

# Package ‘FitDynMix’

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**Title** Estimation of Dynamic Mixtures

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**Description** Estimation of a dynamic lognormal -  
Generalized Pareto mixture via the Approximate Maximum Likelihood and the Cross-  
Entropy methods. See Bee, M. (2023) <[doi:10.1016/j.csda.2023.107764](https://doi.org/10.1016/j.csda.2023.107764)>.

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 AMLEfit

*Estimating a dynamic mixture via AMLE*


---

## Description

This function fits a dynamic mixture via Approximate Maximum Likelihood. Currently only implemented for the lognormal - generalized Pareto case, with Cauchy or exponential weight. The bootstrap estimation of the standard errors of the MLEs (used for finding the supports of the uniform priors) is carried out via parallel computing.

## Usage

```
AMLEfit(yObs, epsilon, k, bootreps, intTol = 1e-04, weight)
```

## Arguments

yObs	numerical vector: observed sample.
epsilon	non-negative scalar: scale parameter of the Markov kernel.
k	non-negative integer: number of samples generated in the AMLE approach, such that $k \cdot \text{epsilon} = \text{ABC sample size}$ .
bootreps	positive integer: number of bootstrap replications.
intTol	non-negative scalar: threshold for stopping the computation of the integral in the normalization constant: if the integral on the interval from $n-1$ to $n$ is smaller than intTol, the approximation procedure stops.
weight	'cau' or 'exp': name of weight distribution.

## Details

For the lognormal and GPD parameters, the support of the uniform prior is set equal to the 99% confidence interval of the bootstrap distribution after discarding the outliers. For the Cauchy parameters, the support is given by the range of the bootstrap distribution after discarding the outliers. Be aware that computing times are large when  $k$  and/or bootreps are large.

**Value**

A list with the following elements:

AMLEpars a list of four 6 or 5-dimensional vectors: approximate maximum likelihood estimates computed via sample mean, maxima of the marginal kernel densities, maximum of the multivariate kernel densities, maximum of the product of the marginal kernel densities.

ABCsam ((k x epsilon) x nc) matrix: ABC sample, where nc is 6 or 5, according to the weight.

MLEpars (np x 1) vector: maximum likelihood estimates and maximized log-likelihood, where np is 7 or 6, according to the weight.

MLEboot (bootreps x nc) matrix: maximum likelihood estimates obtained in each bootstrap replication. nc is 6 or 5, according to the weight.

**References**

Bee M (2023). “Unsupervised mixture estimation via approximate maximum likelihood based on the Cramér - von Mises distance.” *Computational Statistics & Data Analysis*, **185**, 107764.

**See Also**

[AMLEmode](#).

**Examples**

```
k <- 5000
epsilon <- .02
bootreps <- 2
res = AMLEfit(Metro2019, epsilon, k, bootreps, , 'cau')
```

---

AMLEmode

*Approximating the mode of a multivariate empirical distribution*

---

**Description**

This function approximates the mode of a multivariate empirical distribution by means of:

1. the sample mean
2. the product of the maxima of the univariate kernel densities estimated using the marginals
3. the maximum of the multivariate kernel density
4. the maximum of the product of the univariate kernel densities Typically used in connection with AMLEfit (see AMLEfit for examples).

**Usage**

```
AMLEmode(ABCsam)
```

**Arguments**

ABCsam (m x k) matrix: ABC sample, where m is the ABC sample size and k is the number of parameters.

**Details**

The bandwidth is estimated via smoothed cross-validation

**Value**

A list containing the 4 approximate modes.

---

CENoisyFit

*Cross-Entropy estimation*

---

**Description**

This function estimates a dynamic mixture by means of the noisy Cross-Entropy method. Currently only implemented for the lognormal - generalized Pareto case, with Cauchy or exponential weight. This is mainly an auxiliary function, not suitable for the final user. Use CeNoisyFitBoot instead.

**Usage**

```
CENoisyFit(
  x,
  rawdata,
  rho,
  maxiter,
  alpha,
  nsim,
  nrepsInt,
  xiInst,
  betaInst,
  eps,
  r = 5,
  weight
)
```

**Arguments**

x list: sequence of integers 1,...,K, where K is the number of datasets. Set x = 1 in case of a single dataset.

rawdata either a list of vectors or a vector: in the former case, each vector contains a dataset to be used for estimation.

rho real in (0,1): parameter determining the quantile of the log-likelihood values to be used at each iteration.

maxiter	non-negative integer: maximum number of iterations.
alpha	real in (0,1): smoothing parameter.
nsim	non-negative integer: number of replications used in the normal and lognormal updating.
nrepsInt	non-negative integer: number of replications used in the Monte Carlo estimate of the normalizing constant.
xiInst	non-negative real: shape parameter of the instrumental GPD.
betaInst	non-negative real: scale parameter of the instrumental GPD.
eps	non-negative real: tolerance for the stopping criterion of the noisy Cross-Entropy method.
r	positive integer: length of window to be used in the stopping criterion.
weight	'cau' or 'exp': name of weight distribution.

### Details

See Rubinstein and Kroese (2004, chap. 6).

### Value

For each dataset, a list with the following elements is returned:

V (nreps x 12) matrix: updated mean and variance of the distributions used in the stochastic program. nit (positive integer): number of iterations needed for convergence. loglik (scalar): maximized log-likelihood.

### References

Rubinstein RY, Kroese DP (2004). *The Cross-Entropy Method*. Springer.

### See Also

CENoisyFitBoot

### Examples

```
maxiter = 10
alpha = .5
rho = .05
eps = 1e-2
nsim = 1000
nrepsInt = 1000
xiInst = 3
betaInst = 3
r = 5
res <- CENoisyFit(1,Metro2019,rho,maxiter,alpha,nsim,nrepsInt,xiInst,betaInst,eps,r,'exp')
```

**Description**

This function estimates a dynamic mixture by means of the noisy Cross-Entropy method and computes bootstrap standard errors. Currently only implemented for the lognormal - generalized Pareto, with Cauchy or exponential weight. Bootstrap standard errors are computed in parallel.

**Usage**

```
CENoisyFitBoot(
  yObs,
  nboot,
  rho,
  maxiter,
  alpha,
  nsim,
  nrepsInt,
  xiInst,
  betaInst,
  eps,
  r = 5,
  weight
)
```

**Arguments**

yObs	numerical vector: observed random sample from the mixture.
nboot	integer: number of bootstrap replications for computing the standard errors. If nboot = 0, no standard errors are computed.
rho	real in (0,1): parameter determining the quantile of the log-likelihood values to be used at each iteration.
maxiter	non-negative integer: maximum number of iterations.
alpha	real in (0,1): smoothing parameter.
nsim	non-negative integer: number of replications used in the normal and lognormal updating.
nrepsInt	non-negative integer: number of replications used in the Monte Carlo estimate of the normalizing constant.
xiInst	non-negative real: shape parameter of the instrumental GPD.
betaInst	non-negative real: scale parameter of the instrumental GPD.
eps	non-negative real: tolerance for the stopping criterion of the noisy Cross-Entropy method.
r	positive integer: length of window to be used in the stopping criterion.
weight	'cau' or 'exp': name of weight distribution.

**Value**

If nboot > 0, a list with the following elements:  
 estPars: Cross-Entropy estimates.  
 nit: number of iterations needed for convergence.  
 loglik: maximized log-likelihood.  
 bootPars: parameter estimates obtained for each bootstrap sample.  
 stddev: bootstrap standard errors.  
 If nboot = 0, only estPars, nit and loglik are returned.

**Examples**

```
res = CENoisyFitBoot(Metro2019,0,.05,20,.5,500,500,3,3,.01,5,'exp')
```

---

 cvm\_stat\_M

*Computing the Cramér - von Mises distance between two samples*


---

**Description**

This function Computing the Cramér - von Mises distance between two samples.

**Usage**

```
cvm_stat_M(vec1, vec2, power)
```

**Arguments**

vec1	(n x 1) vector: first sample.
vec2	(n x 1) vector: second sample.
power	power of the integrand. Equal to 2 when using the L2 distance

**Details**

This function computes the Cramér - von Mises distance between two samples. See Drovandi and Frazier (2022, p. 7).

**Value**

out scalar: Cramér - von Mises distance between the two samples

**References**

Drovandi C, Frazier DT (2022). “A comparison of likelihood-free methods with and without summary statistics.” *Statistics and Computing*, **32**, Paper No. 42.

**Examples**

```
out = cvm_stat_M(runif(100),rnorm(100),2)
```

---

ddyn *Density of a Lognormal-GPD dynamic mixture*

---

### Description

This function evaluates the density of a Lognormal-GPD dynamic mixture, with Cauchy or exponential weight.

### Usage

```
ddyn(x, pars, intTol, weight)
```

### Arguments

x	non-negative vector: points where the density is evaluated.
pars	numerical vector: if weight is equal to 'cau', values of $\mu_c, \tau, \mu, \sigma, \xi, \beta$ ; if weight is equal to 'exp', values of $\lambda, \mu, \sigma, \xi, \beta$ .
intTol	non-negative scalar: threshold for stopping the computation of the integral in the normalization constant: if the integral on the interval from n-1 to n is smaller than intTol, the approximation procedure stops.
weight	'cau' or 'exp': name of weight distribution.

### Value

density of the lognormal-GPD mixture evaluated at x.

### Examples

```
x <- seq(0,20,length.out=1000)
pars <- c(1,2,0,.5,.25,3.5)
dLNPar <- ddyn(x,pars,1e-04,'cau')
```

---

dynloglik *Log-likelihood of a Lognormal-GPD dynamic mixture*

---

### Description

This function evaluates the log-likelihood of a Lognormal-GPD dynamic mixture, computing the integral in the normalizing constant via quadrature methods.

### Usage

```
dynloglik(x, y, intTol, weight)
```

**Arguments**

x	if weight is equal to 'cau', (6 by 1) numerical vector: values of $\mu_c, \tau, \mu, \sigma, \xi, \beta$ ; if weight is equal to 'exp', (5 by 1) numerical vector: values of $\lambda, \mu, \sigma, \xi, \beta$ .
y	vector: points where the function is evaluated.
intTol	non-negative scalar: threshold for stopping the computation of the integral in the normalization constant: if the integral on the interval from n-1 to n is smaller than intTol, the approximation procedure stops.
weight	'cau' or 'exp': name of weight distribution.

**Value**

log-likelihood of the lognormal-GPD mixture evaluated at y.

**Examples**

```
x <- c(1,2,0,.5,.25,3.5)
y <- rDynMix(100,x,'cau')
fit <- dynloglik(x,y,1e-04,'cau')
```

---

dynloglikMC

*Log-likelihood of a Lognormal-GPD dynamic mixture*

---

**Description**

This function evaluates the log-likelihood of a Lognormal-GPD dynamic mixture, with Cauchy or exponential weight, approximating the normalizing constant via Monte Carlo simulation.

**Usage**

```
dynloglikMC(x, y, nreps, xiInst, betaInst, weight)
```

**Arguments**

x	if weight is equal to 'cau', (6 by 1) numerical vector: values of $\mu_c, \tau, \mu, \sigma, \xi, \beta$ ; if weight is equal to 'exp', (5 by 1) numerical vector: values of $\lambda, \mu, \sigma, \xi, \beta$ .
y	vector: points where the function is evaluated.
nreps	non-negative integer: number of replications to be used in the computation of the integral in the normalizing constant.
xiInst	non-negative real: shape parameter of the instrumental GPD.
betaInst	non-negative real: scale parameter of the instrumental GPD.
weight	'cau' or 'exp': name of weight distribution.

**Value**

Log-likelihood of the lognormal-GPD mixture evaluated at y.

**Examples**

```
llik <- dynloglikMC(c(1,2,0,1,.25,3.5),Metro2019,10000,3,3,'exp')
```

---

Metro2019

*Population estimate of the US metropolitan areas*


---

**Description**

A dataset containing the 2019 population estimate, divided by 10000, of the 415 US metropolitan areas computed by the US Census Bureau.

**Usage**

```
Metro2019
```

**Format**

A numerical vector with 415 rows and 1 column.

**Source**

<https://www.census.gov>

---

MLEBoot

*Bootstrap standard errors of MLEs*


---

**Description**

This function creates bootstrap samples of input data and fits a dynamic mixture via maximum likelihood.

**Usage**

```
MLEBoot(x, y, intTol, weight)
```

**Arguments**

x	list of integers: indices of replications.
y	numerical vector: observed data.
intTol	threshold for stopping the computation of the integral in the normalization constant: if the integral on the interval from n-1 to n is smaller than intTol, the approximation procedure stops.
weight	'cau' or 'exp': name of weight distribution.

**Details**

MLEs are computed by means of the `optim` function. When it breaks down, the sample is discarded and a new one is generated. The function keeps track of the number of times this happens.

**Value**

A list with the following elements:

MLE: maximum likelihood estimates obtained from each bootstrap sample.

errors: number of times the MLE algorithm breaks down.

**Examples**

```
bootMLEs <- MLEBoot(1,Metro2019,1e-04,'exp')
```

---

MLEfit

*Estimating a dynamic mixture via MLE*


---

**Description**

This function fits a dynamic mixture via standard maximum likelihood. Currently only implemented for the lognormal - generalized Pareto case, with Cauchy or exponential weight.

**Usage**

```
MLEfit(yObs, bootreps, intTol = 1e-04, weight)
```

**Arguments**

<code>yObs</code>	numerical vector: observed sample.
<code>bootreps</code>	non-negative integer: number of bootstrap replications. If equal to 0, no standard errors are computed.
<code>intTol</code>	non-negative scalar: threshold for stopping the computation of the integral in the normalization constant: if the integral on the interval from $n-1$ to $n$ is smaller than <code>intTol</code> , the approximation procedure stops.
<code>weight</code>	'cau' or 'exp': name of weight distribution.

**Details**

Starting values for  $\mu$  and  $\sigma$  are the lognormal MLEs computed with the observations below the median. Initial values for  $\xi$  and  $\tau$  are the GPD MLEs obtained with the observations above the median. For the location and scale parameter of the Cauchy, we respectively use the first quartile and  $\text{abs}(\log(\text{sd}(x)/2))$ . For the parameter of the exponential, we use  $\text{abs}(\log(\text{sd}(x)/2))$ .

**Value**

MLEpars vector: maximum likelihood estimates and maximized log-likelihood.

MLEboot matrix: maximum likelihood estimates obtained in each bootstrap replication.

sdMLE vector: bootstrap standard deviation of the MLEs.

**References**

Bee M (2023). “Unsupervised mixture estimation via approximate maximum likelihood based on the Cramér - von Mises distance.” *Computational Statistics & Data Analysis*, **185**, 107764.

**See Also**

[AMLEfit](#)

**Examples**

```
mixFit <- MLEfit(Metro2019,0,1e-04,'cau')
```

---

nConst_MC	<i>Monte carlo approximation of the normalizing constant of a Lognormal-GPD dynamic mixture</i>
-----------	-------------------------------------------------------------------------------------------------

---

**Description**

This function evaluates via Monte Carlo simulation the normalizing constant of a Lognormal-GPD dynamic mixture, with Cauchy or exponential weight.

**Usage**

```
nConst_MC(x, nreps, xiInst = 3, betaInst = 3, weight)
```

**Arguments**

x	if weight is equal to 'cau', (6 by 1) numerical vector: values of $\mu_c, \tau, \mu, \sigma, \xi, \beta$ ; if weight is equal to 'exp', (5 by 1) numerical vector: values of $\lambda, \mu, \sigma, \xi, \beta$ .
nreps	number of replications to be used in the computation of the integral in the normalizing constant.
xiInst	real: value of the shape parameter of the instrumental density. Default value equal to 3.
betaInst	non-negative real: value of the scale parameter of the instrumental density. Default value equal to 3.
weight	'cau' or 'exp': name of weight distribution.

**Value**

Normalizing constant of the density of the lognormal-GPD mixture.

**Examples**

```
nconst <- nConst_MC(c(1,2,0,0.5,0.25,3.5), 10000, 3, 3, 'cau')
```

---

rDynMix

*Simulating a dynamic lognormal-Pareto mixture*


---

**Description**

This function simulates a dynamic mixture. Currently only implemented for the lognormal - generalized Pareto case, with Cauchy or exponential weight.

**Usage**

```
rDynMix(nreps, x, weight)
```

**Arguments**

nreps	integer: number of observations sampled from the mixture.
x	numerical vector: if weight = 'cau', values of $mu_c, \tau, \mu, \sigma, \xi, \beta$ ; if weight = 'exp', values of $\lambda, \mu, \sigma, \xi, \beta$ .
weight	'cau' or 'exp': name of weight distribution.

**Details**

This function simulates a dynamic lognormal-GPD mixture using the algorithm of Frigessi et al. (2002, p. 221).

**Value**

ysim (nreps x 1) vector: nreps random numbers from the lognormal-GPD dynamic mixture.

**References**

Frigessi A, Haug O, Rue H (2002). "A Dynamic Mixture Model for Unsupervised Tail Estimation without Threshold Selection." *Extremes*, **3**(5), 219–235.

**Examples**

```
ysim <- rDynMix(100,c(1,2,0,0.5,0.25,3),'cau')
```

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