cor_mat <- nsCorr( dist1_sq = dist_list$dist1_sq, dist2_sq = dist_list$dist2_sq,
                  dist12 = dist_list$dist12, Sigma11 = Sigma11_vec,
                  Sigma22 = Sigma22_vec, Sigma12 = Sigma12_vec, nu = nu )
Cov_mat <- diag(exp(alpha_vec)) %*% Cor_mat %*% diag(exp(alpha_vec))
D_mat <- diag(exp(delta_vec)^2)
set.seed(110)
data <- as.numeric(mu_vec + t(chol(Cov_mat + D_mat)) %*% rnorm(N))
# Set up constants
constants <- list( nu = 0.5, Sigma_HP1 = 2 )
# Defaults: tau_model = "constant", sigma_model = "constant", mu_model = "constant",
# and Sigma_model = "constant"
Rmodel <- nsgpModel(likelihood = "fullGP", constants = constants, coords = coords, data = data )
conf <- configureMCMC(Rmodel)
Rmcmc <- buildMCMC(conf)
Cmodel <- compileNimble(Rmodel)
Cmcmc <- compileNimble(Rmcmc, project = Rmodel)
samples <- runMCMC(Cmcmc, niter = 200, nburnin = 100)
# Prediction
predCoords <- as.matrix(expand.grid(seq(0,1,l=10),seq(0,1,l=10)))
postpred <- nsgpPredict( model = Rmodel, samples = samples, coords.predict = predCoords )

## End(Not run)
```

---

orderCoordinatesMMD     *Order coordinates according to a maximum-minimum distance criterion.*

---

### Description

orderCoordinatesMMD orders an array of (x,y) spatial coordinates according to the "maximum minimum distance" (MMD), as described in Guinness, 2018. (Points are selected to maximize their minimum distance to already- selected points).

### Usage

```
orderCoordinatesMMD(coords, exact = FALSE)
```

### Arguments

coords            N x 2 array of N 2-dimensional (x,y) spatial coordinates.  
exact             Logical; FALSE uses a fast approximation to MMD ordering (and is almost always recommended), while TRUE uses exact MMD ordering but is infeasible for large number of locations.

### Value

A list of distances matrices, with the following components:

orderedCoords    N x 2 matrix; contains the ordered spatial coordinates as coords.  
orderedIndicesNoNA     N-vector; contains the ordered indices with any NA values removed.

### Examples

```
coords <- cbind(runif(100), runif(100))
orderCoordinatesMMD(coords)
```

---

rmnorm\_gp2Scale            *Function for the evaluating the SGV approximate density.*

---

### Description

dmnorm\_gp2Scale (and rmnorm\_gp2Scale) calculate the usual Gaussian likelihood for a fixed set of parameters (but with sparse matrices). Finally, the distributions must be registered within nimble.

### Usage

```
rmnorm_gp2Scale(n, mean, Cov, N, Nnz)
```

**Arguments**

n	Number of realizations to generate
mean	Vector of mean values
Cov	Matrix of size $N \times N$ ; sparse kernel
N	Number of measurements in x
Nnz	Number of measurements in x

**Value**

Not applicable.

---

 rmnorm\_nngp

*Function for the evaluating the NNGP approximate density.*

---

**Description**

dmnorm\_nngp (and rmnorm\_nngp) calculate the approximate NNGP likelihood for a fixed set of parameters (i.e., A and D matrices). Finally, the distributions must be registered within nimble.

**Usage**

```
rmnorm_nngp(n, mean, AD, nID, N, k)
```

**Arguments**

n	N-vector of data.
mean	N-vector with current values of the mean
AD	$N \times (k+1)$ matrix; the first k columns are the 'A' matrix, and the last column is the 'D' vector.
nID	$N \times k$ matrix of neighbor indices.
N	Scalar; number of data measurements.
k	Scalar; number of nearest neighbors.

**Value**

The NNGP approximate density.

---

rmnorm_sgv	<i>Function for the evaluating the SGV approximate density.</i>
------------	---

---

**Description**

dmnorm\_sgv (and rmnorm\_sgv) calculate the approximate SGV likelihood for a fixed set of parameters (i.e., the U matrix). Finally, the distributions must be registered within nimble.

**Usage**

```
rmnorm_sgv(n, mean, U, N, k)
```

**Arguments**

n	Vector of measurements
mean	Vector of mean values
U	Matrix of size N x 3; representation of a sparse N x N Cholesky of the precision matrix. The first two columns contain row and column indices, respectively, and the last column is the nonzero elements of the matrix.
N	Number of measurements in x
k	Number of neighbors for the SGV approximation.

**Value**

Not applicable.

---

R_sparse_chol	<i>R_sparse_chol</i>
---------------	----------------------

---

**Description**

R\_sparse\_chol

**Usage**

```
R_sparse_chol(i, j, x, n)
```

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
n	Length of the vector

---

`R_sparse_cholesky`      *R\_sparse\_chol*

---

**Description**

`R_sparse_chol`

**Usage**

`R_sparse_cholesky(i, j, x)`

**Arguments**

<code>i</code>	Vector of row indices.
<code>j</code>	Vector of column indices.
<code>x</code>	Vector of values in the matrix.

---

`R_sparse_crossprod`      *nimble\_sparse\_crossprod*

---

**Description**

`nimble_sparse_crossprod`

**Usage**

`R_sparse_crossprod(i, j, x, z, n, subset = -1, transp = 1)`

**Arguments**

<code>i</code>	Vector of row indices.
<code>j</code>	Vector of column indices.
<code>x</code>	Vector of values in the matrix.
<code>z</code>	Vector to calculate the cross-product with.
<code>n</code>	Length of the vector
<code>subset</code>	Optional vector of rows to include in the calculation.
<code>transp</code>	Optional indicator of using the transpose

---

R\_sparse\_solve      *nimble\_sparse\_solve*

---

**Description**

nimble\_sparse\_solve

**Usage**

R\_sparse\_solve(i, j, x, z)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
z	Vector to calculate the cross-product with.

---

R\_sparse\_solveMat      *nimble\_sparse\_crossprod*

---

**Description**

nimble\_sparse\_crossprod

**Usage**

R\_sparse\_solveMat(i, j, x, z, transp = 1)

**Arguments**

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
z	Vector to calculate the cross-product with.
transp	Optional indicator of using the transpose

---

R\_sparse\_tcrossprod    *nimble\_sparse\_tcrossprod*

---

### Description

nimble\_sparse\_tcrossprod

### Usage

```
R_sparse_tcrossprod(i, j, x, subset = -1)
```

### Arguments

i	Vector of row indices.
j	Vector of column indices.
x	Vector of values in the matrix.
subset	Optional vector of rows to include in the calculation.

---

sgvSetup                      *One-time setup wrapper function for the SGV approximation*

---

### Description

sgvSetup is a wrapper function that sets up the SGV approximation. Three objects are required: (1) ordering the locations, (2) identify nearest neighbors, and (3) determine the conditioning set. This function only needs to be run once per SGV analysis.

### Usage

```
sgvSetup(
  coords,
  coords_pred = NULL,
  k = 15,
  seed = NULL,
  pred.seed = NULL,
  order_coords = TRUE,
  order_coords_pred = TRUE
)
```

**Arguments**

coords	Matrix of observed locations.
coords_pred	Optional matrix of prediction locations.
k	Number of neighbors.
seed	Setting the seed for reproducibility of the observed location ordering
pred.seed	Setting the seed for reproducibility of the prediction ordering.
order_coords	Logical; should the coordinates be ordered.
order_coords_pred	Logical; should the coordinates be ordered.

**Value**

A list with the following components:

ord	A vector of ordering position for the observed locations.
ord_pred	A vector of ordering position for the prediction locations (if coords_pred is provided).
ord_all	A concatenated vector of ord and ord_pred.
coords_ord	A matrix of ordered locations (observed and prediction), included for convenience.
nID_ord	A matrix of (ordered) neighbor indices.
condition_on_y_ord	A matrix indicating whether the conditioning set for each (ordered) location is on the latent process (y, 1) or the observed values (z, 0).

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