

# Package ‘COINT’

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**Type** Package

**Title** Unit Root Tests with Structural Breaks and Fully-Modified Estimators

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**Description** Procedures include Phillips (1995) FMVAR <doi:10.2307/2171721>, Kitamura and Phillips (1997) FMGMM <doi:10.1016/S0304-4076(97)00004-3>, Park (1992) CCR <doi:10.2307/2951679>, and so on. Tests with 1 or 2 structural breaks include Gregory and Hansen (1996) <doi:10.1016/0304-4076(96)001685-7>, Zivot and Andrews (1992) <doi:10.2307/1391541>, and Kurozumi (2002) <doi:10.1016/S0304-4076(01)00106-3>.

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## Contents

bartlett . . . . .	2
Bartlett_uni . . . . .	3
bohman . . . . .	4
cauchy . . . . .	5

ccr	6
ccrQ	8
CZa	10
data-sets	11
dchlet	11
fm	12
fmgive	15
fmgmm	17
fmols	19
fmQ	21
fmvar	23
fmvar_forecast	25
fmvar_plag	26
GHansen	28
gw	30
kpss	31
kpss_1br	32
kpss_2br	34
Kurozumi_Bartlett	36
Kurozumi_QS	37
mdchlet	38
parzen	39
Parzen_uni	40
pp	41
qs	42
QS_uni	43
reisz	44
SPC_Bartlett	45
SPC_QS	46
sw	47
tukham	49
tukhan	50
Za	51
ZA_1br	52
ZA_2br	54

<b>Index</b>	<b>57</b>
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---

bartlett

*Bartlett Kernel for Consistent Estimate of Long-run Variance*

---

### Description

Compute the Bartlett kernel to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
bartlett(data,v)
```

**Arguments**

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmar", "fmgmm" and "fmols".
weights	The weights vector, used by function <code>fmgive</code>

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) Time Series Data Analysis and Theory. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
bartlett(e,v=15)
```

---

Bartlett\_uni

*Bartlett Kernel for Consistent Estimate of Long-run Variance*

---

**Description**

Compute the Bartlett kernel to obtain consistent estimate of long-run variance, univariate time series only.

**Usage**

```
Bartlett_uni(e,v)
```

**Arguments**

- e                    A univariate time series for computing consistent long-run variance, normally, regression residuals.
- v                    Number of lag terms used to compute the long-run variance.

**Value**

Return the consistent estimate of long-run variance, that PP and KPSS tests require. This procedure handles single time series only.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) Time Series Data Analysis and Theory. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
y=macro[, "INF"]
e=y-mean(y)
Bartlett_uni(e, v=15)
```

---

 bohman

---

*Bohman Kernel for Consistent Estimate of Long-run Variance*


---

**Description**

Computes the Bohman window to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
bohman(data, v)
```

**Arguments**

- data                Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
- v                    Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) Time Series Data Analysis and Theory. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
bohman(e,v=15)
```

---

cauchy

---

*Cauchy Kernel for Consistent Estimate of Long-run Variance*


---

**Description**

Compute the Cauchy window to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
cauchy(data,v)
```

**Arguments**

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

**Author(s)**

Ho Tsung-wu

**References**

Brillinger, David R. (1981) Time Series Data Analysis and Theory. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
cauchy(e, v=15)
```

---

 ccr

---

*Canonical Cointegrating Regression Estimator*


---

**Description**

Computes Park's (1992) Canonical Cointegrating Regression estimator for cointegrated regression models, using OLS for the first stage regression.

**Usage**

```
ccr(y, x, type=c("const", "trend", "season", "all"),
    v=15, ker_fun="parzen", aband=0, filter=0)
```

**Arguments**

y	The data of dependent variable in a regression.
x	The data of independent variables in a regression.
type	The deterministic parts in the regression. const = intercept; trend = linear trend; season = seasonal dummies, this option requires ts format; all = all three options, which requires ts format.
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.

## Details

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel  
 "bartlett"=Bartlett kernel  
 "dchlet"= Dirichlet kernel  
 "mdchlet"= Modified Dirichlet kernel  
 "tukham"=Tukey-Hamming kernel  
 "tukhan"=Tukey-Hanning kernel  
 "cauchy"=Cauchy kernel  
 "bohman"=Bohman kernel  
 "reisz"=Riesz,Bochner kernel  
 "gw"= Gauss-Weierstrass kernel  
 "qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for ccr procedures, technically different from those used in pp and kpss tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

## Value

coefTable	Coefficients table.
vcov	Variance-covariance matrix for the parameter estimates.
sigma	Standard error of the residuals.
rss	Residual sum of squares.
fit	The fitted values, or conditional mean, of the regression.
resid	Estimated residuals.

## Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

Park, J. Y. (1992) Canonical Cointegrating Regressions. *Econometrica*, 60, 119-144.

## Examples

```
data(macro)
y=macro[,1]
x=macro[,-1]
```

```

out=ccr(y,x,type=c("const","trend","season","all")[2],v=15,ker_fun="bartlett")
out$coefTable
out$vcov
tail(out$fit)
tail(out$resid)

```

---

ccrQ

*Canonical Cointegrating Regression with Time Polynomial*


---

### Description

Computes Park's (1992) canonical cointegrating regression estimator for cointegrated regressions with time polynomial, using OLS for the first stage regression.

### Usage

```

ccrQ(y,
     x,
     type=c("trend","all"),
     v=15,
     q=2,
     ker_fun="parzen",
     aband=0,
     filter=0)

```

### Arguments

y	The data of dependent variable in a regression.
x	The data of independent variables in a regression.
type	The deterministic parts in the regression. const = intercept; trend = linear trend; season = seasonal dummies, this option requires ts format; all = all three options, which requires ts format.
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
q	degree of time polynomial, default=2.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.

## Details

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel  
 "bartlett"=Bartlett kernel  
 "dchlet"= Dirichlet kernel  
 "mdchlet"= Modified Dirichlet kernel  
 "tukham"=Tukey-Hamming kernel  
 "tukhan"=Tukey-Hanning kernel  
 "cauchy"=Cauchy kernel  
 "bohman"=Bohman kernel  
 "reisz"=Riesz,Bochner kernel  
 "gw"= Gauss-Weierstrass kernel  
 "qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for ccrQ procedures, technically different from those used in pp and kpss tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

## Value

coefTable	Coefficients table.
vcov	Variance-covariance matrix for the parameter estimates.
sigma	Standard error of the residuals.
rss	Residual sum of squares.
fit	The fitted values, or conditional mean, of the regression.
resid	Estimated residuals.

## Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

Park, J. Y. (1992) Canonical Cointegrating Regressions. *Econometrica*, 60, 119-144.

## Examples

```
data(macro)
y=macro[,1]
x=macro[,-1]
```

```

out=ccrQ(y,x,q=3,type=c("trend","all")[1],v=15,ker_fun="parzen")
out$coefTable
out$vcov
tail(out$fit)
tail(out$resid)

```

CZa

*Phillips' (1987) Za and Zt test for cointegration***Description**

Test the null hypothesis of no cointegration between  $y$  and  $x$  using Phillips' (1987)  $Z_a$  and  $Z_t$  statistics and Phillips and Ouliaris (1990) limit theory.

**Usage**

```
CZa(y, x, p=1, v=15)
```

**Arguments**

$y$	The data of dependent variable in a regression.
$x$	The data of independent variables in a regression.
$p$	Order of the time polynomial in the cointegrating regression. Critical values are available for $p$ within [1,5]. $p = -1$ : No deterministic term in the cointegrating regression. $p = 0$ : For a constant term. $p = 1$ : For a constant term and trend. Default. $p > 2$ : For time polynomial.
$v$	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.

**Value**

$\alpha$	Estimate of the AR(1) coefficient.
$cza$	$Z_a$ statistic for non-cointegration. Reject the null hypothesis of no cointegration if the $Z$ statistic < critical value.
$cza_{cv}$	Critical values of $cza$ .
$czt$	$Z_t$ statistic for non-cointegration. Reject the null hypothesis of no cointegration if the $Z$ statistic < critical value.
$czt_{cv}$	Critical values of $czt$ .

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

- Phillips, P. C. B. (1987) Time Series Regression with a Unit Root. *Econometrica*, 55, 277-301.  
Phillips, P. C. B. and Ouliaris S. (1990) Asymptotic Properties of Residual Based Tests for Cointegration. *Econometrica*, 58, 165-193.

**Examples**

```
data(macro)
y=macro[,1]
x=macro[,-1]
CZa(y,x,p=1,v=10)
```

---

data-sets

*Macroeconomic Time Series Data Sets, 1967M1-2025M7*

---

**Description**

macro contains monthly observations from 1967M1 to 2025M7 for the unemployment rate (the dependent variable), IC (Initial Claims of unemployment insurance), inflation rate (seasonal growth rate of CPI), industrial growth (seasonal growth rate of the industrial production index).r

**Usage**

```
data(macro)
```

**Value**

macro is an object of class timeSeries.

---

dchlet

*Dirichlet Kernel for Consistent Estimate of Long-run Variance*

---

**Description**

Computes the Dirichlet window to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
dchlet(data,v)
```

**Arguments**

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fivar", "fmgmm" and "fmols".
weights	The weights vector, used by function <code>fmgive</code>

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) Time Series Data Analysis and Theory. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
dchlet(e,v=15)
```

---

 fm

---

*Fully-Modified OLS Estimator*


---

**Description**

Computes the Phillips-Hansen (1990) Fully-Modified estimator for cointegrated regressions, using OLS for the first stage regression.

**Usage**

```
fm(y,
  x,
  type=c("const", "trend", "season", "all"),
  v=15,
  ker_fun="parzen",
  aband=0,
  filter=0,
  sb_start=0.15)
```

**Arguments**

y	The data of dependent variable in a regression.
x	The data of independent variables in a regression.
type	The deterministic parts in the regression. const = intercept; trend = linear trend; season = seasonal dummies, this option requires ts format; all = all three options, which requires ts format.
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to use the automatic bandwidth selection. aband=1. To use. aband=0. Do not use. The default.
filter	Whether to activate an AR(1) filter to compute the spectrum at frequency zero. filter=1. To activate. filter=0. Do not activate. The default.
sb_start	The percentage specifies the beginning of sub-sample for stability test, and the end sample is ( $I-sb\_start$ ).

**Details**

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel  
 "bartlett"=Bartlett kernel  
 "dchlet"= Dirichlet kernel  
 "mdchlet"= Modified Dirichlet kernel  
 "tukham"=Tukey-Hamming kernel  
 "tukhan"=Tukey-Hanning kernel  
 "cauchy"=Cauchy kernel  
 "bohman"=Bohman kernel  
 "reisz"=Riesz,Bochner kernel  
 "gw"= Gauss-Weierstrass kernel  
 "qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for FM procedures, technically different from those used in pp and kpss tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

**Value**

coefTable	Coefficients table.
vcov	Variance-covariance matrix for the parameter estimates.
sigma	Standard error of the residuals.
rss	Residual sum of squares.
fit	The fitted values, or conditional mean, of the regression.
stests	3x1 vector containing Hansen's (1992) Lc, MeanF, and SupF (in this order) statistics for testing the null hypothesis that the cointegrating vector is stable over the sample period.
resid	Estimated residuals.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Andrews, D. W. K. (1991) Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica*, 59: 817-858.

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

Hansen, B. E. (1992) Tests for Parameter Instability in Regressions with I(1) Processes. *Journal of Business and Economic Statistics*, 10, 321-335.

Phillips, P. C. B. and Hansen B. E. (1990) Statistical Inference in Instrumental Variables Regression with I(1) Processes. *Review of Economic Studies*, 57, 99-125.

**Examples**

```
data(macro)
y=macro[,1]
x=macro[,-1]
out=fm(y,x,type=c("const","trend","season","all")[2],v=15,ker_fun="parzen")
out$coefTable
out$vcov
out$stests
tail(out$fit)
tail(out$resid)
```

---

fmgive	<i>Fully-Modified GIVE Estimator</i>
--------	--------------------------------------

---

**Description**

Computes the Kitamura-Phillips (1997) Fully-Modified GIVE estimator for univariate and multivariate cointegrated regression models.

**Usage**

```
fmgive(y,x,z,v,t,ker_fun="parzen",times=5)
```

**Arguments**

y	The data of dependent variable(s) in a regression.
x	The data of independent variables in a regression.
z	Instruments
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
t	number of terms to use in the computation of the WT matrix.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
times	Number of iteration to compute GIVE residuals, default =5.

**Details**

1. Like FMOLS, fmgmm allows both single equation and multivariate system of equations. The multivariate case is a system that many dependent variables to common Xs.
2. Available kernels. Technical details are referred to Brillinger (1981,P.55)

```
"parzen"=Parzen kernel
"bartlett"=Bartlett kernel
"dchlet"= Dirichlet kernel
"mdchlet"= Modified Dirichlet kernel
"tukham"=Tukey-Hamming kernel
"tukhan"=Tukey-Hanning kernel
"cauchy"=Cauchy kernel
"bohman"=Bohman kernel
"reisz"=Riesz,Bochner kernel
"gw"= Gauss-Weierstrass kernel
"qs"= Andrews (1991) Quadratic-Spectral
```

These kernels are written for FM procedures, technically different from those used in pp and kpss tests.

3. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When `aband` is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.
4. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

### Value

<code>beta</code>	Coefficient estimates.
<code>stderr</code>	Standard error of the residuals.
<code>tstat</code>	t-statistics of parameter estimates.
<code>vcov</code>	Variance-covariance matrix for the parameter estimates.
<code>lomega</code>	long-run variance-covariance matrix of residuals.
<code>s1</code>	The first statistic for testing validity of overidentifying restrictions.
<code>s2</code>	The second statistic for testing validity of overidentifying restrictions.
<code>pvalue</code>	The p-value for <code>s1+s2</code> .
<code>fit</code>	The fitted values, or conditional mean, of the regression.
<code>resid</code>	GMM residuals.

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Kitamura, Y. and P. C. B. Phillips (1997) Fully-Modified IV, GIVE and GMM Estimation with Possibly Nonstationary Regressors and Instruments. *Journal of Econometrics*, 80, 85-123.

### Examples

```
data(macro)

y=macro[-1,c(1)]
x=macro[-1,c(2,4)]
z=as.matrix(na.omit(exp(diff(macro)))) #IV

out=fmgive(y,x,z,v=15,t=5,ker_fun="parzen")
out$beta
out$vcov
out$stderr
out$tstat #t-ratio
tail(out$fit)
tail(out$resid)

y=macro[-1,c(1,3)]
```

```
x=macro[-1,c(2,4)]
z=as.matrix(na.omit(exp(diff(macro)))) #IV

out=fmgive(y,x,z,v=15,t=5,ker_fun="parzen")
out$beta
```

fmgmm

*Fully-Modified GMM Estimator***Description**

Computes the Kitamura-Phillips (1997) Fully-Modified GMM estimator for univariate and multivariate cointegrated regression models.

**Usage**

```
fmgmm(y,x,z,v,ker_fun="parzen",times=5)
```

**Arguments**

y	The data of dependent variable(s) in a regression.
x	The data of independent variables in a regression.
z	Instruments
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
times	Number of iteration to compute GMM residuals, default =5.

**Details**

1. Like FMOLS, fmgmm allows both single equation and multivariate system of equations. The multivariate case is a system that many dependent variables to common Xs.

2. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel

"bartlett"=Bartlett kernel

"dchlet"= Dirichlet kernel

"mdchlet"= Modified Dirichlet kernel

"tukham"=Tukey-Hamming kernel

"tukhan"=Tukey-Hanning kernel

"cauchy"=Cauchy kernel

"bohman"=Bohman kernel

"reisz"=Riesz,Bochner kernel

"gw"= Gauss-Weierstrass kernel

"qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for FM procedures, technically different from those used in pp and kpss tests.

3. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When `aband` is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

4. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

### Value

<code>beta</code>	Coefficient estimates.
<code>stderr</code>	Standard error of the residuals.
<code>tstat</code>	t-statistics of parameter estimates.
<code>vcov</code>	Variance-covariance matrix for the parameter estimates.
<code>lromega</code>	long-run variance-covariance matrix of residuals.
<code>s1</code>	The first statistic for testing validity of overidentifying restrictions.
<code>s2</code>	The second statistic for testing validity of overidentifying restrictions.
<code>pvalue</code>	The p-value for <code>s1+s2</code> .
<code>fit</code>	The fitted values, or conditional mean, of the regression.
<code>resid</code>	GMM residuals.

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Kitamura, Y. and P. C. B. Phillips (1997) Fully-Modified IV, GIVE and GMM Estimation with Possibly Nonstationary Regressors and Instruments. *Journal of Econometrics*, 80, 85-123.

### Examples

```
data(macro)

y=macro[-1,c(1)]
x=macro[-1,c(2,3)]
z=as.matrix(na.omit(exp(diff(macro)))) #IV

out=fmgmm(y,x,z,v=15,ker_fun="parzen")
out$beta
out$vcov
out$stderr
out$tstat #t-ratio
```

```
tail(out$fit)
tail(out$resid)
```

---

fmols

---

*Multivariate Fully-Modified OLS Estimator*


---

### Description

Phillips' (1995) Fully-Modified OLS estimator for single equation and multivariate cointegrated regression models.

### Usage

```
fmols(y,
      x,
      type=c("const", "trend", "season", "all"),
      v=15,
      ker_fun="parzen",
      aband=0,
      filter=1)
```

### Arguments

y	The data of dependent variable(s) in a regression.
x	The data of independent variables in a regression.
type	The deterministic parts in the regression. const = intercept; trend = linear trend; season = seasonal dummies, this option requires ts format; all = all three options, which requires ts format.
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. The default. filter=0. Do not use.

### Details

1. This fmols allows both single equation and multivariate system of equations. The multivariate case is a system that many dependent variables to common Xs. 2. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel

"bartlett"=Bartlett kernel

"dchlet"= Dirichlet kernel

"mdchlet"= Modified Dirichlet kernel

"tukham"=Tukey-Hamming kernel

"tukhan"=Tukey-Hanning kernel

"cauchy"=Cauchy kernel

"bohman"=Bohman kernel

"reisz"=Riesz,Bochner kernel

"gw"= Gauss-Weierstrass kernel

"qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for fmols procedures, technically different from those used in pp and kpss tests.

3. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

4. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

### Value

beta	Coefficient estimates.
stderr	Standard error of the residuals.
tstat	t-statistics of parameter estimates.
vcov	Variance-covariance matrix for the parameter estimates.
fit	The fitted values, or conditional mean, of the regression.
resid	Estimated residuals.

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Phillips, P. C. B (1995) Fully Modified Least Squares and Vector Autoregression, *Econometrica*, 63, 1023-1078.

**Examples**

```

data(macro)
y=macro[,1:2]
x=macro[,3:4]
out=fmols(y,x,type=c("const","trend","season","all")[2],v=15,ker_fun="bartlett")
out$beta
out$stderr
out$stat #t-ratio
out$vcov
tail(out$fit)
tail(out$resid)

```

fmQ

*Fully-Modified OLS Estimator with Time Polynomial***Description**

Computes the Phillips-Hansen (1990) Fully-Modified estimator for cointegrated regressions with Time Polynomial, using OLS for the first stage regression.

**Usage**

```

fmQ(y,x,type=c("trend","all"),v=15,
    q=2, ker_fun="parzen",aband=0,filter=0,sb_start=0.15)

```

**Arguments**

y	The data of dependent variable in a regression.
x	The data of independent variables in a regression.
type	The deterministic parts in the regression. trend = linear trend; all = all three options, which requires ts format.
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
q	degree of time polynomial, default=2.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.
sb_start	The percentage specifies the beginning of sub-sample for stability test, and the end sample is $(1-sb\_start)$ .

## Details

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel

"bartlett"=Bartlett kernel

"dchlet"= Dirichlet kernel

"mdchlet"= Modified Dirichlet kernel

"tukham"=Tukey-Hamming kernel

"tukhan"=Tukey-Hanning kernel

"cauchy"=Cauchy kernel

"bohman"=Bohman kernel

"reisz"=Riesz,Bochner kernel

"gw"= Gauss-Weierstrass kernel

"qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for fmQ procedures, technically different from those used in pp and kpss tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

## Value

coefTable	Coefficients table.
vcov	Variance-covariance matrix for the parameter estimates.
sigma	Standard error of the residuals.
rss	Residual sum of squares.
fit	The fitted values, or conditional mean, of the regression.
resid	Estimated residuals.

## Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

Andrews, D. W. K. (1991) Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica*, 59: 817-858.

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

Hansen, B. E. (1992) Tests for Parameter Instability in Regressions with I(1) Processes. *Journal of Business and Economic Statistics*, 10, 321-335.

Phillips, P. C. B. and Hansen B. E.(1990) Statistical Inference in Instrumental Variables Regression with I(1) Processes. *Review of Economic Studies*, 57, 99-125.

### Examples

```
data(macro)
y=macro[,1]
x=macro[,-1]
out=fmQ(y,x,type=c("trend","all")[1],v=15,q=3,ker_fun="parzen")
out$coefTable
out$vcov
out$stests
tail(out$fit)
tail(out$resid)
```

---

fmvar	<i>Fully-Modified VAR Estimator</i>
-------	-------------------------------------

---

### Description

Computes the Phillips' (1995) Fully-Modified" VAR estimator for cointegrated regressions, using OLS for the first stage regression.

### Usage

```
fmvar(data,p=1,q=5,v=15,type=c("const","trend","season","all"),
      ker_fun="parzen",aband=0,filter=0)
```

### Arguments

data	The dependent variables for a VAR system.
p	The number of lags for dependent variables, as in a VAR(p).
q	The number of lagged innovation terms to include in the fitted FMVAR(p,q).
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
type	The deterministic parts in the regression. Please note that fmvar will "de-" before inclusion, for example, trend is to pre-detrend the data. const = intercept; trend = linear trend; season = seasonal dummies, this option requires ts format; all = all three options, which requires ts format.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.

aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.

### Details

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel  
 "bartlett"=Bartlett kernel  
 "dchlet"= Dirichlet kernel  
 "mdchlet"= Modified Dirichlet kernel  
 "tukham"=Tukey-Hamming kernel  
 "tukhan"=Tukey-Hanning kernel  
 "cauchy"=Cauchy kernel  
 "bohman"=Bohman kernel  
 "reisz"=Riesz,Bochner kernel  
 "gw"= Gauss-Weierstrass kernel  
 "qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for FM procedures, technically different from those used in pp and kps tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

### Value

beta	Coefficient estimates.
stderr	Standard error of the residuals.
tstat	t-statistics of parameter estimates.
vcov	Variance-covariance matrix for the parameter estimates.
fit	The fitted values, or conditional mean, of the regression.
resid	Estimated residuals.
data	The data used in fmvar
type	The type used in fmvar
p	The p argument used in fmvar
q	The q argument used in fmvar

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Phillips, P. C. B (1995) Fully Modified Least Squares and Vector Autoregression. *Econometrica*, 63, 1023-1078.

**Examples**

```
data(macro)
out=fmvar(macro,p=1,q=6,v=15,type="trend",ker_fun="parzen",aband=0,filter=0)
out$beta
out$stderr
out$tstat
out$vcov
tail(out$data)
tail(out$resid)

ID1=grep(rownames(out$beta),pattern="_dL")
ID2=grep(rownames(out$beta),pattern="_L")
ID3=rownames(out$beta)[-c(ID1,ID2)]

out$beta[ID1,]; #innovation terms
out$beta[ID2,]; #VAR(1)
out$beta[ID3,]; #deterministic parts
```

---

fmvar\_forecast

*Forecast a FM-VAR System*


---

**Description**

Forecast a VAR generated by fmvar\_forecast.

**Usage**

```
fmvar_forecast(output, n.ahead=6)
```

**Arguments**

output	The output object of fmvar_forecast.
n.ahead	The steps of out-of-sample forecasting.

**Details**

This function recursively computes the n.ahead steps of out-of-sample forecasting.

**Value**

Forecasted values of all endogenous variables.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**Examples**

```
data(macro)
out=fmvar(macro,p=1,q=6,v=10,type="trend",ker_fun="parzen")
fmvar_forecast(out,n.ahead=6)
```

---

 fmvar\_plag

---

*Select the q in a FMVAR(p,q) by Specific Criterion*


---

**Description**

Select the q for a FMVAR(p,q) for cointegrated regressions.

**Usage**

```
fmvar_plag(data,
            p=1,
            lag.max=12,
            v=15,
            type=c("const","trend","season","all"),
            ker_fun="parzen",
            aband=0,
            filter=0)
```

**Arguments**

data	The dependent variables for a VAR system.
p	The number of lags for dependent variables, as in a standard VAR(p).
lag.max	The maxum number of lags used to search for optimal q in fmvar(p,q).
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
type	The deterministic parts in the regression. const = intercept; trend = linear trend; season = seasonal dummies, this option requires ts format; all = all three options, which requires ts format.

ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.

### Details

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel  
 "bartlett"=Bartlett kernel  
 "dchlet"= Dirichlet kernel  
 "mdchlet"= Modified Dirichlet kernel  
 "tukham"=Tukey-Hamming kernel  
 "tukhan"=Tukey-Hanning kernel  
 "cauchy"=Cauchy kernel  
 "bohman"=Bohman kernel  
 "reisz"=Riesz,Bochner kernel  
 "gw"= Gauss-Weierstrass kernel  
 "qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for fm.ols procedures, technically different from those used in pp and kpss tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

### Value

selection	The selected order of q: the selected lagged innovation terms to include in the fitted FMVAR(p,q)
criteria	The matrix of all lags and the values of four criteria: "AIC(q)", "HQ(q)", "SC(q)", "FPE(q)".

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

Phillips, P. C. B (1995) Fully Modified Least Squares and Vector Autoregression. *Econometrica*, 63, 1023-1078.

## Examples

```
data(macro)

Q=fmvar_plag(macro,
             p=1,
             v=15,
             lag.max=16,type="trend",
             ker_fun="parzen")$selection[1]

out=fmvar(macro,p=1,q=Q,v=15,type="trend", ker_fun="parzen")
```

---

GHansen

*Gregory-Hansen Test for Cointegration in Models with Regime Shifts*

---

## Description

Conduct the cointegration analysis with regime shifts, proposed by Gregory-Hansen (1996A).

## Usage

```
GHansen(y, x, model, trim=0.1, use=c("nw", "ba"))
```

## Arguments

<code>y</code>	The data of dependent variable in a regression.
<code>x</code>	The data of independent variables in a regression.
<code>model</code>	Models for the structural breaks: 1 = C (level shift); 2 = C/T (level shift with trend); 3 = C/S (regime shift); 4 = regime and trend shift.
<code>trim</code>	The trimming percentage. Default is 10 percent.
<code>use</code>	Conditions for pp test for the nonstationarity of regression residuals.

## Details

This function calculates three residual-based test for cointegration with regime shifts: ADF, and  $Z_a$ ,  $Z_t$  of pp. Argument use is detailed by the example of pp documentation.

**Value**

result	Comprehensive results of three tests.
teststat	Time series of three sequential tests.
test.reg.adf	Final regression output for ADF test.
test.reg.za	Final regression output for Za test.
test.reg.zt	Final regression output for Zt test.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Gregory, A.W. and Hansen, B. E. (1996A) Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics*, 70, 99-126.  
 Gregory, A.W. and Hansen, B. E. (1996B). Tests for Cointegration in Models with Regime and Trend Shifts. *Oxford Bulletin Economics and Statistics*, 58(3), 555-560.

**Examples**

```
data(macro)
y=macro[1:300,1]
x=macro[1:300,-1]
output=GHansen(y,x,model=1, use=c("nw","ba"))
output$result
summary(output$test.reg.adf)
head(output$teststat)

#Plotting

test.name=rownames(output$result)[1]
stat=output$teststat[,test.name]
CV=output$result[test.name,1:3]
bpoint=output$result[test.name,5]
main=paste(paste(unlist(strsplit(test.name,"_")),collapse = " "),"test")

plot(stat,main=main,ylab="",xlab="",ylim=range(c(max(stat)+3,min(stat)-1,CV)));grid()
  abline(h=CV[1],col="red")
  abline(h=CV[2],col="blue")
  abline(h=CV[3],col="seagreen")
  abline(v=as.POSIXct(time(y)[bpoint]),col="orange",lty=2)

# legend(x=as.POSIXct("2010-01-01"), y=max(stat)+3, legend=c("1% cv", "5% cv", "10% cv"),
#       col=c("red", "blue", "seagreen"),xjust=1, yjust=1, lty=1,
#       horiz=TRUE, cex=0.66, bty="n")

#plot(y,main=colnames(y),ylab="",xlab="");grid()
```

```
#abline(v=time(y[output$bpoint,]),col="orange",lty=2)
```

---

gw	<i>Gauss-Weierstrass Kernel for Consistent Estimate of Long-run Variance</i>
----	--

---

### Description

Computes the Gauss-Weierstrass window to obtain consistent estimate of long-run variance of multivariate time series.

### Usage

```
gw(data,v)
```

### Arguments

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

### Value

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

### Examples

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
gw(e,v=15)
```

---

kpss *KPSS Unit Root Test for the null of stationarity*

---

### Description

Implement the KPSS unit root test for the null of  $I(0)$  stationarity. The test type as deterministic component is specified as `x`, see example below.

### Usage

```
kpss(y, x, lags = c("short", "long", "nil"), use=c("nw", "ba"))
```

### Arguments

<code>y</code>	Vector to be tested for a unit root.
<code>x</code>	data matrix for deterministic component. For example a vector of one for intercept, or trend. The default is "x=NULL", which is the same of a vector of one
<code>lags</code>	Lags used for correction of error term.
<code>use</code>	User specified lags for correction of error term. See section details below. The default is the lag determined by Newey-West bandwidth ("nw") with Bartlett kernel ("ba").

### Details

`lags="short"` sets the number of lags to  $\sqrt[4]{4 \times (n/100)}$ , whereas `lags="long"` sets the number of lags to  $\sqrt[4]{12 \times (n/100)}$ . If `lags="nil"` is chosen, no error correction is made.

Furthermore, "lags" and "use" are mutually exclusive: As long as "use" is not NULL, its argument will be chosen first. One can specify a different number of maximum lags by setting "use" accordingly. Users can input number of your source. This version supports two bandwidth functions: "nw" for Newey-West and "and" for Andrews. The kernel functions are supported: "ba"=Bartlett, "pa"=Parzen, "qs"=Quadratic Spectral

### Value

<code>teststat</code>	The KPSS test statistic.
<code>cval</code>	Critical values.
<code>lag</code>	Number of lags used for kernel function.
<code>resid</code>	Regression residuals.

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

- Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992) Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root? *Journal of Econometrics*, 54,159-178.
- Phillips, P.C.B. and Sainan Jin (2002) The KPSS test with seasonal dummies. *Economics Letters*, 77, 239-243.

## Examples

```
data(macro)
y=macro[, "INF"]
const=rep(1,nrow(y))
trend=seq(nrow(y))/nrow(y)
D=cbind(const,trend) #seasonal dummies can be specified here
KPSS=kpss(y,x=D,lags = c("short", "long", "nil")[2],
          use=c("nw", "ba")) # If argument use isn't NULL, the argument "lags" will be ignored.
KPSS$teststat
KPSS$cval
KPSS$lag

kpss(y,x=D,lags = c("short", "long", "nil")[2],use=15)

kpss(y,x=D,
     lags = c("short", "long", "nil")[2],
     use=NULL) #if "use=NULL", argument "lags" will be chosen as input.
```

---

kpss\_1br

*KPSS Unit Root Test with One Structural Break*


---

## Description

Implement the Kurozumi (2002) sequential kpss test with one structural break.

## Usage

```
kpss_1br(y,
        lags = c("short", "long", "nil"),
        model=c("intercept", "both"),
        use=c("nw", "ba"),
        trim=0.1)
```

**Arguments**

y	Vector to be tested for a unit root.
lags	Lags used for correction of error term.
model	Modelling where the unknown structural change occurs. intercept= structural change occurs at the intercept. both= structural changes occur at both the intercept and the trend.
use	User specified lags for correction of error term. The default is the lag determined by Newey-West bandwidth "nw" with Bartlett "ba" kernel. Users can input your own number. This version supports two bandwidth functions: "nw" for Newey-West, "and" for Andrews. Three kernel functions are supported by both bandwidth functions: "ba"=Bartlett, "pa"=Parzen, "qs"=Quadratic Spectral
trim	The trimming percentage. Default is 10

**Details**

lags="short" sets the number of lags to  $\sqrt[4]{4 \times (n/100)}$ , whereas lags="long" sets the number of lags to  $\sqrt[4]{12 \times (n/100)}$ . If lags="nil" is chosen, then no error correction is made. Furthermore, lags and use are mutually exclusive. As long as use is not set to be NULL, its argument will be chosen first. One can specify a different number of maximum lags by setting use accordingly.

**Value**

teststat	The kpss test statistic with one structural break.
cval	Critical values.
bpoint	The breaking point that corresponds to the teststat.
tstats	The sequential KPSS test statistic.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

- Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992) Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root? *Journal of Econometrics*, 54, 159-178.
- Kurozumi, E. (2002) Testing for stationarity with a break. *Journal of Econometrics*, 108(1), 105-127.
- Phillips, P.C.B. and Sainan Jin (2002) The KPSS test with seasonal dummies. *Economics Letters*, 77, 239-243.

**Examples**

```

data(macro)
y=macro[, "INF"]
KPSS1=kpss_1br(y,model=c("intercept", "both")[2],use=c("nw", "ba"))
KPSS1$teststat
KPSS1$cval
y[KPSS1$bpoint,]
#Plot
plot.ts(KPSS1$tstats,ylim=range(c(KPSS1$tstats,KPSS1$cval)));grid()
abline(h=KPSS1$cval[1],col="red")
abline(h=KPSS1$cval[2],col="blue")
abline(h=KPSS1$cval[3],col="green")
abline(v=KPSS1$bpoint,col="red",lty=2)

```

kpss\_2br

*KPSS Unit Root Test with Two Structural Breaks***Description**

Implement the kpss unit root test with two unknown structural breaks. Carrion-i-Silvestre and Sanso (2007) extends Kurozumi (2002) to two breaks, and create critical values.

**Usage**

```
kpss_2br(y, lags = c("short", "long", "nil"), model=1, use=c("nw", "ba"), trace=TRUE)
```

**Arguments**

y	Vector to be tested for a unit root.
lags	Lags used for correction of error term.
model	Modelling where the unknown structural change occurs. 1 = AAn (Level shift without trend); 2 = AA (Level shift with trend); 3 = BB (Trend shift); 4 = CC (Level and trend shift) Default is 1.
use	User specified lags for correction of error term. The default is the lag determined by Newey-West bandwidth "nw" with Bartlett "ba" kernel. Users can input your own number. This version supports two bandwidth functions: "nw" for Newey-West, "and" for Andrews. Three kernel functions are supported by both bandwidth functions: "ba"=Bartlett, "pa"=Parzen, "qs"=Quadratic Spectral
trace	Logical. If TRUE (default), then screen displays the sequential progress.

**Details**

lags="short" sets the number of lags to  $\sqrt[4]{4 \times (n/100)}$ , whereas lags="long" sets the number of lags to  $\sqrt[4]{12 \times (n/100)}$ . If lags="nil" is chosen, then no error correction is made. Furthermore, lags and use are mutually exclusive. As long as use is not set to be NULL, its argument will be chosen first. One can specify a different number of maximum lags by setting use accordingly.

**Value**

teststat	The test statistic.
cval	The critical values that are tabulated in Carrion-i-Silvestre and Sanso, A. (2007).
bpoint1	The first breaking point that corresponds to the teststat.
bpoint2	The second breaking point that corresponds to the teststat.
timeElapse	Time elapsed for sequential search.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

- Carrion-i-Silvestre, J. L. and Sanso, A. (2006) A guide to the computation of stationarity tests. *Empirical Economics*, 31(2), 433-448.
- Carrion-i-Silvestre, J. L. and Sanso, A. (2007) The KPSS test with two structural breaks, *Spanish Economic Review*, 9(2), 105-127.
- Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992) Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root? *Journal of Econometrics*, 54, 159-178.
- Kurozumi, E. (2002) Testing for stationarity with a break. *Journal of Econometrics*, 108(1), 105-127.

**Examples**

```
data(macro)

y=macro[1:200,"INF"]

KPSS2=kpss_2br(y,model=1,use=c("nw","ba"))
KPSS2$teststat
KPSS2$cval
y[KPSS2$bpoint,]
```

---

`Kurozumi_Bartlett`*Bartlett Kernel for Consistent Estimate of Long-run Variance*

---

**Description**

Compute the Bartlett kernel proposed by Kurozumi (2002) to obtain consistent estimate of long-run variance.

**Usage**

```
Kurozumi_Bartlett(e)
```

**Arguments**

`e` data that needs to compute consistent long-run variance, normally, regression residuals

**Value**

Return the consistent estimate of long-run variance, Bartlett kernel proposed by Kurozumi (2002).

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Kurozumi, E. (2002) Testing for stationarity with a break. *Journal of Econometrics*, 108(1), 105-127.

**Examples**

```
data(macro)
y=macro[, "INF"]
e=y-mean(y)
Kurozumi_Bartlett(e)
```

---

Kurozumi_QS	<i>Quadratic Spectral Kernel for Consistent Estimate of Long-run Variance</i>
-------------	---

---

**Description**

Compute the Quadratic Spectral kernel proposed by Kurozumi (2002) to obtain consistent estimate of long-run variance.

**Usage**

```
Kurozumi_QS(e)
```

**Arguments**

e                    data that needs to compute consistent long-run variance, normally, regression residuals.

**Value**

Return the consistent estimate of long-run variance, Quadratic Spectral kernel proposed by Kurozumi (2002).

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Kurozumi, E. (2002) Testing for stationarity with a break. *Journal of Econometrics*, 108(1), 105-127.

**Examples**

```
data(macro)
y=macro[, "INF"]
e=y-mean(y)
Kurozumi_QS(e)
```

---

mdchlet	<i>Modified Dirichlet Kernel for Consistent Estimate of Long-run Variance</i>
---------	---

---

**Description**

Computes the modified Dirichlet window to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
mdchlet(data, v)
```

**Arguments**

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmvar", "fmgmm" and "fmols".
weights	The weights vector, used by function <code>fmgive</code>

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
mdchlet(e, v=15)
```

---

parzen

*Parzen Kernel for Consistent Estimate of Long-run Variance*

---

### Description

Compute the Parzen window to obtain consistent estimate of long-run variance of multivariate time series.

### Usage

```
parzen(data,v)
```

### Arguments

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

### Value

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fvar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

### Examples

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
parzen(e,v=15)
```

---

Parzen\_uni

*Parzen Kernel for Consistent Estimate of Long-run Variance*

---

**Description**

Compute the Parzen kernel to obtain consistent estimate of long-run variance.

**Usage**

```
Parzen_uni(e,v)
```

**Arguments**

e	A univariate time series for computing consistent long-run variance, normally, regression residuals.
v	Number of lag terms used to compute the long-run variance.

**Value**

Return the consistent estimate of long-run variance, that PP and KPSS tests require. This procedure handles single time series only.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
y=macro[,"INF"]
e=y-mean(y)
Parzen_uni(e,v=15)
```

pp

*Phillips and Perron Unit Root Test***Description**

Implement the Phillips-Perron (1988) PP unit root test, including both  $Z_\alpha$  (Z-alpha) and  $Z_t$  (Z-tau) statistics. This wrapper allows inputting additional deterministic part, for example season dummies, but the asymptotic critical values are not available.

**Usage**

```
pp(y, type=c("none", "const", "trend"), d=NULL, lags=c("short", "long", "nil"), use=c("nw", "ba"))
```

**Arguments**

y	Vector to be tested for a unit root.
type	The deterministic parts in the test regression.
d	Additional deterministic parts in addition to "type" in the test regression.
lags	Lags used for correction of error term. See section "details" below.
use	User specified lags for correction of error term. See section "details" below. The default is the lag determined by Newey-West bandwidth ("nw") with Bartlett kernel ("ba").

**Details**

lags="short" sets the number of lags to  $\sqrt[4]{4 \times (n/100)}$ , whereas lags="long" sets the number of lags to  $\sqrt[4]{12 \times (n/100)}$ . If lags="nil" is chosen, no error correction is made.

Furthermore, "lags" and "use" are mutually exclusive: As long as "use" is not set to be NULL, its argument will be chosen first. One can specify a different number of maximum lags by setting "use" accordingly. Users can input number of your source. This version supports two bandwidth functions: "nw" for Newey-West and "and" for Andrews. The kernel functions are supported: "ba"=Bartlett, "pa"=Parzen, "qs"=Quadratic Spectral

**Value**

Zt	The Z-tau test statistic.
cvZt	Critical values of Zt.
Za	The Z-alpha test statistic.
cvZa	Critical values of Za.
lag	Number of lags used for kernel function.
resid	Regression residuals.

**Note**

This code modifies function `ur.pp` of package `urca`, which does not have relevant critical values for "Z-alpha" test statistic.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Phillips, P.C.B. and Perron, P. (1988) Testing for a unit root in time series regression. *Biometrika*, 75(2), 335-346.  
 MacKinnon, J.G. (1991) Critical Values for Cointegration Tests- Long-Run Economic Relationships, eds. R.F. Engle and C.W.J. Granger, London, Oxford, 267-276.

**Examples**

```
data(macro)
y=macro[, "INF"]
pp(y,
  type=c("none", "const", "trend")[3],
  lags = c("short", "long", "nil")[2],
  use=c("nw", "ba")) # If argument "use" is NOT NULL, argument lags will be ignored.

pp(y, lags = c("short", "long", "nil")[2],
  type=c("none", "const", "trend")[3],
  use=NULL)

pp(y, lags = c("short", "long", "nil")[2],
  type=c("none", "const", "trend")[3],
  use=18)
```

qs

*Quadratic-Spectral Kernel for Consistent Estimate of Long-run Variance*

**Description**

Computes the Andrews (1991) Quadratic-Spectral window to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
qs(data, v)
```

**Arguments**

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fivar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Andrews, D. W. K. (1991) Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica*, 59, 817-858.

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))

qs(e, v=15)
```

---

QS_uni	<i>Quadratic Spectral Kernel for Consistent Estimate of Long-run Variance</i>
--------	---

---

**Description**

Compute the QS kernel to obtain consistent estimate of long-run variance.

**Usage**

```
QS_uni(e, v)
```

**Arguments**

- e                    A univariate time series for computing consistent long-run variance, normally, regression residuals.
- v                    Number of lag terms used to compute the long-run variance.

**Value**

Return the consistent estimate of long-run variance, that PP and KPSS tests require. This procedure handles single time series only.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
y=macro[, "INF"]
e=y-mean(y)
QS_uni(e, v=15)
```

---

reisz

---

*Reisz Kernel for Consistent Estimate of Long-run Variance*


---

**Description**

Computes the Reisz Bochner window to obtain consistent estimate of long-run variance of multivariate time series.

**Usage**

```
reisz(data, v)
```

**Arguments**

- data                Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
- v                    Number of autocovariance terms in the kernel.

**Value**

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fivar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

**Examples**

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
reisz(e,v=15)
```

---

 SPC\_Bartlett

---

*Bartlett Kernel for Consistent Estimate of Long-run Variance*


---

**Description**

Compute the Bartlett kernel proposed by Sul, Phillips and Choi (2003) to obtain consistent estimate of long-run variance.

**Usage**

```
SPC_Bartlett(e,v)
```

**Arguments**

e	data that needs to compute consistent long-run variance, normally, regression residuals.
v	Number of lag terms to use when computing the long-run variance.

**Value**

Return the consistent estimate of long-run variance, Bartlett kernel proposed by Sul, Phillips and Choi (2003).

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

- Carrion-i-Silvestre, J. L. and Sanso, A. (2006) A guide to the computation of stationarity tests. *Empirical Economics*, 31(2), 433-448.
- Carrion-i-Silvestre, J. L. and Sanso, A. (2007) The KPSS test with two structural breaks. *Spanish Economic Review*, 9(2), 105-127.
- Sul, D., Phillips, P.C.B., and Choi, C.Y.(2005) Prewhitening Bias in HAC Estimation. *Oxford Bulletin of Economics and Statistics*, 67(4), 517-546.

**Examples**

```
data(macro)
y=macro[, "INF"]
e=y-mean(y)
SPC_Bartlett(e, v=15)
```

---

 SPC\_QS

*Quadratic Spectral Kernel for Consistent Estimate of Long-run Variance*

---

**Description**

Compute the Quadratic Spectral kernel of Sul, Phillips and Choi (2003) to obtains consistent estimate of long-run variance.

**Usage**

```
SPC_QS(e, v)
```

**Arguments**

- e                    data that needs to compute consistent long-run variance, normally, regression residuals.
- v                    Number of lag terms to use when computing the long-run variance.

**Value**

Return the consistent estimate of long-run variance.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

- Carrion-i-Silvestre, J. L. and Sanso, A. (2006) A guide to the computation of stationarity tests. *Empirical Economics*, 31(2), 433-448.
- Carrion-i-Silvestre, J. L. and Sanso, A. (2007) The KPSS test with two structural breaks. *Spanish Economic Review*, 9(2), 105-127.
- Sul, D., Phillips, P.C.B., and Choi, C.Y.(2005) Prewhitening Bias in HAC Estimation. *Oxford Bulletin of Economics and Statistics*, 67(4), 517-546.

## Examples

```
data(macro)
y=macro["INF"]
e=y-mean(y)
SPC_QS(e, v=15)
```

---

 sw

*Stock-Watson Common Trends Statistic*


---

## Description

Computes Stock and Watson (1988) common trends statistic for the null hypothesis that the data is a noncointegrated system (after allowing for a p-th order polynomial time trend).

## Usage

```
sw(data, p, v=15, ker_fun="parzen", aband=0, filter=0)
```

## Arguments

data	Matrix of k-time series variables.
p	Order of the time polynomial in the null hypothesis.
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.

## Details

1. Available kernels. Technical details are referred to Brillinger (1981,P.55)

"parzen"=Parzen kernel

"fejer"=Bartlett kernel

"dchlet"= Dirichlet kernel

"mdchlet"= Modified Dirichlet kernel

"tukham"=Tukey-Hamming kernel

"tukhan"=Tukey-Hanning kernel

"cauchy"=Cauchy kernel

"bohman"=Bohman kernel

"reisz"=Riesz,Bochner kernel

"gw"= Gauss-Weierstrass kernel

"qs"= Andrews (1991) Quadratic-Spectral

These kernels are written for FM procedures, technically different from those used in pp and kpss tests.

2. Andrews (1991) has developed data based (or automatic) bandwidth procedures for computing the spectrum. COINT implements these procedures for the Parzen, Bartlette, Tukey-Hamming, and the Quadratic-Spectral kernels. When aband is active, COINT ignores the value you specify for the band-width parameter and automatically substitutes the data-based value.

3. The aim of the AR(1) filter is to flatten the spectrum of residual around the zero frequency, thereby making it easier to estimate the true spectrum by simple averaging of the periodogram.

## Value

sw\_stat            Test statistic. Reject the null of a unit root if the SW statistic < critical value

sw\_cv             Critical values.

## Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

Stock, J. & M. K. Watson (1988) Testing for Common Trends. *Journal of the American Statistical Association*, 83, 1097-1107.

## Examples

```
data(macro)
sw(macro,p=1,v=15)
```

---

tukham

*Tukey-Hamming Kernel for Consistent Estimate of Long-run Variance*

---

### Description

Compute the Tukey-Hamming window to obtain consistent estimate of long-run variance of multivariate time series.

### Usage

```
tukham(data,v)
```

### Arguments

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

### Value

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmar", "fmgmm" and "fmols".
weights	The weights vector, used by function fmgive

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Brillinger, David R. (1981) Time Series Data Analysis and Theory. San Francisco, CA: Holden-Day.

### Examples

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
tukham(e,v=15)
```

---

tukhan

*Tukey-Hanning Kernel for Consistent Estimate of Long-run Variance*

---

### Description

Compute the Tukey-Hanning window to obtain consistent estimate of long-run variance of multivariate time series.

### Usage

```
tukhan(data,v)
```

### Arguments

data	Data matrix for computing consistent long-run variance, normally, multivariate regression residuals.
v	Number of autocovariance terms in the kernel.

### Value

amat	Return the consistent estimate of long-run variance. This procedure handles both multivariate and single time series, which is basically designed for "fmar", "fmgmm" and "fmols".
weights	The weights vector, used by function <code>fmgive</code>

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Brillinger, David R. (1981) *Time Series Data Analysis and Theory*. San Francisco, CA: Holden-Day.

### Examples

```
data(macro)
e=apply(macro, 2, function(x) x-mean(x))
tukhan(e,v=15)
```

---

Za *Phillips' (1987) Za and Zt Test for Unit Root*

---

**Description**

Compute Phillips' (1987) Za and Zt statistics for the null hypothesis that y has a unit root.

**Usage**

```
Za(y, p=1, v=15, ker_fun="parzen", aband=0, filter=0)
```

**Arguments**

y	The data of dependent variable in a regression.
p	Order of the time polynomial in the cointegrating regressio. p = -1: No deterministic term in the cointegrating regression. p = 0: For a constant term. p = 1: For a constant term and trend. Default. P > 2: For time polynomial. Critical values are available for p within [1,5].
v	Number of autocovariance terms to compute the spectrum at frequency zero, default=15.
ker_fun	Set kernel function to one of the available kernels, default="parzen". See section details below.
aband	Whether to activate the automatic bandwidth selection. aband=1. To activate. aband=0. Do not activate. The default.
filter	Whether to use an AR(1) filter to compute the spectrum at frequency zero. filter=1. To use. filter=0. Do not use. The default.

**Value**

alpha	Estimate of the AR(1) coefficient.
za	Za statistic for the series under the null has a unit root. Reject the null hypothesis of a unit root if the test statistic < critical value.
za_cv	Critical values of Za.
zt	Zt statistic for the series under the null has a unit root. Reject the null hypothesis of a unit root if the test statistic < critical value.
zt_cv	Critical values of Zt.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

## References

Ouliaris, S., J. Y. Park, and P. C. B. Phillips (1989) Testing for a Unit Root in the Presence of a Maintained Trend. Ch. 1 in Baldev Raj (ed.), *Advances in Econometrics and Modelling*. Netherlands: Kluwer Academic Publishers.

Phillips, P. C. B. (1987) Time Series Regression with a Unit Root. *Econometrica*, 55, 277-301.

## Examples

```
data(macro)
y=macro[,1]
Za(y,p=1,v=10)
```

---

ZA\_1br

*Zivot-Andrews unit root test with unknown one structural break.*

---

## Description

This function implements Zivot-Andrews sequential ADF unit root test with unknown one structural break. Handling two outlier models: "Innovational outlier" and "Additive outlier".

## Usage

```
ZA_1br(y,
       model=c("intercept", "trend", "both"),
       outlier=1,
       pmax=8,
       ic=c("AIC", "BIC"),
       fixed=FALSE,
       trim=0.1,
       eq=1,
       season=FALSE)
```

## Arguments

y	Univariate time series data, a preferable format is ts when season=TRUE.
model	Modelling where the unknown structural change occurs. intercept= structural change occurs at the intercept. trend= structural change occurs at the trend. both= structural changes occur at both the intercept and the trend.
outlier	The outlier model. 1=Innovational outlier model. 2=Additive outlier model.
pmax	The maximal lags that are either included in the test regression or lag selection searches its optimal lag via "ic".

ic	Information criteria, "AIC" or "BIC". The default is "AIC".
fixed	Logical. If TRUE, pmax is the fixed inputed lags. If FALSE, pmax is the maximal lags where lag selection searches its optimal lag.
trim	The trimming percentage. Default is 10
eq	The type of dependent variable in ADF equation. 1= original y. 2= differenced y.
season	Logical. If TRUE, then seasonal dummies will be included in the test regression.

**Value**

teststat	The Zivot-Andrews test statistic, which is the $\min(tstats)$ .
cval	The critical values that are tabulated in Zivot and Andrews(1992)
p	The number of lags that are included in the test regression.
bpoint	The breaking point that corresponds to the teststat.
tstats	The sequential ADF test statistic.
testreg	The $lm()$ output that corresponds to teststat.
timeElapse	Time elapsed for sequential search.

**Note**

This code modifies function `ur.za` of package `urca`. We add "season", "eq", "outlier", and "trim". Specifically, "outlier" is crucial, "season" is left to advanced research.

**Author(s)**

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

**References**

Zivot, E. and Andrews, W.K. (1992) Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. *Journal of Business & Economic Statistics*, 10(3), 251-270.

**Examples**

```
data(macro) #US inflation rate, 1958M1-2025M7
y=macro[1:200,"INF"]
za1=ZA_1br(y,
           ic=c("AIC","BIC")[2],
           outlier=1,
           pmax=8,
           fixed=TRUE,
           model=c("intercept","trend","both")[1],
           trim=0.01,
           eq=1,
           season=TRUE)
za1$timeElapse[3]
za1$teststat
```

```

za1$cval
y[za1$bpoint,]
za1$tstats
za1$p

#plotting
plot.ts(za1$tstats,ylim=range(c(za1$tstats,za1$cval)))
abline(h=za1$cval[1],col="red")
abline(h=za1$cval[2],col="blue")
abline(h=za1$cval[3],col="green")
abline(v=za1$bpoint,col="red",lty=2)

```

---

ZA\_2br

*Zivot-Andrews unit root test with unknown one structural break.*


---

### Description

This function implements Zivot-Andrews sequential ADF unit root test with one unknown structural break. Handling two outlier models: "Innovational outlier" and "Additive outlier".

### Usage

```

ZA_2br(y,
       model=c("intercept", "both"),
       pmax=8,
       ic=c("AIC", "BIC"),
       fixed=TRUE,
       trim=0.1,
       eq=1,
       trace=TRUE,
       season=FALSE)

```

### Arguments

y	Univariate time series data, a preferable format is ts when "season=TRUE".
model	Modelling where the unknown structural change occurs. intercept= structural change occurs at the intercept. trend= structural change occurs at the trend. both= structural changes occur at both the intercept and the trend.
pmax	The maximal lags that are either included in the test regression or lag selection searches its optimal lag via "ic".
ic	Information criteria, "AIC" or "BIC". The default is "AIC".
fixed	Logical. If TRUE, pmax is the fixed inputed lags, and the default is TRUE. If FALSE, pmax is the maximal lags where lag selection searches its optimal lag.

trim	The trimming percentage. Default is 10
eq	The type of dependent variable in ADF equation. 1= original y. 2= differenced y.
trace	Logical. If TRUE, then screen displays the sequential progress.
season	Logical. If TRUE, then seasonal dummies will be included in the test regression, and y must be in ts format.

### Details

This code extends Zivot-Andrews (1992) sequential procedure to two unknown structural changes. Critical values are from Narayan and Popp (2010).

### Value

teststat	The ADF test statistic in the presence of two structural breaks.
cval	The critical values that are tabulated in Narayan and Popp (2010).
p	The number of lags that are included in the test regression.
bpoint1	The first breaking point that corresponds to the teststat.
bpoint2	The second breaking point that corresponds to the teststat.
timeElapse	Time elapsed for sequential search.

### Note

This code modifies function `ur.za` of package `urca`. We add "season", "eq", "outlier", and "trim". Specifically, "outlier" is crucial, "season" is left to advanced research.

### Author(s)

Ho Tsung-wu <tsungwu@ntnu.edu.tw>, College of Management, National Taiwan Normal University.

### References

Narayan, P. K. and Popp, S. (2010) A new unit root test with two structural breaks in level and slope at unknown time. *Journal of Applied Statistics*, 37, 1425-1438.

Zivot, E. and Andrews, W.K. (1992), Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. *Journal of Business & Economic Statistics*, 10(3), 251-270.

### Examples

```
data(macro) # US macro data, 1967M1-2025M7

# It takes time
```

```
y=macro[1:200,"INF"]
za2=ZA_2br(y,
           ic=c("AIC","BIC")[2],
           pmax=8,
           fixed=TRUE,
           model=c("intercept","trend","both")[1],
           trim=0.1,
           eq=1,
           season=FALSE)
za2$timeElapse[3]/60
za2$teststat
za2$cval
y[za2$point1,] #The first dated strictural change
y[za2$point2,] #The second dated strictural change
```

# Index

## \* cointegration

- ccr, 6
- ccrQ, 8
- CZa, 10
- fm, 12
- fmgive, 15
- fmgmm, 17
- fmoIs, 19
- fmQ, 21
- fmvar, 23
- fmvar\_forecast, 25
- fmvar\_plag, 26
- GHansen, 28
- sw, 47

## \* datasets

- data-sets, 11

## \* kernel

- bartlett, 2
- Bartlett\_uni, 3
- bohman, 4
- cauchy, 5
- dchlet, 11
- gw, 30
- Kurozumi\_Bartlett, 36
- Kurozumi\_QS, 37
- mdchlet, 38
- parzen, 39
- Parzen\_uni, 40
- qs, 42
- QS\_uni, 43
- reisz, 44
- SPC\_Bartlett, 45
- SPC\_QS, 46
- tukham, 49
- tukhan, 50

## \* unit root test

- kpss, 31
- kpss\_1br, 32
- kpss\_2br, 34

- pp, 41
- Za, 51
- ZA\_1br, 52
- ZA\_2br, 54

- bartlett, 2
- Bartlett\_uni, 3
- bohman, 4

- cauchy, 5
- ccr, 6
- ccrQ, 8
- CZa, 10

- data-sets, 11
- dchlet, 11

- fm, 12
- fmgive, 15
- fmgmm, 17
- fmoIs, 19
- fmQ, 21
- fmvar, 23
- fmvar\_forecast, 25
- fmvar\_plag, 26

- GHansen, 28
- gw, 30

- kpss, 31
- kpss\_1br, 32
- kpss\_2br, 34
- Kurozumi\_Bartlett, 36
- Kurozumi\_QS, 37

- macro (data-sets), 11
- mdchlet, 38

- parzen, 39
- Parzen\_uni, 40
- pp, 41

qs, [42](#)  
QS\_uni, [43](#)

reisz, [44](#)

SPC\_Bartlett, [45](#)  
SPC\_QS, [46](#)  
sw, [47](#)

tukham, [49](#)  
tukhan, [50](#)

Za, [51](#)  
ZA\_1br, [52](#)  
ZA\_2br, [54](#)