

Package ‘GFGM.copula’

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Description Compute bivariate dependence measures and perform bivariate competing risks analysis under the generalized Farlie-Gumbel-Morgenstern (FGM) copula. See Shih and Emura (2018) <[doi:10.1007/s00180-018-0804-0](https://doi.org/10.1007/s00180-018-0804-0)> and Shih and Emura (2019) <[doi:10.1007/s00362-016-0865-5](https://doi.org/10.1007/s00362-016-0865-5)> for details.

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GFGM.copula-package *Generalized Farlie-Gumbel-Morgenstern Copula*

Description

Compute bivariate dependence measures and perform bivariate competing risks analysis under the generalized Farlie-Gumbel-Morgenstern (FGM) copula. See Shih and Emura (2016, 2018) for details.

Details

The functions in this package are based on latent failure time models with competing risks in Shih and Emura (2018). However, they can be adapted to dependent censoring models in Emura and Chen (2018). See `MLE.GFGM.spline` for example.

Author(s)

Jia-Han Shih

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References

Shih J-H, Emura T (2016) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, *Statistical Papers*, doi: 10.1007/s00362-016-0865-5.

Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, *Computational Statistics*, doi: 10.1007/s00180-018-0804-0.

Emura T, Chen Y-H (2018) *Analysis of Survival Data with Dependent Censoring, Copula-Based Approaches*, JSS Research Series in Statistics, Springer, in press.

CvM.GFGM.BurrIII *The Cramer-von Mises type statistics under the generalized FGM copula*

Description

Compute the Cramer-von Mises type statistics under the generalized FGM copula.

Usage

```

CvM.GFGM.BurrIII(
  t.event,
  event1,
  event2,
  Alpha,
  Beta,
  Gamma,
  g1,
  g2,
  p,
  q,
  theta,
  eta = 0,
  Sdist.plot = TRUE
)

```

Arguments

t.event	Vector of the observed failure times.
event1	Vector of the indicators for the failure cause 1.
event2	Vector of the indicators for the failure cause 2.
Alpha	Positive shape parameter for the Burr III margin (failure cause 1).
Beta	Positive shape parameter for the Burr III margin (failure cause 2).
Gamma	Common positive shape parameter for the Burr III margins.
g1	Splines coefficients for the failure cause 1.
g2	Splines coefficients for the failure cause 2.
p	Copula parameter that greater than or equal to 1.
q	Copula parameter that greater than 1 (integer).
theta	Copula parameter with restricted range.
eta	Location parameter with default value 0.
Sdist.plot	Plot sub-distribution functions if TRUE.

Details

The copula parameter q is restricted to be a integer due to the binominal theorem. The admissible range of θ is given in `Dependence.GFGM`.

Value

S.overall	Cramer-von Mises type statistic based on parametric and non-parametric estimators of sub-distribution functions for testing overall model.
S.GFGM	Cramer-von Mises type statistic based on semi-parametric and non-parametric estimators of sub-distribution functions for testing the generalized FGM copula.

References

Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, *Computational Statistics*, 33:1293-1323.

See Also

[Dependence.GFGM](#), [MLE.GFGM.BurrIII](#), [MLE.GFGM.spline](#)

Examples

```
con = c(16,224,16,80,128,168,144,176,176,568,392,576,128,56,112,160,384,600,40,416,
408,384,256,246,184,440,64,104,168,408,304,16,72,8,88,160,48,168,80,512,
208,194,136,224,32,504,40,120,320,48,256,216,168,184,144,224,488,304,40,160,
488,120,208,32,112,288,336,256,40,296,60,208,440,104,528,384,264,360,80,96,
360,232,40,112,120,32,56,280,104,168,56,72,64,40,480,152,48,56,328,192,
168,168,114,280,128,416,392,160,144,208,96,536,400,80,40,112,160,104,224,336,
616,224,40,32,192,126,392,288,248,120,328,464,448,616,168,112,448,296,328,56,
80,72,56,608,144,408,16,560,144,612,80,16,424,264,256,528,56,256,112,544,
552,72,184,240,128,40,600,96,24,184,272,152,328,480,96,296,592,400,8,280,
72,168,40,152,488,480,40,576,392,552,112,288,168,352,160,272,320,80,296,248,
184,264,96,224,592,176,256,344,360,184,152,208,160,176,72,584,144,176)
uncon = c(368,136,512,136,472,96,144,112,104,104,344,246,72,80,312,24,128,304,16,320,
560,168,120,616,24,176,16,24,32,232,32,112,56,184,40,256,160,456,48,24,
200,72,168,288,112,80,584,368,272,208,144,208,114,480,114,392,120,48,104,272,
64,112,96,64,360,136,168,176,256,112,104,272,320,8,440,224,280,8,56,216,
120,256,104,104,8,304,240,88,248,472,304,88,200,392,168,72,40,88,176,216,
152,184,400,424,88,152,184)
cen = rep(630,44)

t.event = c(con,uncon,cen)
event1 = c(rep(1,length(con)),rep(0,length(uncon)),rep(0,length(cen)))
event2 = c(rep(0,length(con)),rep(1,length(uncon)),rep(0,length(cen)))

library(GFGM.copula)
#res.BurrIII = MLE.GFGM.BurrIII(t.event,event1,event2,5000,3,2,0.75,eta = -71)
#Alpha = res.BurrIII$Alpha[1]
#Beta = res.BurrIII$Beta[1]
#Gamma = res.BurrIII$Gamma[1]
#res.spline = MLE.GFGM.spline(t.event,event1,event2,3,2,0.75)
#g1 = res.spline$g1
#g2 = res.spline$g2
#CvM.GFGM.BurrIII(t.event,event1,event2,Alpha,Beta,Gamma,g1,g2,3,2,0.75,eta = -71)
```

Dependence.GFGM

Bivariate dependence measures under the generalized FGM copula

Description

Compute Kendall's tau and Spearman's rho with their boundaries under the generalized FGM copula.

Usage

Dependence.GFGM(p, q, theta)

Arguments

p Copula parameter that greater than or equal to 1.
 q Copula parameter that greater than 1.
 theta Copula parameter with restricted range.

Details

The admissible range of theta (θ) is

$$-\min \left\{ 1, \frac{1}{p^{2q}} \left(\frac{1+pq}{q-1} \right)^{2q-2} \right\} \leq \theta \leq \frac{1}{p^q} \left(\frac{1+pq}{q-1} \right)^{q-1}.$$

See also Shih and Emura (2019) for details.

Value

theta Dependence parameter.
 tau Kendall's tau.
 rho Spearman's rho.

References

Shih J-H, Emura T (2019) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, *Statistical Papers*, 60:1101-1118.

Examples

```
library(GFGM.copula)
Dependence.GFGM(3, 2, 0.75)
```

GFGM.BurrIII	<i>Generate samples from the generalized FGM copula with the Burr III margins</i>
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Description

Generate samples from the generalized FGM copula with the Burr III margins.

Usage

GFGM.BurrIII(n, p, q, theta, Alpha, Beta, Gamma)

Arguments

n	Sample size.
p	Copula parameter that greater than or equal to 1.
q	Copula parameter that greater than 1.
theta	Copula parameter with restricted range.
Alpha	Positive shape parameter for the Burr III margin.
Beta	Positive shape parameter for the Burr III margin.
Gamma	Common positive shape parameter for the Burr III margins.

Details

The admissible range of theta is given in [Dependence.GFGM](#).

Value

X	X is associated with the parameter Alpha.
Y	Y is associated with the parameter Beta.

References

Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, *Computational Statistics*, 33:1293-1323.

Shih J-H, Emura T (2019) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, *Statistical Papers*, 60:1101-1118.

See Also

[Dependence.GFGM](#)

Examples

```
library(GFGM.copula)
GFGM.BurrIII(5, 3, 2, 0.75, 1, 1, 1)
```

MLE.GFGM.BurrIII

Maximum likelihood estimation for bivariate dependent competing risks data under the generalized FGM copula with the Burr III margins

Description

Maximum likelihood estimation for bivariate dependent competing risks data under the generalized FGM copula with the Burr III margins.

Usage

```
MLE.GFGM.BurrIII(
  t.event,
  event1,
  event2,
  D,
  p,
  q,
  theta,
  eta = 0,
  Gamma.0 = 1,
  epsilon.0 = 1e-05,
  epsilon.1 = 1e-10,
  epsilon.2 = 1e-300,
  r.1 = 1,
  r.2 = 1,
  r.3 = 1
)
```

Arguments

t.event	Vector of the observed failure times.
event1	Vector of the indicators for the failure cause 1.
event2	Vector of the indicators for the failure cause 2.
D	Positive tuning parameter in the NR algorithm.
p	Copula parameter that greater than or equal to 1.
q	Copula parameter that greater than 1 (integer).
theta	Copula parameter with restricted range.
eta	Location parameter with default value 0.
Gamma.0	Initial guess for the common shape parameter gamma with default value 1.
epsilon.0	Positive tuning parameter in the NR algorithm with default value 1e-5.
epsilon.1	Positive tuning parameter in the NR algorithm with default value 1e-10.
epsilon.2	Positive tuning parameter in the NR algorithm with default value 1e-300.
r.1	Positive tuning parameter in the NR algorithm with default value 1.
r.2	Positive tuning parameter in the NR algorithm with default value 1.
r.3	Positive tuning parameter in the NR algorithm with default value 1.

Details

The copula parameter q is restricted to be a integer due to the binominal theorem. The admissible range of θ is given in Dependence.GFGM.

Value

n	Sample size.
count	Iteration number.
random	Randomization number.
Alpha	Positive shape parameter for the Burr III margin (failure cause 1).
Beta	Positive shape parameter for the Burr III margin (failure cause 2).
Gamma	Common shape parameter for the Burr III margins.
MeanX	Mean lifetime due to failure cause 1.
MeanY	Mean lifetime due to failure cause 2.
logL	Log-likelihood value under the fitted model.

References

Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, *Computational Statistics*, 33:1293-1323.

Shih J-H, Emura T (2019) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, *Statistical Papers*, 60:1101-1118.

See Also

[Dependence.GFGM](#)

Examples

```

con = c(16,224,16,80,128,168,144,176,176,568,392,576,128,56,112,160,384,600,40,416,
        408,384,256,246,184,440,64,104,168,408,304,16,72,8,88,160,48,168,80,512,
        208,194,136,224,32,504,40,120,320,48,256,216,168,184,144,224,488,304,40,160,
        488,120,208,32,112,288,336,256,40,296,60,208,440,104,528,384,264,360,80,96,
        360,232,40,112,120,32,56,280,104,168,56,72,64,40,480,152,48,56,328,192,
        168,168,114,280,128,416,392,160,144,208,96,536,400,80,40,112,160,104,224,336,
        616,224,40,32,192,126,392,288,248,120,328,464,448,616,168,112,448,296,328,56,
        80,72,56,608,144,408,16,560,144,612,80,16,424,264,256,528,56,256,112,544,
        552,72,184,240,128,40,600,96,24,184,272,152,328,480,96,296,592,400,8,280,
        72,168,40,152,488,480,40,576,392,552,112,288,168,352,160,272,320,80,296,248,
        184,264,96,224,592,176,256,344,360,184,152,208,160,176,72,584,144,176)
uncon = c(368,136,512,136,472,96,144,112,104,104,344,246,72,80,312,24,128,304,16,320,
          560,168,120,616,24,176,16,24,32,232,32,112,56,184,40,256,160,456,48,24,
          200,72,168,288,112,80,584,368,272,208,144,208,114,480,114,392,120,48,104,272,
          64,112,96,64,360,136,168,176,256,112,104,272,320,8,440,224,280,8,56,216,
          120,256,104,104,8,304,240,88,248,472,304,88,200,392,168,72,40,88,176,216,
          152,184,400,424,88,152,184)
cen = rep(630,44)

t.event = c(con,uncon,cen)
event1 = c(rep(1,length(con)),rep(0,length(uncon)),rep(0,length(cen)))
event2 = c(rep(0,length(con)),rep(1,length(uncon)),rep(0,length(cen)))

library(GFGM.copula)
MLE.GFGM.BurrIII(t.event,event1,event2,5000,3,2,0.75,eta = -71)

```

MLE.GFGM.spline	<i>Maximum likelihood estimation for bivariate dependent competing risks data under the generalized FGM copula with the marginal distributions approximated by splines</i>
-----------------	--

Description

Maximum likelihood estimation for bivariate dependent competing risks data under the generalized FGM copula with the marginal distributions approximated by splines.

Usage

```
MLE.GFGM.spline(t.event, event1, event2, p, q, theta, h.plot = TRUE)
```

Arguments

t.event	Vector of the observed failure times.
event1	Vector of the indicators for the failure cause 1.
event2	Vector of the indicators for the failure cause 2.
p	Copula parameter that greater than or equal to 1.
q	Copula parameter that greater than 1 (integer).
theta	Copula parameter with restricted range.
h.plot	Plot hazard functions if TRUE.

Details

The copula parameter q is restricted to be a integer due to the binominal theorem. The admissible range of θ is given in `Dependence.GFGM`.

To adapt our functions to dependent censoring models in Emura and Chen (2018), one can simply set `event2 = 1-event1`.

Value

n	Sample size.
g1	Maximum likelihood estimator of the splines coefficients for the failure cause 1.
g2	Maximum likelihood estimator of the splines coefficients for the failure cause 2.
g1.var	Covariance matrix of splines coefficients estimates for the failure cause 1.
g2.var	Covariance matrix of splines coefficients estimates for the failure cause 2.

References

- Emura T, Chen Y-H (2018) Analysis of Survival Data with Dependent Censoring, Copula-Based Approaches, JSS Research Series in Statistics, Springer, Singapore.
- Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, Computational Statistics, 33:1293-1323.
- Shih J-H, Emura T (2019) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, Statistical Papers, 60:1101-1118.

See Also

[Dependence.GFGM](#)

Examples

```
con = c(16, 224, 16, 80, 128, 168, 144, 176, 176, 568, 392, 576, 128, 56, 112, 160, 384, 600, 40, 416,
        408, 384, 256, 246, 184, 440, 64, 104, 168, 408, 304, 16, 72, 8, 88, 160, 48, 168, 80, 512,
        208, 194, 136, 224, 32, 504, 40, 120, 320, 48, 256, 216, 168, 184, 144, 224, 488, 304, 40, 160,
        488, 120, 208, 32, 112, 288, 336, 256, 40, 296, 60, 208, 440, 104, 528, 384, 264, 360, 80, 96,
        360, 232, 40, 112, 120, 32, 56, 280, 104, 168, 56, 72, 64, 40, 480, 152, 48, 56, 328, 192,
        168, 168, 114, 280, 128, 416, 392, 160, 144, 208, 96, 536, 400, 80, 40, 112, 160, 104, 224, 336,
        616, 224, 40, 32, 192, 126, 392, 288, 248, 120, 328, 464, 448, 616, 168, 112, 448, 296, 328, 56,
        80, 72, 56, 608, 144, 408, 16, 560, 144, 612, 80, 16, 424, 264, 256, 528, 56, 256, 112, 544,
        552, 72, 184, 240, 128, 40, 600, 96, 24, 184, 272, 152, 328, 480, 96, 296, 592, 400, 8, 280,
        72, 168, 40, 152, 488, 480, 40, 576, 392, 552, 112, 288, 168, 352, 160, 272, 320, 80, 296, 248,
        184, 264, 96, 224, 592, 176, 256, 344, 360, 184, 152, 208, 160, 176, 72, 584, 144, 176)
uncon = c(368, 136, 512, 136, 472, 96, 144, 112, 104, 104, 344, 246, 72, 80, 312, 24, 128, 304, 16, 320,
          560, 168, 120, 616, 24, 176, 16, 24, 32, 232, 32, 112, 56, 184, 40, 256, 160, 456, 48, 24,
          200, 72, 168, 288, 112, 80, 584, 368, 272, 208, 144, 208, 114, 480, 114, 392, 120, 48, 104, 272,
          64, 112, 96, 64, 360, 136, 168, 176, 256, 112, 104, 272, 320, 8, 440, 224, 280, 8, 56, 216,
          120, 256, 104, 104, 8, 304, 240, 88, 248, 472, 304, 88, 200, 392, 168, 72, 40, 88, 176, 216,
          152, 184, 400, 424, 88, 152, 184)
cen = rep(630, 44)

t.event = c(con, uncon, cen)
event1 = c(rep(1, length(con)), rep(0, length(uncon)), rep(0, length(cen)))
event2 = c(rep(0, length(con)), rep(1, length(uncon)), rep(0, length(cen)))

library(GFGM.copula)
MLE.GFGM.spline(t.event, event1, event2, 3, 2, 0.75)
```

Sdist.GFGM.BurrIII

Sub-distribution functions under the generalized FGM copula with the Burr III margins

Description

Sub-distribution functions under the generalized FGM copula with the Burr III margins.

Usage

```
Sdist.GFGM.BurrIII(time, Alpha, Beta, Gamma, p, q, theta, eta = 0)
```

Arguments

time	Vector of times.
Alpha	Positive shape parameter for the Burr III margin (failure cause 1).
Beta	Positive shape parameter for the Burr III margin (failure cause 2).
Gamma	Common positive shape parameter for the Burr III margins.
p	Copula parameter that greater than or equal to 1.
q	Copula parameter that greater than 1 (integer).
theta	Copula parameter with restricted range.
eta	Location parameter with default value 0.

Details

The copula parameter q is restricted to be a integer due to the binominal theorem. The admissible range of theta is given in `Dependence.GFGM`.

Value

time	Failure times
Sdist.1	Probability of an object fails due to the failure cause 1.
Sdist.2	Probability of an object fails due to the failure cause 2.

References

Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, *Computational Statistics*, 33:1293-1323.

Shih J-H, Emura T (2019) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, *Statistical Papers*, 60:1101-1118.

See Also

[MLE.GFGM.BurrIII](#), [Dependence.GFGM](#)

Examples

```
library(GFGM.copula)
Sdist.GFGM.BurrIII(c(1:5),1,1,1,3,2,0.75,eta = 1)
```

Sdist.GFGM.spline	<i>Sub-distribution functions under the generalized FGM copula with the marginal distributions approximated by splines</i>
-------------------	--

Description

Sub-distribution functions under the generalized FGM copula with the marginal distributions approximated by splines.

Usage

```
Sdist.GFGM.spline(time, g1, g2, tmin, tmax, p, q, theta)
```

Arguments

time	Vector of times.
g1	Splines coefficients for the failure cause 1.
g2	Splines coefficients for the failure cause 2.
tmin	Lower bound of times.
tmax	upper bound of times.
p	Copula parameter that greater than or equal to 1.
q	Copula parameter that greater than 1 (integer).
theta	Copula parameter with restricted range.

Details

The splines coefficients g_1 and g_2 are usually computed by `MLE.GFGM.spline`. The copula parameter q is restricted to be a integer due to the binominal theorem. The admissible range of θ is given in `Dependence.GFGM`.

Value

time	Failure times
Sdist.1	Probability of an object fails due to the failure cause 1.
Sdist.2	Probability of an object fails due to the failure cause 2.

References

Shih J-H, Emura T (2018) Likelihood-based inference for bivariate latent failure time models with competing risks under the generalized FGM copula, *Computational Statistics*, 33:1293-1323.

Shih J-H, Emura T (2019) Bivariate dependence measures and bivariate competing risks models under the generalized FGM copula, *Statistical Papers*, 60:1101-1118.

See Also

[MLE.GFGM.spline](#), [Dependence.GFGM](#)

Examples

```
library(GFGM.copula)
Sdist.GFGM.spline(seq(1,5,1),rep(0.1,5),rep(0.1,5),1,5,3,2,0.75)
```

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