

Package ‘Copula.surv’

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Type Package

Title Analysis of Bivariate Survival Data Based on Copulas

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Description Simulating bivariate survival data from copula models.

Estimation of the association parameter in copula models.

Two different ways to estimate the association parameter in copula models are implemented.

A goodness-of-fit test for a given copula model is implemented.

See Emura, Lin and Wang (2010) <[doi:10.1016/j.csda.2010.03.013](https://doi.org/10.1016/j.csda.2010.03.013)> for details.

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Copula.surv-package *Analysis of Bivariate Survival Data*

Description

Simulating bivariate survival data from copula models (Emura et al. 2019). Estimation of the association parameter in copula models. Two different ways to estimate the association parameter in copula models are implemented. A goodness-of-fit test for a given copula model is implemented. See Emura, Lin and Wang (2010) <doi:10.1016/j.csda.2010.03.013> for details. Also, Weibull regression is implemented (Section 2.6.3 of Emura et al. (2019)).

Details

Details are seen from the references.

Author(s)

Takeshi Emura Maintainer: Takeshi Emura <takeshiemura@gmail.com>

References

- Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43
- Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

| | |
|----------|--|
| simu.BB1 | <i>Simulating data from the BB1 copula</i> |
|----------|--|

Description

n pairs of (U,V) are generated from the BB1 copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($scale1=scale2=shape1=shape2=1$) give the unit exponential distributions.

Usage

```
simu.BB1(n,alpha,delta=0,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter |
| delta | BB1 copula's departure parameter from the Clayton (0 is the default) |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.BB1(n=n,alpha=1,delta=2,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.BB1reg

Simulating data from the BB1 copula regression model

Description

n pairs of (U,V) are generated from the BB1 copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($scale1=scale2=shape1=shape2=1$) give the unit exponential distributions.

Usage

```
simu.BB1reg(n,alpha,delta=0,scale1=1,scale2=1,shape1=1,shape2=1,
  beta1=0,beta2=0,beta12=0,Z.dist=runif,...)
```

Arguments

| | |
|---------------------|--|
| <code>n</code> | sample size |
| <code>alpha</code> | association (copula) parameter |
| <code>delta</code> | BB1 copula's departure parameter from the Clayton (0 is the default) |
| <code>scale1</code> | scale parameter for X |
| <code>scale2</code> | scale parameter for Y |
| <code>shape1</code> | shape parameter for X |
| <code>shape2</code> | shape parameter for Y |
| <code>beta1</code> | regression coefficient for X |
| <code>beta2</code> | regression coefficient for Y |
| <code>beta12</code> | regression coefficient for copula |
| <code>Z.dist</code> | distribution for covariates |
| <code>...</code> | parameters for Z.dist |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |
| Z | Covariates |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=10
simu.BB1reg(n=n,alpha=1,delta=2,scale1=1,scale2=2,shape1=0.5,beta1=1,beta2=-1,beta12=2,shape2=2)
```

simu.CC

Simulating data from the Celebioglu-Cuadras (CC) copula

Description

n pairs of (U,V) are generated from the CC copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.CC(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|---|
| n | sample size |
| alpha | association (copula) parameter, $-1 \leq \alpha \leq 1$ |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.CC(n=n,alpha=-1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Clayton

Simulating data from the Clayton copula

Description

n pairs of (U,V) are generated from the Clayton copula. n paris of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Clayton(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Clayton(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[,"U"],Dat[,"V"])
cor(Dat[,"U"],Dat[,"V"],method="kendall")
plot(Dat[,"X"],Dat[,"Y"])
cor(Dat[,"X"],Dat[,"Y"],method="kendall")
```

simu.FGM

*Simulating data from the FGM copula***Description**

n pairs of (U, V) are generated from the FGM copula. n pairs of (X, Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($\text{scale1}=\text{scale2}=\text{shape1}=\text{shape2}=1$) give the unit exponential distributions.

Usage

```
simu.FGM(n, alpha, scale1=1, scale2=1, shape1=1, shape2=1, Print=FALSE)
```

Arguments

| | |
|---------------------|---|
| <code>n</code> | sample size |
| <code>alpha</code> | association (copula) parameter; $-1 \leq \alpha \leq 1$ |
| <code>scale1</code> | scale parameter for X |
| <code>scale2</code> | scale parameter for Y |
| <code>shape1</code> | shape parameter for X |
| <code>shape2</code> | shape parameter for Y |
| <code>Print</code> | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|----------------|---|
| <code>U</code> | uniformly distributed on $(0,1)$ |
| <code>V</code> | uniformly distributed on $(0,1)$ |
| <code>X</code> | Weibull distributed (scale1 , shape1) |
| <code>Y</code> | Weibull distributed (scale2 , shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.FGM(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Frank

*Simulating data from the Frank copula***Description**

n pairs of (U,V) are generated from the Frank copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Frank(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Frank(n=n,alpha=10,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.GB

Simulating data from the Gumbel-Barnett (GB) copula

Description

n pairs of (U,V) are generated from the GB copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.GB(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter, $0 \leq \alpha \leq 1$ |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.GB(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Gumbel

Simulating data from the Gumbel copula

Description

n pairs of (U,V) are generated from the Gumbel copula. n paris of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Gumbel(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Gumbel(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Joe

Simulating data from the Joe copula

Description

n pairs of (U,V) are generated from the Joe copula. n paris of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Joe(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Joe(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.t *Simulating data from the t-copula*

Description

n pairs of (U,V) are generated from the t -copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($scale1=scale2=shape1=shape2=1$) give the unit exponential distributions.

Usage

```
simu.t(n,alpha,df=1,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

| | |
|--------|--|
| n | sample size |
| alpha | association (copula) parameter |
| df | degrees of freedom (d=1 is the default) |
| scale1 | scale parameter for X |
| scale2 | scale parameter for Y |
| shape1 | shape parameter for X |
| shape2 | shape parameter for Y |
| Print | print Kendall's tau and means of X and Y if "TRUE" |

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

| | |
|---|--------------------------------------|
| U | uniformly distributed on (0,1) |
| V | uniformly distributed on (0,1) |
| X | Weibull distributed (scale1, shape1) |
| Y | Weibull distributed (scale2, shape2) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```

n=100
Dat=simu.t(n=n,alpha=0.8,df=1,scale1=1,scale2=2,shape1=0.5,shape2=2,Print=TRUE)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")

```

Test.Clayton

A goodness-of-fit test for the Clayton copula

Description

Perform a goodness-of-fit test for the Clayton copula based on Emura, Lin and Wang (2010). The test is asymptotically equivalent to the test of Shih (1998).

Usage

```
Test.Clayton(x.obs,y.obs,dx,dy,lower=0.001,upper=50,U.plot=TRUE)
```

Arguments

| | |
|--------|---|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| lower | lower bound for the association parameter |
| upper | upper bound for the association parameter |
| U.plot | if TRUE, draw the plot of $U_1(\theta)$ |

Details

See the references.

Value

| | |
|--------|--|
| theta1 | association parameter by the pseudo-likelihood estimator |
| theta2 | association parameter by the unweighted estimator |
| Stat | $\log(\theta_1+1)-\log(\theta_2+1)$ |
| Z | Z-value of the goodness-of-fit for the Clayton copula |
| P | P-value of the goodness-of-fit for the Clayton copula |

Author(s)

Takeshi Emura

References

- Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43
- Shih JH (1998) A goodness-of-fit test for association in a bivariate survival model. *Biometrika* 85: 189-200

Examples

```
n=20
theta_true=2 ## association parameter ##
r1_true=2 ## hazard for X
r2_true=2 ## hazard for Y

set.seed(1)
V1=runif(n)
V2=runif(n)
X=-1/r1_true*log(1-V1)
W=(1-V1)^(-theta_true)
Y=1/theta_true/r2_true*log( 1-W+W*(1-V2)^(-theta_true/(theta_true+1)) )
C=runif(n,min=0,max=5)

x.obs=pmin(X,C)
y.obs=pmin(Y,C)
dx=X<=C
dy=Y<=C

Test.Clayton(x.obs,y.obs,dx,dy)
```

Test.Gumbel

A goodness-of-fit test for the Gumbel copula

Description

Perform a goodness-of-fit test for the Gumbel copula based on Emura, Lin and Wang (2010).

Usage

```
Test.Gumbel(x.obs,y.obs,dx,dy,lower=0.01,upper=50,U.plot=TRUE)
```

Arguments

| | |
|--------|---|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| lower | lower bound for the association parameter |
| upper | upper bound for the association parameter |
| U.plot | if TRUE, draw the plot of $U_1(\theta)$ and $U_2(\theta)$ |

Details

See the references.

Value

| | |
|--------|--|
| theta1 | association parameter by the pseudo-likelihood estimator |
| theta2 | association parameter by the unweighted estimator |
| Stat | $\log(\text{theta1}) - \log(\text{theta2})$ |
| Z | Z-value of the goodness-of-fit for the Clayton copula |
| P | P-value of the goodness-of-fit for the Clayton copula |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
x.obs=c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
y.obs=c(2,1,4,5,6,8,3,7,10,9,11,12,13,14,15)
dx=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
dy=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
Test.Gumbel(x.obs,y.obs,dx,dy)
```

U1.Clayton

Estimation of an association parameter via the pseudo-likelihood

Description

Estimate the association parameter of the Clayton copula using bivariate survival data. The estimator was derived by Clayton (1978) and reformulated by Emura, Lin and Wang (2010).

Usage

```
U1.Clayton(x.obs,y.obs,dx,dy,lower=0.001,upper=50,U.plot=TRUE)
```

Arguments

| | |
|--------|---|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| lower | lower bound for the association parameter |
| upper | upper bound for the association parameter |
| U.plot | if TRUE, draw the plot of $U_1(\theta)$ |

Details

Details are seen from the references.

Value

| | |
|-------|--|
| theta | association parameter |
| tau | Kendall's tau ($=\theta/(\theta+2)$) |

Author(s)

Takeshi Emura

References

- Clayton DG (1978). A model for association in bivariate life tables and its application to epidemiological studies of familial tendency in chronic disease incidence. *Biometrika* 65: 141-51.
- Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
n=200
theta_true=2 ## association parameter ##
r1_true=1 ## hazard for X
r2_true=1 ## hazard for Y

set.seed(1)
V1=runif(n)
V2=runif(n)
X=-1/r1_true*log(1-V1)
W=(1-V1)^(-theta_true)
Y=1/theta_true/r2_true*log( 1-W+W*(1-V2)^(-theta_true/(theta_true+1)) )
C=runif(n,min=0,max=5)

x.obs=pmin(X,C)
y.obs=pmin(Y,C)
dx=X<=C
dy=Y<=C
```

```
U1.Clayton(x.obs,y.obs,dx,dy)
```

U1.Gumbel

Estimation of an association parameter via the unweighted estimator

Description

Estimate the association parameter of the Gumbel copula using bivariate survival data. The estimator was derived by Emura, Lin and Wang (2010).

Usage

```
U1.Gumbel(x.obs,y.obs,dx,dy,lower=0.01,upper=50,U.plot=TRUE)
```

Arguments

| | |
|--------|---|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| lower | lower bound for the association parameter |
| upper | upper bound for the association parameter |
| U.plot | if TRUE, draw the plot of $U_1(\theta)$ |

Details

Details are seen from the references.

Value

| | |
|-------|--|
| theta | association parameter |
| tau | Kendall's tau ($=\theta/(\theta+2)$) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
x.obs=c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
y.obs=c(2,1,4,5,6,8,3,7,10,9,11,12,13,14,15)
dx=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
dy=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
U1.Gumbel(x.obs,y.obs,dx,dy)
```

U2.Clayton

*Estimation of an association parameter via the unweighted estimator***Description**

Estimate the association parameter of the Clayton copula using bivariate survival data. The estimator was defined as the unweighted estimator in Emura, Lin and Wang (2010).

Usage

```
U2.Clayton(x.obs,y.obs,dx,dy,lower=0.001,upper=50,U.plot=TRUE)
```

Arguments

| | |
|--------|--|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| lower | lower bound for the association parameter |
| upper | upper bound for the association parameter |
| U.plot | if TRUE, draw the plot of U ₂ (theta) |

Details

Details are seen from the references.

Value

| | |
|-------|----------------------------------|
| theta | association parameter |
| tau | Kendall's tau (=theta/(theta+2)) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring. *Compt Stat Data Anal* 54: 3033-43

Examples

```

n=200
theta_true=2 ## association parameter ##
r1_true=1 ## hazard for X
r2_true=1 ## hazard for Y

set.seed(1)
V1=runif(n)
V2=runif(n)
X=-1/r1_true*log(1-V1)
W=(1-V1)^(-theta_true)
Y=1/theta_true/r2_true*log( 1-W+W*(1-V2)^(-theta_true/(theta_true+1)) )
C=runif(n,min=0,max=5)

x.obs=pmin(X,C)
y.obs=pmin(Y,C)
dx=X<=C
dy=Y<=C

U2.Clayton(x.obs,y.obs,dx,dy)

```

U2.Gumbel

Estimation of an association parameter via the pseudo-likelihood

Description

Estimate the association parameter of the Gumbel copula using bivariate survival data. The estimator was derived by Emura, Lin and Wang (2010).

Usage

```
U2.Gumbel(x.obs,y.obs,dx,dy,lower=0.01,upper=50,U.plot=TRUE)
```

Arguments

| | |
|--------|---|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| lower | lower bound for the association parameter |
| upper | upper bound for the association parameter |
| U.plot | if TRUE, draw the plot of $U_2(\theta)$ |

Details

Details are seen from the references.

Value

| | |
|-------|--|
| theta | association parameter |
| tau | Kendall's tau ($=\text{theta}/(\text{theta}+1)$) |

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
x.obs=c(1,2,3,4,5)
y.obs=c(2,1,4,5,6)
dx=c(1,1,1,1,1)
dy=c(1,1,1,1,1)
U2.Gumbel(x.obs,y.obs,dx,dy)
```

Weib.reg.BB1

Weibull regression under the BB1 copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.BB1(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

| | |
|-----------------|----------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|--------|-------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |
| alpha | copula parameter |
| delta | copula parameter |
| tau | Kendall's tau |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

#TBA

Weib.reg.BB1.0

Weibull regression under the BB1 copula with known "delta"

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

Weib.reg.BB1.0(x.obs,y.obs,dx,dy,zx,zy,delta=0,convergence.par=FALSE)

Arguments

| | |
|-----------------|---------------------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |
| delta | known copula parameter ($d \geq 0$) |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|--------|-------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |
| alpha | copula parameter |
| tau | Kendall's tau |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

Weib.reg.cBB1

Weibull regression under the conditional BBI copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.cBB1(x.obs,y.obs,dx,dy,zx,zy,zxy,convergence.par=FALSE)
```

Arguments

| | |
|-----------------|---------------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |
| zxy | matrix of covariates for copula |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|---------|------------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |
| beta_xy | regression coefficients for copula |
| alpha | copula parameter |
| delta | copula parameter |
| tau | Kendall's tau given beta_xy=0 |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

#TBA

| | |
|-----------------|---|
| Weib.reg.cBB1.0 | <i>Weibull regression under the conditional BBI copula with known "delta"</i> |
|-----------------|---|

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.cBB1.0(x.obs,y.obs,dx,dy,zx,zy,zxy,delta=0,convergence.par=FALSE)
```

Arguments

| | |
|-------|----------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |

| | |
|-----------------|---------------------------------------|
| zxy | matrix of covariates for copula |
| delta | known copula parameter ($d \geq 0$) |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|---------|------------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |
| beta_xy | regression coefficients for copula |
| alpha | copula parameter |
| tau | Kendall's tau given $\beta_{xy}=0$ |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

Weib.reg.Clayton

Weibull regression under the Clayton copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.Clayton(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

| | |
|-----------------|----------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|--------|-------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

Weib.reg.Frank

Weibull regression under the Frank copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.Frank(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

| | |
|-----------------|----------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|--------|-------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

Weib.reg.Gumbel

Weibull regression under the Gumbel copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.Gumbel(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

| | |
|-----------------|----------------------------|
| x.obs | censored times for X |
| y.obs | censored times for Y |
| dx | censoring indicators for X |
| dy | censoring indicators for Y |
| zx | matrix of covariates for X |
| zy | matrix of covariates for Y |
| convergence.par | if TRUE, show the details |

Details

Details are seen from the references.

Value

| | |
|--------|-------------------------------|
| beta_x | regression coefficients for X |
| beta_y | regression coefficients for Y |

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

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