

# Package ‘FuzzyPovertyR’

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**Title** Estimation of Fuzzy Poverty Measures

**Version** 3.0.2

**Description** Estimates fuzzy measures of poverty and deprivation. It also estimates the sampling variance of these measures using bootstrap or jackknife repeated replications.

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eq_predicate	<i>Calculation of Equivalized Poverty Predicate</i>
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## Description

This function takes as input a numeric vector representing a predicate variable and turns it into its equivalised version using different equivalence scales.

## Usage

```
eq_predicate(
  predicate,
  ncomp,
  age = NULL,
  scale.eq = "modifiedOECD",
  newscale,
  data = NULL
)
```

## Arguments

predicate	A numeric vector (or the variable name) representing the poverty predicate (i.e. income or expenditure)
ncomp	A numerical vector (or the variable name) of the total number of components for the j-th family.
age	A numerical vector (or the variable name) of the number of components for the j-th family less than 16 years-old to be defined only for OECD scales
scale.eq	The equivalence scale. Options are: "carbonaro", "n.par" (non parametric), "OECD7050", "modifiedOECD" (Default) or "new"
newscale	a data.frame with two columns: "ncomp" defining the number of components and "s.eq" that define the corresponding
data	An optional data frame containing the variables to be used

**Value**

A data.frame containing the equivalised predicate variable.

**References**

Bernini, C., Emili, S., & Ferrante, M. R. (2024). Regional disparities in the sensitivity of wellbeing to poverty measures. In *Spatial Inequalities and Wellbeing* (pp. 136-157). Edward Elgar Publishing.

Betti, G. (1999). Nonparametric equivalence scales with application to Poland. *Statistics Research Report*.

Chanfreau, J., & Burchardt, T. (2008). *Equivalence scales: rationales, uses and assumptions*. Edinburgh: Scottish Government.

**Examples**

```
#Using OECD scale

eq_predicate(predicate = "HY022", ncomp = "ncomp", age = "age16",
             scale.eq = "OECD7050", data = eusilc) #OECD7050

eq_predicate(predicate = "HY022", ncomp = "ncomp", age = "age16",
             scale.eq = "modifiedOECD", data = eusilc) #modifiedOECD

#Define a new scale

newscal <- data.frame("ncomp" = c(1:7), "s.eq" = runif(7,1,10) ) # new

eq_predicate(predicate = "HY022", ncomp = "ncomp", scale.eq = "new",
             newscale = newscal, data = eusilc)
```

---

eusilc

*Eusilc data*

---

**Description**

Eusilc data

**Usage**

```
data(eusilc)
```

**Format**

An object of class "data.frame"

**HB020** Country of residence

**ID** ID

**HY022** Total disposable household income before social transfer

**HS040** Capacity to afford paying for one week annual holiday

**HS050** Capacity to afford a meal with meat

**HS060** Capacity to face unexpected financial expenses

**HS070** Ownership of a telephone

**HS080** Ownership of a color TV

**HS090** Ownership of a computer

**HS100** Ownership of a washing machine

**HS110** Ownership of a car

**HS120** Ability to make ends meet

**HS160** Problems with the dwelling: too dark, not enough light

**HS170** Noise from neighbors or from the street

**HS180** Pollution, crime or other environmental problems

**HS190** Crime violence or vandalism in the area

**HH010** Dwelling type

**HH020** Tenure Status

**HH040** Leaking roof, damp walls,floors,foundation

**HH050** Ability to keep home adequately warm

**HH081** Bath or shower in dwelling

**HH091** Indoor flushing toilet for sole use of household

**HX040** Household size

**DB090** Household cross-sectional weight

**db040** Sub-domain

**stratum** Stratum

**psu** Primary selection unit

**ncomp** Size of the household

**age16** Number of household members aged less than 16 year

**eq\_income** Equivalised income

**Source**

Created by authors following the EU-SILC structure

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fm_construct	<i>Fuzzy monetary poverty estimation</i>
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---

## Description

fm\_construct constructs fuzzy monetary poverty estimates.

## Usage

```
fm_construct(
  predicate,
  weight = NULL,
  fm = "verma",
  ID = NULL,
  HCR,
  interval = c(1, 10),
  alpha = NULL,
  hh.size,
  z_min,
  z_max,
  z1,
  z2,
  b,
  z,
  breakdown = NULL,
  data = NULL,
  verbose = FALSE
)
```

## Arguments

predicate	A numeric vector representing the poverty predicate (i.e. income or expenditure)
weight	A numeric vector of sampling weights of the same length of predicate. if NULL weights will set equal to n (n = sample size)
fm	The membership function (default is "verma". Other options are "ZBM", "belhadj2015", "belhadj2011", "chakravarty", "cerioli", "verma1999" and "TFR". See Betti et. al., 2023)
ID	A numeric or character vector of IDs. if NULL (the default) it is set as the row sequence
HCR	If fm="verma" or fm="verma1999" or fm="TFR" . The value of the head count ratio used to compute alpha so that the membership function equals the HCR
interval	If fm="verma" or fm="verma1999" or fm="TFR". A numeric vector of length two to look for the value of alpha (if not supplied)
alpha	The value of the exponent in equations of "verma", "verma1999" and "TFR". If NULL it is calculated so that it equates the expectation of the membership function to HCR.

hh.size	If fm="ZBM". A numeric vector of household size
z_min	A parameter of the membership function if fm="belhadj2011", i.e. the $z_{\min}$ : $\mu=1$ for $0 < y_i < z_{\min}$ (see: See Betti et al., 2023)
z_max	A parameter of the membership function if fm="belhadj2011", i.e. the $z_{\max}$ : $\mu=0$ for $y_i > z_{\max}$ (see: See Betti et al., 2023)
z1	A parameter of the membership function if fm="belhadj2015" or fm="cerioli". For "belhadj2015" $z1$ : $\mu=1$ for $y_i < z1$ while for "cerioli" $\mu=1$ for $0 < y_i < z1$ (see: See Betti et al., 2023)
z2	A parameter of the membership function if fm="belhadj2015" or fm="cerioli". For "belhadj2015" $z2$ : $\mu=0$ for $y_i > z2$ while for "cerioli" the $z1$ : $\mu=0$ for $y_i > z2$ (see: See Betti et al., 2023)
b	A parameter of the membership function if fm="belhadj2015". The shape parameter (if $b=1$ the mf is linear between $z1$ and $z2$ )
z	A parameter of the membership function if fm="chakravarty", i.e. $\mu=0$ for $y_i \geq z$ (see: See Betti et al., 2023)
breakdown	A factor of sub-domains to calculate estimates for (using the same alpha)
data	An optional data frame containing the variables to be used
verbose	Logical. whether to print the proceeding of the procedure

### Details

It implements the fuzzy set approach to monetary poverty measurement where the usual dichotomy poor (1) not-poor(0) is replaced with a continuum score in  $(0,1)$

### Value

an object of class FuzzyMonetary containing the (fuzzy) membership function for each individual in the sample, the estimated expected value (estimate) of the function and the parameters of the membership functions (supplied or calculated). If breakdown is supplied it gives an output for each level.

### References

- Belhadj, B. (2011). A new fuzzy unidimensional poverty index from an information theory perspective. *Empirical Economics*, 40(1):687–704.
- Belhadj, B. (2015). Employment measure in developing countries via minimum wage and poverty new fuzzy approach. *Opsearch*, 52(1):329–339.
- Betti, G., Cheli, B., Lemmi, A., and Verma, V. (2006). Multidimensional and longitudinal poverty: an integrated fuzzy approach. In Betti, G. and Lemmi, A., editors, *Fuzzy set approach to multidimensional poverty measurement*, pages 115–137. Springer, Boston, USA.
- Betti, G., D'Agostino, A., Lemmi, A., & Neri, L. (2023). The fuzzy approach to poverty measurement. In *Research Handbook on Measuring Poverty and Deprivation* Edited by Silber, J. (pp. 489-500). Edward Elgar Publishing.
- Betti, G. and Verma, V. (1999). Measuring the degree of poverty in a dynamic and comparative context: a multi-dimensional approach using fuzzy set theory. In *Proceedings, iccs-vi*, volume 11, pages 289–300.

Cerioli, A. and Zani, S. (1990). A fuzzy approach to the measurement of poverty. In *Income and Wealth Distribution, Inequality and Poverty: Proceedings of the Second International Conference on Income Distribution by Size: Generation, Distribution, Measurement and Applications.*, 272–284. Springer, Boston, USA.

Chakravarty, S. R. (2006). An Axiomatic Approach to Multidimensional Poverty Measurement via Fuzzy Sets. *Fuzzy Set Approach to Multidimensional Poverty Measurement*, 49-72.

Cheli, B. and Lemmi, A. (1995). A 'totally' fuzzy and relative approach to the multidimensional analysis of poverty. *24(1):115–134*.

Zedini, A. and Belhadj, B. (2015). A new approach to unidimensional poverty analysis: Application to the Tunisian case. *Review of Income and Wealth*, 61(3):465–476.

## Examples

```
#The following examples are based on the dataset eusilc
#include in the package.

#fm = "verma"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             fm = "verma", HCR = 0.154, ID = eusilc$ID)

#fm = "verma1999"
#In this example we set alpha=4.5

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             fm = "verma1999", alpha = 4.5, ID = eusilc$ID)

#fm = "TFR"
#In this example we do not use the sample weights. alpha = 4.5

fm_construct(predicate = eusilc$eq_income,
             fm = "TFR", alpha = 4.5)

#fm = "belhadj2015"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z1=100, z2=15000, b=2,
             fm = "belhadj2015")

#fm = "cerioli"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z1=100, z2=10000, fm= "cerioli")

#fm = "belhadj2011"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z_min=1000, z_max=8000, fm= "belhadj2011")

#fm = "chakravarty"
```

```

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z=8000, fm= "chakravarty")

#fm = "ZBM"
#For this index have to use the household size

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             hh.size=eusilc$ncomp , fm= "ZBM")

#####
##Including breakdown##
#####

#fm = "verma"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             fm = "verma", HCR = 0.154, ID = eusilc$ID,
             breakdown = eusilc$db040)

#fm = "verma1999"
#In this example we set alpha=4.5

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             fm = "verma1999", alpha = 4.5, ID = eusilc$ID,
             breakdown = eusilc$db040)

#fm = "TFR"
#In this example we do not use the sample weights. alpha = 4.5

fm_construct(predicate = eusilc$eq_income,
             fm = "TFR", alpha = 4.5,
             breakdown = eusilc$db040)

#fm = "belhadj2015"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z1=100, z2=15000, b=2,
             fm = "belhadj2015", breakdown = eusilc$db040)

#fm = "cerioli"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z1=100, z2=10000, fm= "cerioli", breakdown = eusilc$db040)

#fm = "belhadj2011"

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z_min=1000, z_max=8000, fm= "belhadj2011",
             breakdown = eusilc$db040)

#fm = "chakravarty"

```

```

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             z=8000, fm= "chakravarty", breakdown = eusilc$db040)

#fm = "ZBM"
#For this index we have to use the household size

fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
             hh.size=eusilc$ncomp , fm= "ZBM",
             breakdown = eusilc$db040)

```

---

fm\_var

*Fuzzy monetary poverty estimation*


---

## Description

This function estimates the variance of the fuzzy monetary poverty index

## Usage

```

fm_var(
  predicate,
  weight,
  fm,
  ID = NULL,
  type = "bootstrap_naive",
  R = 100,
  M = NULL,
  stratum,
  psu,
  f = 0.01,
  verbose = FALSE,
  HCR,
  interval = c(1, 10),
  alpha = NULL,
  hh.size,
  z_min,
  z_max,
  z1,
  z2,
  b,
  z,
  Xs,
  total,
  breakdown = NULL,
  fixed = FALSE,
  data = NULL
)

```

**Arguments**

predicate	A numeric vector representing the poverty predicate (i.e. income or expenditure)
weight	A numeric vector of sampling weights of the same length of predicate. if NULL weights will set equal to n (n = sample size)
fm	The membership function (default is "verma". Other options are "ZBM", "belhadj2015", "belhadj2011", "chakravarty", "cerioli", "verma1999" and "TFR". See Betti et. al., 2023)
ID	A numeric or character vector of IDs. if NULL (the default) it is set as the row sequence
type	The variance estimation method chosen. One between bootstrap_naive (default), bootstrap_calibrated or jackknife
R	The number of bootstrap replicates. Default is 500
M	The size of bootstrap samples. Default is nrow(data)
stratum	The vector identifying the stratum (if 'jackknife' is chosen as variance estimation technique)
psu	The vector identifying the psu (if 'jackknife' is chosen as variance estimation technique)
f	The finite population correction fraction (if 'jackknife' is chosen as variance estimation technique)
verbose	Logical. whether to print the proceeding of the variance estimation procedure
HCR	If fm="verma" or fm="verma1999" or fm="TFR" . The value of the head count ratio used to compute alpha so that the membership function equals the HCR
interval	If fm="verma" or fm="verma1999" or fm="TFR". A numeric vector of length two to look for the value of alpha (if not supplied)
alpha	The value of the exponent in equations of "verma", "verma1999" and "TFR". If NULL it is calculated so that it equates the expectation of the membership function to HCR.
hh.size	If fm="ZBM". A numeric vector of household size
z_min	A parameter of the membership function if fm="belhadj2011", i.e. the z_min: $\mu=1$ for $0 < y_i < z_{\min}$ (see: See Betti et. al, 2023)
z_max	A parameter of the membership function if fm="belhadj2011", i.e. the z_max: $\mu=0$ for $y_i > z_{\max}$ (see: See Betti et. al, 2023)
z1	A parameter of the membership function if fm="belhadj2015" or fm="cerioli". For "belhadj2015" z1: $\mu=1$ for $y_i < z1$ while for "cerioli" $\mu=1$ for $0 < y_i < z1$ (see: See Betti et. al, 2023)
z2	A parameter of the membership function if fm="belhadj2015" or fm="cerioli". For "belhadj2015" z2: $\mu=0$ for $y_i > z2$ while for "cerioli" the z1: $\mu=0$ for $y_i > z2$ (see: See Betti et. al, 2023)
b	A parameter of the membership function if fm="belhadj2015". The shape parameter (if b=1 the mf is linear between z1 and z2)
z	A parameter of the membership function if fm="chakravarty", i.e. $\mu=0$ for $y_i \geq z$ (see: See Betti et. al, 2023)

Xs	A matrix (i x j) of calibration variables. i number of units, j number of variables
total	A Vector of population totals of dimension 1 x j
breakdown	A factor of sub-domains to calculate estimates for (using the same alpha). If numeric will be coerced to a factor
fixed	Whether the membership function needs to be re-calculated at each bootstrap or jackknife replicate (default is FALSE)
data	An optional data frame containing the variables to be used

### Value

An object of class FuzzyMonetary containing the estimate of variance with the method selected. If breakdown is not NULL, the variance is estimated for each sub-domain.

### References

- Belhadj, B. (2011). A new fuzzy unidimensional poverty index from an information theory perspective. *Empirical Economics*, 40(1):687–704.
- Belhadj, B. (2015). Employment measure in developing countries via minimum wage and poverty new fuzzy approach. *Opsearch*, 52(1):329–339.
- Betti, G., Cheli, B., Lemmi, A., and Verma, V. (2006). Multidimensional and longitudinal poverty: an integrated fuzzy approach. In Betti, G. and Lemmi, A., editors, *Fuzzy set approach to multidimensional poverty measurement*, pages 115–137. Springer, Boston, USA.
- Betti, G., D’Agostino, A., Lemmi, A., & Neri, L. (2023). The fuzzy approach to poverty measurement. In *Research Handbook on Measuring Poverty and Deprivation* Edited by Silber, J. (pp. 489-500). Edward Elgar Publishing.
- Betti, G. and Verma, V. (1999). Measuring the degree of poverty in a dynamic and comparative context: a multi-dimensional approach using fuzzy set theory. In *Proceedings, iccs-vi*, volume 11, pages 289–300.
- Ceroli, A. and Zani, S. (1990). A fuzzy approach to the measurement of poverty. In *Income and Wealth Distribution, Inequality and Poverty: Proceedings of the Second International Conference on Income Distribution by Size: Generation, Distribution, Measurement and Applications.*, 272–284. Springer, Boston, USA.
- Chakravarty, S. R. (2006). An Axiomatic Approach to Multidimensional Poverty Measurement via Fuzzy Sets. *Fuzzy Set Approach to Multidimensional Poverty Measurement*, 49-72.
- Cheli, B. and Lemmi, A. (1995). A ‘totally’ fuzzy and relative approach to the multidimensional analysis of poverty. 24(1):115–134.
- Zedini, A. and Belhadj, B. (2015). A new approach to unidimensional poverty analysis: Application to the Tunisian case. *Review of Income and Wealth*, 61(3):465–476.
- Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

### Examples

```
#The following examples are based on the dataset eusilc
#include in the package.
```

```

#Example 1 using bootstrap and breakdown

#fm = "verma"

fm_var(predicate = eusilc$eq_income, weight = eusilc$DB090,
        fm = "verma", breakdown = NULL, type = "bootstrap_calibrated",
        alpha = 4, Xs = eusilc[,4:6], total = c(20, 30, 40))

#fm = "belhadj2015"

fm_var(predicate = eusilc$eq_income, weight = eusilc$DB090,
        fm = "belhadj2015", breakdown = eusilc$db040, type = "bootstrap_naive",
        z1 = 100, z2 = 15000, b = 2)

#Example 2 using jackknife without breakdown

#fm = "verma1999"

fm_var(predicate = eusilc$eq_income, weight = eusilc$DB090,
        fm = "verma1999", type = "jackknife",
        stratum = eusilc$stratum, psu = eusilc$psu,
        alpha = 4)

#fm = "cerioli"

fm_var(predicate = eusilc$eq_income, weight = eusilc$DB090,
        fm = "cerioli", type = "jackknife",
        stratum = eusilc$stratum, psu = eusilc$psu,
        z1 = 1000, z2 = 12000)

```

---

fs\_construct

*Fuzzy supplementary poverty estimation (Step 7)*


---

## Description

Step 7. Constructs the fuzzy supplementary poverty measure based on Steps1-6.

## Usage

```
fs_construct(steps4_5, weight, alpha, breakdown = NULL)
```

## Arguments

steps4_5	The results from fs_equate.
weight	A numeric vector of sampling weights of length nrow(step1). if NULL weights will set equal to n (n = sample size)

alpha	The value of the exponent in the FM equation. If NULL it is calculated so that it equates the expectation of the membership function to HCR.
breakdown	A Dimension of sub-domains to calculate estimates for (using the same alpha). If numeric will be coerced to a Dimension.

### Value

An object of class FuzzySupplementary containing the fuzzy membership function for each unit, the point estimate (i.e. the expected value of the function), and the alpha parameter.

### References

- Betti, G., Gagliardi, F., Lemmi, A., & Verma, V. (2015). Comparative measures of multidimensional deprivation in the European Union. *Empirical Economics*, 49(3), 1071-1100.
- Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

### Examples

```
#This example is based on the dataset eusilc included in the package
#The FS index is compute without and with breakdown and using an HCR = 0.12
#The step 2-5 are the following (step 1 is the eusilc dataset)
#For more on each step see the ad hoc function included in the package

#Step 2

step2 = fs_transform(eusilc[,4:23], weight = eusilc$DB090, ID = eusilc$ID)

#Step 3 is the definition of the dimension.
#For more about the step see Betti et al. (2018)

dimensions = c(1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5)

#Step 4-5 finding weights

steps4_5 = fs_weight(dimensions, step2 = step2, rho = NULL)

#Step 6 computation of alpha parameter

alpha <- fs_equate(steps4_5 = steps4_5,
                  weight = eusilc$DB090, HCR = 0.12,
                  interval = c(1,10))

#Step 7 the FS index without breakdown

fs_results = fs_construct(steps4_5 = steps4_5,
                        weight = eusilc$DB090, alpha = alpha, breakdown = NULL)

#Step 7 the FS index with breakdown

fs_results = fs_construct(steps4_5 = steps4_5,
```

```
weight = eusilc$DB090, alpha = alpha, breakdown = eusilc$db040)
```

---

```
fs_construct_all      Fuzzy supplementary poverty estimation (all steps)
```

---

### Description

Step 1-7. Constructs the fuzzy supplementary poverty measure based without step-by-step functions.

### Usage

```
fs_construct_all(
  data,
  weight = NULL,
  ID = NULL,
  dimensions,
  rho = NULL,
  HCR,
  interval = c(1, 10),
  alpha = NULL,
  breakdown = NULL
)
```

### Arguments

data	A matrix or a data frame of identified items (see Step 1 of Betti et. al, 2018)
weight	A numeric vector of sampling weights. if NULL weights will set equal to n (n = sample size)
ID	A numeric or character vector of IDs. if NULL (the default) it is set as the row sequence
dimensions	A numeric vector (of length ncol(data)) of assignments of items in data to dimensions
rho	Optional critical value to be used for calculation of weights in the Kendall correlation matrix. If NULL rho is set equal to the point of largest gap between the ordered set of correlation values encountered (see Betti and Verma, 2008)
HCR	The value of the head count ratio used to compute alpha so that the expected value of the membership function equals HCR
interval	A numeric vector of length two to look for the value of alpha (if not supplied)
alpha	The value of the exponent in equations of "verma", "verma1999" and "TFR". If NULL it is calculated so that it equates the expectation of the membership function to HCR.
breakdown	A Dimension of sub-domains to calculate estimates for (using the same alpha). If numeric will be coerced to a Dimension

**Value**

An object of class FuzzySupplementary containing the fuzzy membership function for each unit, the point estimate (i.e. the expected value of the function), and the alpha parameter.

**References**

Betti, G., & Verma, V. (2008). Fuzzy measures of the incidence of relative poverty and deprivation: a multi-dimensional perspective. *Statistical Methods and Applications*, 17, 225-250.

Betti, G., Gagliardi, F., Lemmi, A., & Verma, V. (2015). Comparative measures of multidimensional deprivation in the European Union. *Empirical Economics*, 49(3), 1071-1100.

Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

**Examples**

```
#This example is based on the dataset eusilc included in the package
#The FS index is compute with breakdown and using an HCR = 0.12
#with summary and plot

FS <- fs_construct_all(data = eusilc[,4:23], weight = eusilc$DB090, # step 2
                      dimensions = c(1,1,1,1,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5), # step 3
                      rho = NULL, # steps 4 and 5
                      HCR = .12, # step 6
                      breakdown = eusilc$db040) # step 7 with breakdowns

summary(FS)
plot(FS)
```

---

fs_equate	<i>Fuzzy supplementary poverty estimation, finding the alpha parameter (step 6)</i>
-----------	---

---

**Description**

Step 6. This function solves  $E(\mu)^{(\alpha-1)} = HCR$  for alpha.

**Usage**

```
fs_equate(steps4_5, weight, HCR, interval = c(1, 10), verbose = TRUE)
```

**Arguments**

steps4_5	The results obtained from fs_weight.
weight	A numeric vector of sampling weights. if NULL weights will set equal to n (n = sample size)
HCR	The value of the head count ratio used to compute alpha so that the membership function equals the HCR

interval            The range to look for the value of alpha.  
 verbose            Logical. whether to print the proceeding of the procedure.

### Value

The alpha parameter that solves the non-linear equation  $E(\mu) = HCR$

### References

Betti, G., Gagliardi, F., Lemmi, A., & Verma, V. (2015). Comparative measures of multidimensional deprivation in the European Union. *Empirical Economics*, 49(3), 1071-1100.

Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

### Examples

```
#This example is based on the dataset eusilc included in the package
#The Step 6 of the FS index is computed
#The step 2-5 are the following (step 1 is the eusilc dataset)
#For more on each step see the ad hoc function included in the package

#Step 2

step2 = fs_transform(eusilc[,4:23], weight = eusilc$DB090, ID = eusilc$ID)

#Step 3 is the definition of the dimension.
#For more about the step see Betti et al. (2018)

dimensions = c(1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5)

#Step 4-5 finding weights

steps4_5 = fs_weight(dimensions, step2 = step2, rho = NULL)

#Step 6 computation of alpha parameter

fs_equate(steps4_5 = steps4_5,
          weight = eusilc$DB090,
          HCR = 0.12, interval = c(1,10))
```

---

fs\_order

*Fuzzy monetary poverty estimation (Step 1)*

---

### Description

Detects and inverts deprivation items for FS

### Usage

```
fs_order(data, vec_order)
```

**Arguments**

data	a data-set of n columns with the considered items
vec_order	a vector of length n with TRUE or FALSE. True if the order of the variable is to be inverted, False otherwise

**Value**

A data.frame with the same item of data with inverted order for those with vec\_order==TRUE

**References**

Betti, G., Gagliardi, F., Lemmi, A., & Verma, V. (2015). Comparative measures of multidimensional deprivation in the European Union. *Empirical Economics*, 49(3), 1071-1100.

Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

**Examples**

```
#Create data
data=data.frame("X"=rep(c(1,2,3,4),20), "Y"=rep(c(7,8,9,1),20))

#Crete vec_order
vec_order=c(TRUE,FALSE)

fs_order(data=data, vec_order)
```

---

fs_transform	<i>Fuzzy supplementary poverty estimation (Step 2)</i>
--------------	--

---

**Description**

Step 2. This function maps a set of answers to binary or categorical items to the (0,1) interval.

**Usage**

```
fs_transform(data, weight = NULL, ID = NULL, depr.score = "s", ...)
```

**Arguments**

data	A matrix or a data frame of identified items (see Step 1 of Betti et. al, 2018)
weight	A numeric vector of sampling weights of length nrow(step1). if NULL weights will set equal to n (n = sample size)
ID	A numeric or character vector of IDs. if NULL (the default) it is set as the row sequence
depr.score	The deprivation score to be used (see d or s in Betti et al (2018))
...	other parameters

**Details**

The function calculates deprivation score. To obtain consistent measures of supplementary poverty it is important that items are in the right order. Lower levels of the items have to correspond to more deprivation while higher levels of the items to a less deprivation.

**Value**

An object of class FuzzySupplementary containing a matrix of the same dimension of data with items mapped into the (0,1) interval

**References**

Betti, G., Gagliardi, F., Lemmi, A., & Verma, V. (2015). Comparative measures of multidimensional deprivation in the European Union. *Empirical Economics*, 49(3), 1071-1100.

Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

**Examples**

```
#This example is based on the dataset eusilc included in the package
#step 1 is the choice of the eusilc dataset

#Step 2

step2 = fs_transform(eusilc[,4:23], weight = eusilc$DB090, ID = eusilc$ID)
```

---

 fs\_var

*Fuzzy supplementary poverty estimation.*


---

**Description**

Fuzzy supplementary poverty estimation.

**Usage**

```
fs_var(
  data,
  weight = NULL,
  ID = NULL,
  dimensions,
  HCR,
  breakdown = NULL,
  alpha,
  rho = NULL,
  type = "bootstrap_naive",
  R = 500,
```

```

M = NULL,
stratum,
psu,
f = 0.01,
Xs,
total,
fixed = FALSE,
verbose = TRUE
)

```

### Arguments

data	A matrix or data frame of items
weight	A numeric vector of sampling weights of length nrow(step1). if NULL weights will set equal to n (n = sample size)
ID	A numeric or character vector of IDs. if NULL (the default) it is set as the row sequence
dimensions	A numeric vector (of length ncol(data)) of assignments of items in data to dimensions
HCR	The value of the head count ratio used to compute alpha so that the expected value of the membership function equals HCR
breakdown	A factor of sub-domains to calculate estimates for (using the same alpha). If numeric will be coerced to a factor
alpha	The value of the exponent in equations of "verma", "verma1999" and "TFR". If NULL it is calculated so that it equates the expectation of the membership function to HCR.
rho	Optional critical value to be used for calculation of weights in the Kendall correlation matrix. If NULL rho is set equal to the point of largest gap between the ordered set of correlation values encountered (see Betti and Verma, 2008)
type	The variance estimation method chosen. One between bootstrap_naive (default), bootstrap_calibrated or jackknife
R	The number of bootstrap replicates. Default is 500
M	The size of bootstrap samples. Default is nrow(data)
stratum	The vector identifying the stratum (if 'jackknife' is chosen as variance estimation technique)
psu	The vector identifying the psu (if 'jackknife' is chosen as variance estimation technique)
f	The finite population correction fraction (if 'jackknife' is chosen as variance estimation technique)
Xs	A matrix (i x j) of calibration variables. i number of units, j number of variables
total	A Vector of population totals of dimension 1 x j
fixed	Whether the membership function needs to be re-calculated at each bootstrap or jackknife replicate (default is FALSE)
verbose	Logical. whether to print the proceeding of the variance estimation procedure

**Value**

An object of class FuzzySupplementary containing the estimated variance.

**References**

Betti, G., & Verma, V. (2008). Fuzzy measures of the incidence of relative poverty and deprivation: a multi-dimensional perspective. *Statistical Methods and Applications*, 17, 225-250.

Betti, G., Gagliardi, F., Lemmi, A., & Verma, V. (2015). Comparative measures of multidimensional deprivation in the European Union. *Empirical Economics*, 49(3), 1071-1100.

Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

**Examples**

```
#This example is based on the dataset eusilc included in the package
#The variance of the FS index is compute without breakdown
#and using an alpha = 2

#####
##Bootstrap##
#####

fs_var(data = eusilc[,4:23], weight = eusilc$DB090, ID = NULL,
        dimensions = c(1,1,1,1,2,2,2,2,2,3,3,3,3,3,4,4,4,4,5,5,5),
        breakdown = NULL, alpha = 2,
        rho = NULL, type = 'bootstrap_naive', M = NULL, R = 2, verbose = TRUE)
```

---

fs\_weight

*Fuzzy supplementary poverty estimation (Steps 4 and 5)*


---

**Description**

Step 4 and Step 5. Calculates the weights of dimensions discovered after Dimension analysis.

**Usage**

```
fs_weight(dimensions, step2, rho = NULL)
```

**Arguments**

dimensions	A numeric vector (of length ncol(data)) of assignments of items in data to dimensions
step2	The data frame resulting from step2

rho                    Optional critical value to be used for calculation of weights in the Kendall correlation matrix. If NULL rho is set equal to the point of largest gap between the ordered set of correlation values encountered (see Betti and Verma, 2008)

### Details

This function calculates the two set of weights  $w_a$  and  $w_b$  (see References)

### Value

An object of class FuzzySupplementary with calculated weights and deprivation scores in each dimension identified.

### References

Betti, G., & Verma, V. (2008). Fuzzy measures of the incidence of relative poverty and deprivation: a multi-dimensional perspective. *Statistical Methods and Applications*, 17, 225-250.

Betti, G., Gagliardi, F., & Verma, V. (2018). Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *International Statistical Review*, 86(1), 68-86.

### Examples

```
#This example is based on the dataset eusilc included in the package
#The step 2-3 are the following (step 1 is the eusilc dataset)
#For more on each step see the ad hoc function included in the package

#Step 2

step2 = fs_transform(eusilc[,4:23], weight = eusilc$DB090, ID = eusilc$ID)

#Step 3 is the definition of the dimension.
#For more about the step see Betti et al. (2018)

dimensions = c(1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5)

#Step 4-5 finding weights

steps4_5 = fs_weight(dimensions, step2 = step2, rho = NULL)
```

---

FuzzyPoverty

*s3 class fuzzy poverty*

---

### Description

s3 class fuzzy poverty

### Usage

FuzzyPoverty(x)

**Arguments**

x                    an object

**Value**

an object of class FuzzyPoverty

---

FuzzySupplementary    *s3 class fuzzy poverty*

---

**Description**

s3 class fuzzy poverty

**Usage**

```
FuzzySupplementary(x)
```

**Arguments**

x                    an object

**Value**

an object of class FuzzyPoverty

---

HCR                    *Head Count Ratio (HCR)*

---

**Description**

This function calculates the head count ratio.

**Usage**

```
HCR(predicate, weight = NULL, p = 0.5, q = 0.6, poverty.line = NULL)
```

**Arguments**

predicate	A numeric vector of a predicate variable (i.e. income or expenditure)
weight	A numeric vector of sampling weights. if NULL simple random sampling weights will be used
p	The quantile to be calculated from the predicate variable. Default is the median
q	The percentage of the quantile to be used in determining the poverty line. default is 0.6
poverty.line	The poverty line. If it is NULL it is estimated from data.

**Details**

The head count ration is defined as the sum of the sampling weight of statistical units whose value of the predicate variable is below the poverty line. The poverty line is usually defined as a fraction of a weighted quantile (in official statistics the median) of the predicate distribution

**Value**

A list containing the classification of the units as poor (TRUE) and not-poor (FALSE), the estimated Head Count Ratio, and the poverty line

**Examples**

```
N <- 100
p <- 0.5
q <- 0.6
predicate <- rchisq(N, 15) # predicate variable
HCR(predicate)
```

---

plot.FuzzyMonetary      *The plot of a FuzzyMonetary object*

---

**Description**

plot method for class "FuzzyMonetary"

**Usage**

```
## S3 method for class 'FuzzyMonetary'
plot(x, ...)
```

**Arguments**

x                      An object of class "FuzzyMonetary"  
...                     Additional options

**Value**

The plot

**Examples**

```
#The following example is based on the dataset eusilc
#included in the package.
```

```
#fm = "verma"
```

```

index = fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,
                    fm = "verma", HCR = 0.154, ID = eusilc$ID)

plot(index)

```

---

plot.FuzzySupplementary

*The plot of a FuzzySupplementary object*

---

### Description

plot method for class "FuzzySupplementary"

### Usage

```

## S3 method for class 'FuzzySupplementary'
plot(x, ...)

```

### Arguments

x	An object of class "FuzzySupplementary"
...	Additional options

### Value

The plot

### Examples

```

#This example is based on the dataset eusilc included in the package
#The plot of the FS index is compute with breakdown and using an HCR = 0.12

FS <- fs_construct_all(data = eusilc[,4:23], weight = eusilc$DB090, # step 2
                      dimensions = c(1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5), # step 3
                      rho = NULL, # steps 4 and 5
                      HCR = .12, # step 6
                      breakdown = eusilc$db040) # step 7 with breakdowns

plot(FS)

```

---

summary.FuzzyMonetary *The summary of a FuzzyMonetary object*

---

**Description**

Summary method for class "FuzzyMonetary"

**Usage**

```
## S3 method for class 'FuzzyMonetary'  
summary(object, ...)
```

**Arguments**

object	An object of class "FuzzyMonetary"
...	Additional options

**Value**

The summary method for class "FuzzyMonetary"

**Examples**

```
#The following example is based on the dataset eusilc  
#included in the package.  
  
#fm = "verma"  
  
index = fm_construct(predicate = eusilc$eq_income, weight = eusilc$DB090,  
                    fm = "verma", HCR = 0.154, ID = eusilc$ID)  
  
summary(index)
```

---

summary.FuzzySupplementary  
*The summary of a FuzzySupplementary object*

---

**Description**

Summary method for class "FuzzySupplementary"

**Usage**

```
## S3 method for class 'FuzzySupplementary'  
summary(object, ...)
```

**Arguments**

```
object      An object of class "FuzzySupplementary"  
...        Additional options
```

**Value**

The summary method for class "FuzzySupplementary"

**Examples**

```
#This example is based on the dataset eusilc included in the package  
#The summary of FS index is compute with breakdown and using an HCR = 0.12  
  
FS <- fs_construct_all(data = eusilc[,4:23], weight = eusilc$DB090, # step 2  
                      dimensions = c(1,1,1,1,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5), # step 3  
                      rho = NULL, # steps 4 and 5  
                      HCR = .12, # step 6  
                      breakdown = eusilc$db040) # step 7 with breakdowns  
  
summary(FS)
```

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