Package ‘BenfordTests’

May 6, 2026

Type Package

Title Statistical Tests for Evaluating Conformity to Benford's Law

Version 1.2.0

Date 2015-08-04

Depends R (>= 3.0.0), grDevices, graphics, stats

Maintainer Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

Description Several specialized statistical tests and support functions for determining if numerical data could conform to Benford's law.

License GPL-3

URL https://cran.r-project.org/package=BenfordTests, https://www.researchgate.net/profile/Dieter_Joenssen

Author Dieter William Joenssen [aut, cre, cph], Thomas Muellerleile [ctb]

NeedsCompilation yes

Repository CRAN

Date/Publication 2015-08-04 18:25:11

Contents

`BenfordTests-package` *Statistical Tests for Benford's Law*

Description

This package contains several specialized statistical tests and support functions for determining if numerical data could conform to Benford's law.

Details

Package:	BenfordTests
Type:	Package
Version:	1.2.0
Date:	2015-07-18
License:	GPL-3

`BenfordTests` is the implementation of eight goodness-of-fit (GOF) tests to assess if data conforms to Benford's law.
Tests include:
Pearson χ^2 statistic (Pearson, 1900)
Kolmogorov-Smirnov D statistic (Kolmogorov, 1933)
Freedman's modification of Watson's U^2 statistic (Freedman, 1981; Watson, 1961)
Chebyshev distance m statistic (Leemis, 2000)
Euclidean distance d statistic (Cho and Gaines, 2007)
Judge-Schechter mean deviation a^* statistic (Judge and Schechter, 2009)
Joenssen's J_P^2 statistic, a Shapiro-Francia type correlation test (Shapiro and Francia, 1972)
Joint Digit Test T^2 statistic, a Hotelling type test (Hotelling, 1931)

All tests may be performed using more than one leading digit. All tests simulate the specific p-values required for statistical inference, while p-values for the χ^2, D, a^*, and T^2 statistics may also be determined using their asymptotic distributions.

Author(s)

Dieter William Joenssen

Maintainer: Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Cho, W.K.T. and Gaines, B.J. (2007) Breaking the (Benford) Law: Statistical Fraud Detection in Campaign Finance. *The American Statistician*. **61**, 218–223.

Freedman, L.S. (1981) Watson's Un2 Statistic for a Discrete Distribution. *Biometrika*. **68**, 708–711.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

Judge, G. and Schechter, L. (2009) Detecting Problems in Survey Data using Benford's Law. *Journal of Human Resources*. **44**, 1–24.

Kolmogorov, A.N. (1933) Sulla determinazione empirica di una legge di distibuzione. *Giornale dell'Istituto Italiano degli Attuari*. **4**, 83–91.

Leemis, L.M., Schmeiser, B.W. and Evans, D.L. (2000) Survival Distributions Satisfying Benford's law. *The American Statistician*. **54**, 236–241.

Newcomb, S. (1881) Note on the Frequency of Use of the Different Digits in Natural Numbers. *American Journal of Mathematics*. **4**, 39–40.

Pearson, K. (1900) On the Criterion that a Given System of Deviations from the Probable in the Case of a Correlated System of Variables is Such that it can be Reasonably Supposed to have Arisen from Random Sampling. *Philosophical Magazine Series 5*. **50**, 157–175.

Shapiro, S.S. and Francia, R.S. (1972) An Approximate Analysis of Variance Test for Normality. *Journal of the American Statistical Association*. **67**, 215–216.

Watson, G.S. (1961) Goodness-of-Fit Tests on a Circle. *Biometrika*. **48**, 109–114.

Hotelling, H. (1931). The generalization of Student's ratio. *Annals of Mathematical Statistics*. **2**, 360–378.

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Look at sample
X
#Look at the first digits of the sample
signifd(X)

#Perform a Chi-squared Test on the sample's first digits using defaults
chisq.benftest(X)
#p-value = 0.648
```

`chisq.benftest` *Pearson's Chi-squared Goodness-of-Fit Test for Benford's Law*

Description

`chisq.benftest` takes any numerical vector reduces the sample to the specified number of significant digits and performs Pearson's chi-square goodness-of-fit test to assert if the data conforms to Benford's law.

Usage

```
chisq.benftest(x = NULL, digits = 1, pvalmethod = "asymptotic", pvalsims = 10000)
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`pvalmethod`	Method used for calculating the p-value. Either `"asymptotic"` or `"simulate"`.
`pvalsims`	An integer specifying the number of replicates to use if `pvalmethod` = "simulate".

Details

A χ^2 goodness-of-fit test is performed on `signifd(x,digits)` versus `pbenf(digits)`. Specifically:

$$\chi^2 = n \cdot \sum_{i=10^{k-1}}^{10^k-1} \frac{(f_i^o - f_i^e)^2}{f_i^e}$$

where f_i^o denotes the observed frequency of digits i, and f_i^e denotes the expected frequency of digits i. `x` is a numeric vector of arbitrary length. Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "`htest`" containing the following components:

`statistic`	the value of the χ^2 test statistic
`p.value`	the p-value for the test
`method`	a character string indicating the type of test performed
`data.name`	a character string giving the name of the data

Author(s)

Dieter William Joenssen <`Dieter.Joenssen@googlemail.com`>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

Pearson, K. (1900) On the Criterion that a Given System of Deviations from the Probable in the Case of a Correlated System of Variables is Such that it can be Reasonably Supposed to have Arisen from Random Sampling. *Philosophical Magazine Series 5*. **50**, 157–175.

See Also

pbenf, simulateH0

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform a Chi-squared Test on the sample's
#first digits using defaults but determine
#the p-value by simulation
chisq.benftest(X,pvalmethod ="simulate")
#p-value = 0.6401
```

---

edist.benftest — *Euclidean Distance Test for Benford's Law*

Description

`edist.benftest` takes any numerical vector reduces the sample to the specified number of significant digits and performs a goodness-of-fit test based on the Euclidean distance between the first digits' distribution and Benford's distribution to assert if the data conforms to Benford's law.

Usage

```
edist.benftest(x = NULL, digits = 1, pvalmethod = "simulate", pvalsims = 10000)
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`pvalmethod`	Method used for calculating the p-value. Currently only `"simulate"` is available.
`pvalsims`	An integer specifying the number of replicates used if `pvalmethod = "simulate"`.

Details

A statistical test is performed utilizing the Euclidean distance between `signifd(x,digits)` and `pbenf(digits)`. Specifically:

$$d = \sqrt{n} \cdot \sqrt{\sum_{i=10^{k-1}}^{10^k-1} (f_i^o - f_i^e)^2}$$

where f_i^o denotes the observed frequency of digits i, and f_i^e denotes the expected frequency of digits i. `x` is a numeric vector of arbitrary length. Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "`htest`" containing the following components:

`statistic`	the value of the Euclidean distance test statistic
`p.value`	the p-value for the test
`method`	a character string indicating the type of test performed
`data.name`	a character string giving the name of the data

Author(s)

Dieter William Joenssen <`Dieter.Joenssen@googlemail.com`>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Cho, W.K.T. and Gaines, B.J. (2007) Breaking the (Benford) Law: Statistical Fraud Detection in Campaign Finance. *The American Statistician*. **61**, 218–223.

Morrow, J. (2010) *Benford's Law, Families of Distributions and a Test Basis*. [available under http://www.johnmorrow.info/projects/benford/benfordMain.pdf]

See Also

`pbenf`, `simulateH0`

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform a Euclidean Distance Test on the
#sample's first digits using defaults
edist.benftest(X,pvalmethod ="simulate")
#p-value = 0.6085
```

jointdigit.benftest — *A Hotelling* T-square *Type Test for Benford's Law*

Description

`jointdigit.benftest` takes any numerical vector reduces the sample to the specified number of significant digits and performs a Hotelling T-square type goodness-of-fit test to assert if the data conforms to Benford's law.

Usage

```
jointdigit.benftest(x = NULL, digits = 1, eigenvalues="all", tol = 1e-15,
pvalmethod = "asymptotic", pvalsims = 10000)
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`eigenvalues`	How are the eigenvalues, which are used in testing, selected.
`tol`	Tolerance in detecting values that are essentially zero.
`pvalmethod`	Method used for calculating the p-value. Currently only `"asymptotic"` is available.
`pvalsims`	An integer specifying the number of replicates used if `pvalmethod = "simulate"`.

Details

A Hotelling T^2 type goodness-of-fit test is performed on `signifd(x,digits)` versus `pbenf(digits)`. `x` is a numeric vector of arbitrary length. **argument:** `eigenvalues` can be defined as:

- *numeric*, a vector containing which eigenvalues should be used
- *string length = 1*, eigenvalue selection scheme:
  - *"all"*, use all non-zero eigenvalues
  - *"kaiser"*, use all eigenvalues larger than the mean of all non-zero eigenvalues

Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "`htest`" containing the following components:

`statistic`	the value of the T^2 test statistic
`p.value`	the p-value for the test
`method`	a character string indicating the type of test performed

`data.name`	a character string giving the name of the data
`eigenvalues_tested`	a vector containing the index numbers of the eigenvalues used in testing.
`eigen_val_vect`	the eigen values and vectors of the null distribution. computed using `eigen`.

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Hotelling, H. (1931). The generalization of Student's ratio. *Annals of Mathematical Statistics*. **2**, 360–378.

See Also

pbenf

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform  Test
#on the sample's first digits using defaults
jointdigit.benftest(X)
#p-value = 0.648
#Perform  Test
#using only the two largest eigenvalues
jointdigit.benftest(x=X,eigenvalues=1:2)
#p-value = 0.5176
#Perform  Test
#using the kaiser selection criterion
jointdigit.benftest(x=X,eigenvalues="kaiser")
#p-value = 0.682
```

---

jpsq.benftest — *Joenssen's* JP-square *Test for Benford's Law*

Description

`jpsq.benftest` takes any numerical vector reduces the sample to the specified number of significant digits and performs a goodness-of-fit test based on the correlation between the first digits' distribution and Benford's distribution to assert if the data conforms to Benford's law.

Usage

```
jpsq.benftest(x = NULL, digits = 1, pvalmethod = "simulate", pvalsims = 10000)
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`pvalmethod`	Method used for calculating the p-value. Currently only `"simulate"` is available.
`pvalsims`	An integer specifying the number of replicates used if `pvalmethod = "simulate"`.

Details

A statistical test is performed utilizing the sign-preserved squared correlation between `signifd(x,digits)` and `pbenf(digits)`. Specifically:

$$J_P^2 = sgn\left(cor\left(f^o, f^e\right)\right) \cdot cor\left(f^o, f^e\right)^2$$

where f^o denotes the observed frequencies and f^e denotes the expected frequency of digits $10^{k-1}, 10^{k-1}+1, \ldots, 10^k - 1$. `x` is a numeric vector of arbitrary length. Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "`htest`" containing the following components:

`statistic`	the value of the J_P^2 test statistic
`p.value`	the p-value for the test
`method`	a character string indicating the type of test performed
`data.name`	a character string giving the name of the data

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Joenssen, D.W. (2013) A New Test for Benford's Distribution. In: *Abstract-Proceedings of the 3rd Joint Statistical Meeting DAGStat, March 18-22, 2013*; Freiburg, Germany.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

Shapiro, S.S. and Francia, R.S. (1972) An Approximate Analysis of Variance Test for Normality. *Journal of the American Statistical Association*. **67**, 215–216.

See Also

pbenf, simulateH0

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform Joenssen's \emph{JP-square} Test
#on the sample's first digits using defaults
jpsq.benftest(X)
#p-value = 0.3241
```

---

ks.benftest *Kolmogorov-Smirnov Test for Benford's Law*

Description

`ks.benftest` takes any numerical vector reduces the sample to the specified number of significant digits and performs the Kolmogorov-Smirnov goodness-of-fit test to assert if the data conforms to Benford's law.

Usage

```
ks.benftest(x = NULL, digits = 1, pvalmethod = "simulate", pvalsims = 10000)
```

Arguments

x	A numeric vector.
digits	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
pvalmethod	Method used for calculating the p-value. Currently only "simulate" is available.
pvalsims	An integer specifying the number of replicates used if pvalmethod = "simulate".

Details

A Kolmogorov-Smirnov test is performed between `signifd(x,digits)` and `pbenf(digits)`. Specifically:

$$D = \sup_{i=10^{k-1},\dots,10^k-1} \left| \sum_{j=1}^{i} (f_j^o - f_j^e) \right| \cdot \sqrt{n}$$

where f_i^o denotes the observed frequency of digits i, and f_i^e denotes the expected frequency of digits i. `x` is a numeric vector of arbitrary length. Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "htest" containing the following components:

statistic	the value of the Kolmogorov-Smirnov *D* test statistic
p.value	the p-value for the test
method	a character string indicating the type of test performed
data.name	a character string giving the name of the data

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

Kolmogorov, A.N. (1933) Sulla determinazione empirica di una legge di distibuzione. *Giornale dell'Istituto Italiano degli Attuari*. **4**, 83–91.

See Also

pbenf, simulateH0

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform a Kolmogorov-Smirnov Test on the
#sample's first digits using defaults
ks.benftest(X)
#0.7483
```

---

mdist.benftest — *Chebyshev Distance Test (maximum norm) for Benford's Law*

Description

mdist.benftest takes any numerical vector reduces the sample to the specified number of significant digits and performs a goodness-of-fit test based on the Chebyshev distance between the first digits' distribution and Benford's distribution to assert if the data conforms to Benford's law.

Usage

```
mdist.benftest(x = NULL, digits = 1, pvalmethod = "simulate", pvalsims = 10000)
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`pvalmethod`	Method used for calculating the p-value. Currently only `"simulate"` is available.
`pvalsims`	An integer specifying the number of replicates used if `pvalmethod = "simulate"`.

Details

A statistical test is performed utilizing the Chebyshev distance between `signifd(x,digits)` and `pbenf(digits)`. Specifically:

$$m = \max_{i=10^{k-1},\ldots,10^k-1} |f_i^o - f_i^e| \cdot \sqrt{n}$$

where f_i^o denotes the observed frequency of digits i, and f_i^e denotes the expected frequency of digits i. `x` is a numeric vector of arbitrary length. Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "`htest`" containing the following components:

`statistic`	the value of the Chebyshev distance (maximum norm) test statistic
`p.value`	the p-value for the test
`method`	a character string indicating the type of test performed
`data.name`	a character string giving the name of the data

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Leemis, L.M., Schmeiser, B.W. and Evans, D.L. (2000) Survival Distributions Satisfying Benford's law. *The American Statistician*. **54**, 236–241.

Morrow, J. (2010) *Benford's Law, Families of Distributions and a Test Basis*. [available under http://www.johnmorrow.info/projects/benford/benfordMain.pdf]

See Also

pbenf, simulateH0

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform a Chebyshev Distance Test on the
#sample's first digits using defaults
mdist.benftest(X)
#p-value = 0.6421
```

---

meandigit.benftest *Judge-Schechter Mean Deviation Test for Benford's Law*

Description

meandigit.benftest takes any numerical vector reduces the sample to the specified number of significant digits and performs a goodness-of-fit test based on the deviation in means of the first digits' distribution and Benford's distribution to assert if the data conforms to Benford's law.

Usage

```
meandigit.benftest(x = NULL, digits = 1, pvalmethod = "asymptotic", pvalsims = 10000)
```

Arguments

x	A numeric vector.
digits	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
pvalmethod	Method used for calculating the p-value. Either "asymptotic" or "simulate".
pvalsims	An integer specifying the number of replicates used if pvalmethod = "simulate".

Details

A statistical test is performed utilizing the deviation between the mean digit of signifd(x,digits) and pbenf(digits). Specifically:

$$a^* = \frac{|\mu_k^o - \mu_k^e|}{(9 \cdot 10^{k-1}) - \mu_k^e}$$

where μ_k^o is the observed mean of the chosen k number of digits, and μ_k^e is the expected/true mean value for Benford's predictions. a^* conforms asymptotically to a truncated normal distribution under the null-hypothesis, i.e.,

$$a^* \sim truncnorm\left(\mu = 0, \sigma = \sigma_B, a = 0, b = \infty\right)$$

x is a numeric vector of arbitrary length. Values of x should be continuous, as dictated by theory, but may also be integers. digits should be chosen so that signifd(x,digits) is not influenced by previous rounding.

Value

A list with class "htest" containing the following components:

statistic	the value of the a^* test statistic
p.value	the p-value for the test
method	a character string indicating the type of test performed
data.name	a character string giving the name of the data

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Judge, G. and Schechter, L. (2009) Detecting Problems in Survey Data using Benford's Law. *Journal of Human Resources*. **44**, 1–24.

See Also

pbenf, simulateH0

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform a Judge-Schechter Mean Deviation Test
#on the sample's first digits using defaults
meandigit.benftest(X)
#p-value = 0.1458
```

---

pbenf *Probability Mass Function for Benford's Distribution*

Description

Returns the complete probability mass function for Benford's distribution for a given number of first digits.

Usage

```
pbenf(digits = 1)
```

Arguments

`digits`	An integer determining the number of first digits for which the pdf is returned, i.e. 1 for 1:9, 2 for 10:99 etc.

Details

Benford's distribution has the following probability mass function:

$$P(d_k) = log_{10}\left(1 + d_k^{-1}\right)$$

where $d_k \in \left(10^{k-1}, 10^{k-1}+1, \ldots, 10^k - 1\right)$ for any chosen k number of digits.

Value

Returns an object of class "`table`" containing the expected density of Benford's distribution for the given number of digits.

Author(s)

Dieter William Joenssen <`Dieter.Joenssen@googlemail.com`>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under `http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf`]

See Also

`qbenf`; `rbenf`

Examples

```
#show Benford's predictions for the frequencies of the first digit values
pbenf(1)
```

`qbenf` *Quantile Function for Benford's Distribution*

Description

Returns the complete quantile function for Benford's distribution with a given number of first digits.

Usage

```
qbenf(digits = 1)
```

Arguments

`digits`	An integer determining the number of first digits for which the qdf is returned, i.e. 1 for 1:9, 2 for 10:99 etc.

Value

Returns an object of class `"table"` containing the expected quantile function of Benford's distribution with a given number of digits.

Author(s)

Dieter William Joenssen <`Dieter.Joenssen@googlemail.com`>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

See Also

`pbenf`; `rbenf`

Examples

```
qbenf(1)

qbenf(1)==cumsum(pbenf(1))
```

rbenf *Random Sample Satisfying Benford's Law*

Description

Returns a random sample with length n satisfying Benford's law.

Usage

```
rbenf(n)
```

Arguments

n Number of observations.

Details

This distribution has the density:

$$f(x) = \frac{1}{x \cdot ln(10)} \forall x \in [1, 10]$$

Value

Returns a random sample with length n satisfying Benford's law.

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

See Also

qbenf; pbenf

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Look at sample
X
#should be
# [1] 6.159420 1.396476 5.193371 2.064033 7.001284 5.006184
#7.950332 4.822725 3.386809 1.619609 2.080063 2.242473 1.944697 5.460581
#[15] 6.443031 2.662821 2.079283 3.703353 1.364175 3.354136
```

signifd *First Digits Function*

Description

Applies the first digits function to each element of a given vector.

Usage

```
signifd(x = NULL, digits = 1)
```

Arguments

x	A numeric vector.
digits	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.

Details

The first digits function can be written as:

$$D_k(x) = \lfloor |x| \cdot 10^{(-1 \cdot \lfloor log_{10}|x| \rfloor + k - 1)} \rfloor$$

with k being the number of first digits that should be extracted. x is a numeric vector of arbitrary length. Unlike other solutions, this function will work reliably with all real numbers.

Value

Returns a vector of integers the same length as the input vector x.

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

See Also

chisq.benftest; ks.benftest; usq.benftest; mdist.benftest; edist.benftest; meandigit.benftest; jpsq.benftest

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Look at the first digits of the sample
signifd(X)
#should be:
#[1] 6 1 5 2 7 5 7 4 3 1 2 2 1 5 6 2 2 3 1 3
```

`signifd.analysis` *Graphical Analysis of First Significant Digits*

Description

`signifd.analysis` takes any numerical vector reduces the sample to the specified number of significant digits. The (relative) frequencies are then plotted so that a subjective analysis may be performed.

Usage

```
signifd.analysis(x = NULL, digits = 1, graphical_analysis = TRUE, freq = FALSE,
alphas = 20, tick_col = "red", ci_col = "darkgreen", ci_lines = c(.05))
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`graphical_analysis`	Boolean value indicating if results should be plotted.
`freq`	Boolean value indicating if absolute frequencies should be used.
`alphas`	Either a vector containing the significance levels([0,1]) that will be shaded, or an integer defining the number of evenly spaced confidence intervals.
`tick_col`	Color code or name that will be passed to "`points`" for plotting.
`ci_col`	Color code or name that will be passed to "`polygon`" for shading the different confidence intervals. May be more than one color.
`ci_lines`	Boolean or fractional value(s) indicating significance levels where lines are drawn

Details

Confidence intervals are calculated from the normal distribution with $\mu_i = np_i$ and $\sigma^2 = np_i(1 - p_i)$, where i represents the considered digit. Be aware that the normal approximation only holds for "large" n.

Value

A list containing the following components:

summary	the summary printed below the graph, a matrix of digits, their (relative) frequencies and individual p-values
CIs	confidence intervals used for plotting as defined by parameter "ci_lines" or "alphas" if ci_lines==FALSE

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Freedman, L.S. (1981) Watson's Un2 Statistic for a Discrete Distribution. *Biometrika*. **68**, 708–711.

See Also

pbenf

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Analyze the first digits using the the defaults
signifd.analysis(X)
#Turn off plot
signifd.analysis(X,graphical_analysis=FALSE)
#Use absolute frequencies
signifd.analysis(X,graphical_analysis=FALSE,freq=TRUE)
#Use five evenly spaced confidence intervals, no lines
#alphas is used for shadeing
signifd.analysis(X,graphical_analysis=TRUE,alphas=5,freq=TRUE,ci_lines=FALSE)
#Use fifty evenly spaced, gray confidence intervals, blue ticks, and lines at
#the 1 and 5 percent confidence intervals
signifd.analysis(X,graphical_analysis=TRUE,alphas=50,freq=TRUE,tick_col="blue",
ci_col="gray",ci_lines=c(.01,.05))
```

signifd.seq — *Sequence of Possible Leading Digits*

Description

Returns a vector containing all possible significant digits for a given number of places.

Usage

```
signifd.seq(digits = 1)
```

Arguments

digits	An integer determining the number of first digits to be returned, i.e. 1 for 1:9, 2 for 10:99 etc.

Value

Returns an integer vector.

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

Examples

```
signifd.seq(1)
seq(from=1,to=9)==signifd.seq(1)

signifd.seq(2)
seq(from=10,to=99)==signifd.seq(2)
```

simulateH0 — *Function for Simulating the H0-Distributions needed for BenfordTests*

Description

simulateH0 is a wrapper function that calculates the specified test statistic under the null hypothesis a certain number of times.

Usage

```
simulateH0(teststatistic="chisq", n=10, digits=1, pvalsims=10)
```

Arguments

teststatistic	Which test statistic should be used: "chisq", "edist", "jpsq", "ks", "mdist", "meandigit", or "usq".
n	Sample size of interest.
digits	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
pvalsims	An integer specifying the number of replicates to be used in simulation.

Details

Wrapper function that directly outputs the distributions of the specified test statistic under the null hypothesis.

Value

A vector of length equal to "pvalsims".

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

See Also

pbenf, chisq.benftest, edist.benftest, jpsq.benftest, ks.benftest, mdist.benftest, \
meandigit.benftest, usq.benftest

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)

#calculate critical value for chisquare test via simulation
quantile(simulateH0(teststatistic="chisq", n=100,digits=1,pvalsims=100000),probs=.95)

#calculate the "real" critical value
qchisq(.95,df=8)

#alternatively look at critical values for the jpsq statistic
#for different sample sizes (notice the low value for pvalsims)
set.seed(421)
apply(sapply((1:9)*10,FUN=simulateH0,teststatistic="jpsq", digits=1, pvalsims=100),
MARGIN=2,FUN=quantile,probs=.05)
```

`usq.benftest` *Freedman-Watson U-square Test for Benford's Law*

Description

`usq.benftest` takes any numerical vector reduces the sample to the specified number of significant digits and performs the Freedman-Watson test for discreet distributions between the first digits' distribution and Benford's distribution to assert if the data conforms to Benford's law.

Usage

```
usq.benftest(x = NULL, digits = 1, pvalmethod = "simulate", pvalsims = 10000)
```

Arguments

`x`	A numeric vector.
`digits`	An integer determining the number of first digits to use for testing, i.e. 1 for only the first, 2 for the first two etc.
`pvalmethod`	Method used for calculating the p-value. Currently only `"simulate"` is available.
`pvalsims`	An integer specifying the number of replicates used if `pvalmethod = "simulate"`.

Details

A Freedman-Watson test for discreet distributions is performed between `signifd(x,digits)` and `pbenf(digits)`. Specifically:

$$U^2 = \frac{n}{9 \cdot 10^{k-1}} \cdot \left[\sum_{i=10^{k-1}}^{10^k-2} \left(\sum_{j=1}^{i} (f_j^o - f_j^e) \right)^2 - \frac{1}{9 \cdot 10^{k-1}} \cdot \left(\sum_{i=10^{k-1}}^{10^k-2} \sum_{j=1}^{i} (f_i^o - f_i^e) \right)^2 \right]$$

where f_i^o denotes the observed frequency of digits i, and f_i^e denotes the expected frequency of digits i. `x` is a numeric vector of arbitrary length. Values of `x` should be continuous, as dictated by theory, but may also be integers. `digits` should be chosen so that `signifd(x,digits)` is not influenced by previous rounding.

Value

A list with class "`htest`" containing the following components:

`statistic`	the value of the U^2 test statistic
`p.value`	the p-value for the test
`method`	a character string indicating the type of test performed
`data.name`	a character string giving the name of the data

Author(s)

Dieter William Joenssen <Dieter.Joenssen@googlemail.com>

References

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*. **78**, 551–572.

Freedman, L.S. (1981) Watson's Un2 Statistic for a Discrete Distribution. *Biometrika*. **68**, 708–711.

Joenssen, D.W. (2013) Two Digit Testing for Benford's Law. *Proceedings of the ISI World Statistics Congress, 59th Session in Hong Kong*. [available under http://www.statistics.gov.hk/wsc/CPS021-P2-S.pdf]

Watson, G.S. (1961) Goodness-of-Fit Tests on a Circle. *Biometrika*. **48**, 109–114.

See Also

pbenf, simulateH0

Examples

```
#Set the random seed to an arbitrary number
set.seed(421)
#Create a sample satisfying Benford's law
X<-rbenf(n=20)
#Perform Freedman-Watson U-squared Test on
#the sample's first digits using defaults
usq.benftest(X)
#p-value = 0.4847
```

Index