

# Package ‘KernSmooth’

May 7, 2026

**Priority** recommended

**Version** 2.23-26

**Date** 2024-12-10

**Title** Functions for Kernel Smoothing Supporting Wand & Jones (1995)

**Note** Maintainers are not available to give advice on using a package they did not author.

**Depends** R ( $\geq 2.5.0$ ), stats

**Suggests** MASS, carData

**Description** Functions for kernel smoothing (and density estimation) corresponding to the book:  
Wand, M.P. and Jones, M.C. (1995) ``Kernel Smoothing".

**License** Unlimited

**ByteCompile** yes

**NeedsCompilation** yes

**Author** Matt Wand [aut],  
Cleve Moler [ctb] (LINPACK routines in src/d\*),  
Brian Ripley [trl, cre, ctb] (R port and updates)

**Maintainer** Brian Ripley <Brian.Ripley@R-project.org>

**Repository** CRAN

**Date/Publication** 2025-01-01 10:25:36 UTC

## Contents

bkde . . . . .	2
bkde2D . . . . .	3
bkfe . . . . .	4
dpih . . . . .	6
dpik . . . . .	7
dpill . . . . .	9
locpoly . . . . .	10

<b>Index</b>	<b>13</b>
--------------	-----------

---

 bkde

---

*Compute a Binned Kernel Density Estimate*


---

### Description

Returns x and y coordinates of the binned kernel density estimate of the probability density of the data.

### Usage

```
bkde(x, kernel = "normal", canonical = FALSE, bandwidth,
     gridsize = 401L, range.x, truncate = TRUE)
```

### Arguments

x	numeric vector of observations from the distribution whose density is to be estimated. Missing values are not allowed.
bandwidth	the kernel bandwidth smoothing parameter. Larger values of bandwidth make smoother estimates, smaller values of bandwidth make less smooth estimates. The default is a bandwidth computed from the variance of x, specifically the ‘oversmoothed bandwidth selector’ of Wand and Jones (1995, page 61).
kernel	character string which determines the smoothing kernel. kernel can be: "normal" - the Gaussian density function (the default). "box" - a rectangular box. "epanech" - the centred beta(2,2) density. "biweight" - the centred beta(3,3) density. "triweight" - the centred beta(4,4) density. This can be abbreviated to any unique abbreviation.
canonical	length-one logical vector: if TRUE, canonically scaled kernels are used.
gridsize	the number of equally spaced points at which to estimate the density.
range.x	vector containing the minimum and maximum values of x at which to compute the estimate. The default is the minimum and maximum data values, extended by the support of the kernel.
truncate	logical flag: if TRUE, data with x values outside the range specified by range.x are ignored.

### Details

This is the binned approximation to the ordinary kernel density estimate. Linear binning is used to obtain the bin counts. For each x value in the sample, the kernel is centered on that x and the heights of the kernel at each datapoint are summed. This sum, after a normalization, is the corresponding y value in the output.

### Value

a list containing the following components:

x	vector of sorted x values at which the estimate was computed.
y	vector of density estimates at the corresponding x.

## Background

Density estimation is a smoothing operation. Inevitably there is a trade-off between bias in the estimate and the estimate's variability: large bandwidths will produce smooth estimates that may hide local features of the density; small bandwidths may introduce spurious bumps into the estimate.

## References

Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing*. Chapman and Hall, London.

## See Also

[density](#), [dpik](#), [hist](#), [ksmooth](#).

## Examples

```
data(geyser, package="MASS")
x <- geyser$duration
est <- bkde(x, bandwidth=0.25)
plot(est, type="l")
```

---

 bkde2D

---

*Compute a 2D Binned Kernel Density Estimate*


---

## Description

Returns the set of grid points in each coordinate direction, and the matrix of density estimates over the mesh induced by the grid points. The kernel is the standard bivariate normal density.

## Usage

```
bkde2D(x, bandwidth, gridsize = c(51L, 51L), range.x, truncate = TRUE)
```

## Arguments

x	a two-column numeric matrix containing the observations from the distribution whose density is to be estimated. Missing values are not allowed.
bandwidth	numeric vector of length 2, containing the bandwidth to be used in each coordinate direction.
gridsize	vector containing the number of equally spaced points in each direction over which the density is to be estimated.
range.x	a list containing two vectors, where each vector contains the minimum and maximum values of x at which to compute the estimate for each direction. The default minimum in each direction is minimum data value minus 1.5 times the bandwidth for that direction. The default maximum is the maximum data value plus 1.5 times the bandwidth for that direction.
truncate	logical flag: if TRUE, data with x values outside the range specified by range.x are ignored.

**Value**

a list containing the following components:

x1	vector of values of the grid points in the first coordinate direction at which the estimate was computed.
x2	vector of values of the grid points in the second coordinate direction at which the estimate was computed.
fhat	matrix of density estimates over the mesh induced by x1 and x2.

**Details**

This is the binned approximation to the 2D kernel density estimate. Linear binning is used to obtain the bin counts and the Fast Fourier Transform is used to perform the discrete convolutions. For each x1,x2 pair the bivariate Gaussian kernel is centered on that location and the heights of the kernel, scaled by the bandwidths, at each datapoint are summed. This sum, after a normalization, is the corresponding fhat value in the output.

**References**

Wand, M. P. (1994). Fast Computation of Multivariate Kernel Estimators. *Journal of Computational and Graphical Statistics*, **3**, 433-445.

Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing*. Chapman and Hall, London.

**See Also**

[bkde](#), [density](#), [hist](#).

**Examples**

```
data(geyser, package="MASS")
x <- cbind(geyser$duration, geyser$waiting)
est <- bkde2D(x, bandwidth=c(0.7, 7))
contour(est$x1, est$x2, est$fhat)
persp(est$fhat)
```

---

 bkfe

---

*Compute a Binned Kernel Functional Estimate*


---

**Description**

Returns an estimate of a binned approximation to the kernel estimate of the specified density functional. The kernel is the standard normal density.

**Usage**

```
bkfe(x, drv, bandwidth, gridsize = 401L, range.x, binned = FALSE,
      truncate = TRUE)
```

**Arguments**

x	numeric vector of observations from the distribution whose density is to be estimated. Missing values are not allowed.
drv	order of derivative in the density functional. Must be a non-negative even integer.
bandwidth	the kernel bandwidth smoothing parameter. Must be supplied.
gridsize	the number of equally-spaced points over which binning is performed.
range.x	vector containing the minimum and maximum values of x at which to compute the estimate. The default is the minimum and maximum data values, extended by the support of the kernel.
binned	logical flag: if TRUE, then x and y are taken to be grid counts rather than raw data.
truncate	logical flag: if TRUE, data with x values outside the range specified by range.x are ignored.

**Details**

The density functional of order `drv` is the integral of the product of the density and its `drv`th derivative. The kernel estimates of such quantities are computed using a binned implementation, and the kernel is the standard normal density.

**Value**

the (scalar) estimated functional.

**Background**

Estimates of this type were proposed by Sheather and Jones (1991).

**References**

Sheather, S. J. and Jones, M. C. (1991). A reliable data-based bandwidth selection method for kernel density estimation. *Journal of the Royal Statistical Society, Series B*, **53**, 683–690.

Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing*. Chapman and Hall, London.

**Examples**

```
data(geyser, package="MASS")
x <- geyser$duration
est <- bkfe(x, drv=4, bandwidth=0.3)
```

---

 dpih

*Select a Histogram Bin Width*


---

### Description

Uses direct plug-in methodology to select the bin width of a histogram.

### Usage

```
dpih(x, scalest = "minim", level = 2L, gridsize = 401L,
     range.x = range(x), truncate = TRUE)
```

### Arguments

x	numeric vector containing the sample on which the histogram is to be constructed.
scalest	estimate of scale. "stdev" - standard deviation is used. "iqr" - inter-quartile range divided by 1.349 is used. "minim" - minimum of "stdev" and "iqr" is used.
level	number of levels of functional estimation used in the plug-in rule.
gridsize	number of grid points used in the binned approximations to functional estimates.
range.x	range over which functional estimates are obtained. The default is the minimum and maximum data values.
truncate	if truncate is TRUE then observations outside of the interval specified by range.x are omitted. Otherwise, they are used to weight the extreme grid points.

### Details

The direct plug-in approach, where unknown functionals that appear in expressions for the asymptotically optimal bin width and bandwidths are replaced by kernel estimates, is used. The normal distribution is used to provide an initial estimate.

### Value

the selected bin width.

### Background

This method for selecting the bin width of a histogram is described in Wand (1995). It is an extension of the normal scale rule of Scott (1979) and uses plug-in ideas from bandwidth selection for kernel density estimation (e.g. Sheather and Jones, 1991).

**References**

- Scott, D. W. (1979). On optimal and data-based histograms. *Biometrika*, **66**, 605–610.
- Sheather, S. J. and Jones, M. C. (1991). A reliable data-based bandwidth selection method for kernel density estimation. *Journal of the Royal Statistical Society, Series B*, **53**, 683–690.
- Wand, M. P. (1995). Data-based choice of histogram binwidth. *The American Statistician*, **51**, 59–64.

**See Also**

[hist](#)

**Examples**

```
data(geyser, package="MASS")
x <- geyser$duration
h <- dpik(x)
bins <- seq(min(x)-h, max(x)+h, by=h)
hist(x, breaks=bins)
```

---

 dpik

*Select a Bandwidth for Kernel Density Estimation*


---

**Description**

Use direct plug-in methodology to select the bandwidth of a kernel density estimate.

**Usage**

```
dpik(x, scalest = "minim", level = 2L, kernel = "normal",
     canonical = FALSE, gridsize = 401L, range.x = range(x),
     truncate = TRUE)
```

**Arguments**

x	numeric vector containing the sample on which the kernel density estimate is to be constructed.
scalest	estimate of scale. "stdev" - standard deviation is used. "iqr" - inter-quartile range divided by 1.349 is used. "minim" - minimum of "stdev" and "iqr" is used.
level	number of levels of functional estimation used in the plug-in rule.
kernel	character string which determines the smoothing kernel. kernel can be: "normal" - the Gaussian density function (the default). "box" - a rectangular box. "epanech" - the centred beta(2,2) density. "biweight" - the centred beta(3,3) density. "triweight" - the centred beta(4,4) density. This can be abbreviated to any unique abbreviation.

canonical	logical flag: if TRUE, canonically scaled kernels are used
gridsize	the number of equally-spaced points over which binning is performed to obtain kernel functional approximation.
range.x	vector containing the minimum and maximum values of $x$ at which to compute the estimate. The default is the minimum and maximum data values.
truncate	logical flag: if TRUE, data with $x$ values outside the range specified by <code>range.x</code> are ignored.

### Details

The direct plug-in approach, where unknown functionals that appear in expressions for the asymptotically optimal bandwidths are replaced by kernel estimates, is used. The normal distribution is used to provide an initial estimate.

### Value

the selected bandwidth.

### Background

This method for selecting the bandwidth of a kernel density estimate was proposed by Sheather and Jones (1991) and is described in Section 3.6 of Wand and Jones (1995).

### References

Sheather, S. J. and Jones, M. C. (1991). A reliable data-based bandwidth selection method for kernel density estimation. *Journal of the Royal Statistical Society, Series B*, **53**, 683–690.

Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing*. Chapman and Hall, London.

### See Also

[bkde](#), [density](#), [ksmooth](#)

### Examples

```
data(geyser, package="MASS")
x <- geyser$duration
h <- dpik(x)
est <- bkde(x, bandwidth=h)
plot(est, type="l")
```

---

 dpill

*Select a Bandwidth for Local Linear Regression*


---

### Description

Use direct plug-in methodology to select the bandwidth of a local linear Gaussian kernel regression estimate, as described by Ruppert, Sheather and Wand (1995).

### Usage

```
dpill(x, y, blockmax = 5, divisor = 20, trim = 0.01, proptrun = 0.05,
      gridsize = 401L, range.x, truncate = TRUE)
```

### Arguments

x	numeric vector of x data. Missing values are not accepted.
y	numeric vector of y data. This must be same length as x, and missing values are not accepted.
blockmax	the maximum number of blocks of the data for construction of an initial parametric estimate.
divisor	the value that the sample size is divided by to determine a lower limit on the number of blocks of the data for construction of an initial parametric estimate.
trim	the proportion of the sample trimmed from each end in the x direction before application of the plug-in methodology.
proptrun	the proportion of the range of x at each end truncated in the functional estimates.
gridsize	number of equally-spaced grid points over which the function is to be estimated.
range.x	vector containing the minimum and maximum values of x at which to compute the estimate. For density estimation the default is the minimum and maximum data values with 5% of the range added to each end. For regression estimation the default is the minimum and maximum data values.
truncate	logical flag: if TRUE, data with x values outside the range specified by range.x are ignored.

### Details

The direct plug-in approach, where unknown functionals that appear in expressions for the asymptotically optimal bandwidths are replaced by kernel estimates, is used. The kernel is the standard normal density. Least squares quartic fits over blocks of data are used to obtain an initial estimate. Mallows's  $C_p$  is used to select the number of blocks.

### Value

the selected bandwidth.

**Warning**

If there are severe irregularities (i.e. outliers, sparse regions) in the x values then the local polynomial smooths required for the bandwidth selection algorithm may become degenerate and the function will crash. Outliers in the y direction may lead to deterioration of the quality of the selected bandwidth.

**References**

Ruppert, D., Sheather, S. J. and Wand, M. P. (1995). An effective bandwidth selector for local least squares regression. *Journal of the American Statistical Association*, **90**, 1257–1270.

Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing*. Chapman and Hall, London.

**See Also**

[ksmooth](#), [locpoly](#).

**Examples**

```
data(geyser, package = "MASS")
x <- geyser$duration
y <- geyser$waiting
plot(x, y)
h <- dpill(x, y)
fit <- locpoly(x, y, bandwidth = h)
lines(fit)
```

---

locpoly

---

*Estimate Functions Using Local Polynomials*


---

**Description**

Estimates a probability density function, regression function or their derivatives using local polynomials. A fast binned implementation over an equally-spaced grid is used.

**Usage**

```
locpoly(x, y, drv = 0L, degree, kernel = "normal",
        bandwidth, gridsize = 401L, bwdisc = 25,
        range.x, binned = FALSE, truncate = TRUE)
```

**Arguments**

x                    numeric vector of x data. Missing values are not accepted.

bandwidth           the kernel bandwidth smoothing parameter. It may be a single number or an array having length `gridsize`, representing a bandwidth that varies according to the location of estimation.

y	vector of y data. This must be same length as x, and missing values are not accepted.
drv	order of derivative to be estimated.
degree	degree of local polynomial used. Its value must be greater than or equal to the value of drv. The default value is of degree is drv + 1.
kernel	"normal" - the Gaussian density function. Currently ignored.
gridsize	number of equally-spaced grid points over which the function is to be estimated.
bwdisc	number of logarithmically-equally-spaced bandwidths on which bandwidth is discretised, to speed up computation.
range.x	vector containing the minimum and maximum values of x at which to compute the estimate.
binned	logical flag: if TRUE, then x and y are taken to be grid counts rather than raw data.
truncate	logical flag: if TRUE, data with x values outside the range specified by range.x are ignored.

### Value

if y is specified, a local polynomial regression estimate of  $E[Y|X]$  (or its derivative) is computed. If y is missing, a local polynomial estimate of the density of x (or its derivative) is computed.

a list containing the following components:

x	vector of sorted x values at which the estimate was computed.
y	vector of smoothed estimates for either the density or the regression at the corresponding x.

### Details

Local polynomial fitting with a kernel weight is used to estimate either a density, regression function or their derivatives. In the case of density estimation, the data are binned and the local fitting procedure is applied to the bin counts. In either case, binned approximations over an equally-spaced grid is used for fast computation. The bandwidth may be either scalar or a vector of length `gridsize`.

### References

Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing*. Chapman and Hall, London.

### See Also

[bkde](#), [density](#), [dpill](#), [ksmooth](#), [loess](#), [smooth](#), [supsmu](#).

**Examples**

```
data(geyser, package = "MASS")
# local linear density estimate
x <- geyser$duration
est <- locpoly(x, bandwidth = 0.25)
plot(est, type = "l")

# local linear regression estimate
y <- geyser$waiting
plot(x, y)
fit <- locpoly(x, y, bandwidth = 0.25)
lines(fit)
```

# Index

**\* distribution**

bkde, 2  
bkde2D, 3

**\* regression**

locpoly, 10

**\* smooth**

bkde, 2  
bkde2D, 3  
bkfe, 4  
dpih, 6  
dpik, 7  
dpill, 9  
locpoly, 10

bkde, 2, 4, 8, 11

bkde2D, 3

bkfe, 4

density, 3, 4, 8, 11

dpih, 6

dpik, 3, 7

dpill, 9, 11

hist, 3, 4, 7

ksmooth, 3, 8, 10, 11

locpoly, 10, 10

loess, 11

smooth, 11

supsmu, 11