

# Package ‘Kernelheaping’

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**Type** Package

**Title** Kernel Density Estimation for Heaped and Rounded Data

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mvtnorm

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**Description** In self-reported or anonymised data the user often encounters heaped data, i.e. data which are rounded (to a possibly different degree of coarseness). While this is mostly a minor problem in parametric density estimation the bias can be very large for non-parametric methods such as kernel density estimation. This package implements a partly Bayesian algorithm treating the true unknown values as additional parameters and estimates the rounding parameters to give a corrected kernel density estimate. It supports various standard bandwidth selection methods. Varying rounding probabilities (depending on the true value) and asymmetric rounding is estimable as well: Gross, M. and Rendtel, U. (2016) (<[doi:10.1093/jssam/smw011](https://doi.org/10.1093/jssam/smw011)>). Additionally, bivariate non-parametric density estimation for rounded data, Gross, M. et al. (2016) (<[doi:10.1111/rssa.12179](https://doi.org/10.1111/rssa.12179)>), as well as data aggregated on areas is supported.

**License** GPL-2 | GPL-3

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---

createSim.Kernelheaping  
*Create heaped data for Simulation*

---

### Description

Create heaped data for Simulation

### Usage

```
createSim.Kernelheaping(
  n,
  distribution,
  rounds,
  thresholds,
  offset = 0,
  downbias = 0.5,
  Beta = 0,
  ...
)
```

### Arguments

n	sample size
distribution	name of the distribution where random sampling is available, e.g. "norm"
rounds	rounding values

thresholds	rounding thresholds (for Beta=0)
offset	certain value added to all observed random samples
downbias	bias parameter
Beta	acceleration paramter
...	additional attributes handed over to "rdistribution" (i.e. rnorm, rgamma,..)

**Value**

List of heaped values, true values and input parameters

---

dbivr	<i>Bivariate kernel density estimation for rounded data</i>
-------	---

---

**Description**

Bivariate kernel density estimation for rounded data

**Usage**

```
dbivr(
  xrounded,
  roundvalue,
  burnin = 2,
  samples = 5,
  adaptive = FALSE,
  gridsize = 200
)
```

**Arguments**

xrounded	rounded values from which to estimate bivariate density, matrix with 2 columns (x,y)
roundvalue	rounding value (side length of square in that the true value lies around the rounded one)
burnin	burn-in sample size
samples	sampling iteration size
adaptive	set to TRUE for adaptive bandwidth
gridsize	number of evaluation grid points

**Value**

The function returns a list object with the following objects (besides all input objects):

Mestimates	kde object containing the corrected density estimate
gridx	Vector Grid on which density is evaluated (x)
gridy	Vector Grid on which density is evaluated (y)
resultDensity	Array with Estimated Density for each iteration
resultX	Matrix of true latent values X estimates
delaigle	Matrix of Delaigle estimator estimates

**Examples**

```
# Create Mu and Sigma -----
mu1 <- c(0, 0)
mu2 <- c(5, 3)
mu3 <- c(-4, 1)
Sigma1 <- matrix(c(4, 3, 3, 4), 2, 2)
Sigma2 <- matrix(c(3, 0.5, 0.5, 1), 2, 2)
Sigma3 <- matrix(c(5, 4, 4, 6), 2, 2)
# Mixed Normal Distribution -----
mus <- rbind(mu1, mu2, mu3)
Sigmas <- rbind(Sigma1, Sigma2, Sigma3)
props <- c(1/3, 1/3, 1/3)
## Not run: xtrue=rmvnorm.mixt(n=1000, mus=mus, Sigmas=Sigmas, props=props)
roundvalue=2
xrounded=plyr::round_any(xtrue,roundvalue)
est <- dbivr(xrounded,roundvalue=roundvalue,burnin=5,samples=10)

#Plot corrected and Naive distribution
plot(est,trueX=xtrue)
#for comparison: plot true density
dens=dmvnorm.mixt(x=expand.grid(est$Mestimates$eval.points[[1]],est$Mestimates$eval.points[[2]]),
  mus=mus, Sigmas=Sigmas, props=props)
dens=matrix(dens,nrow=length(est$gridx),ncol=length(est$gridy))
contour(dens,x=est$Mestimates$eval.points[[1]],y=est$Mestimates$eval.points[[2]],
  xlim=c(min(est$gridx),max(est$gridx)),ylim=c(min(est$gridy),max(est$gridy)),main="True Density")
## End(Not run)
```

---

dclass

*Kernel density estimation for classified data*


---

**Description**

Kernel density estimation for classified data

**Usage**

```
dclass(
  xclass,
  burnin = 2,
  samples = 5,
  boundary = FALSE,
  bw = "nrd0",
  evalpoints = 200,
  adjust = 1,
  dFunc = NULL
)
```

**Arguments**

xclass	classified values; matrix with two columns: lower and upper value
burnin	burn-in sample size
samples	sampling iteration size
boundary	TRUE for positive only data (no positive density for negative values)
bw	bandwidth selector method, defaults to "nrd0" see density for more options
evalpoints	number of evaluation grid points
adjust	as in density, the user can multiply the bandwidth by a certain factor such that bw=adjust*bw
dFunc	character optional density (with "d", "p" and "q" functions) function name for parametric estimation such as "norm" "gamma" or "lnorm"

**Value**

The function returns a list object with the following objects (besides all input objects):

Mestimates	kde object containing the corrected density estimate
gridx	Vector Grid on which density is evaluated
resultDensity	Matrix with Estimated Density for each iteration
resultX	Matrix of true latent values X estimates

**Examples**

```
x=rlnorm(500, meanlog = 8, sdlog = 1)
classes <- c(0,500,1000,1500,2000,2500,3000,4000,5000,6000,8000,10000,15000,Inf)
xclass <- cut(x,breaks=classes)
xclass <- cbind(classes[as.numeric(xclass)], classes[as.numeric(xclass) + 1])
densityEst <- dclass(xclass=xclass, burnin=20, samples=50, evalpoints=1000)
plot(densityEst$Mestimates~densityEst$gridx ,lwd=2, type = "l")
```

dheaping

*Kernel density estimation for heaped data***Description**

Kernel density estimation for heaped data

**Usage**

```
dheaping(
  xheaped,
  rounds,
  burnin = 5,
  samples = 10,
  setBias = FALSE,
  weights = NULL,
  bw = "nrd0",
  boundary = FALSE,
  unequal = FALSE,
  random = FALSE,
  adjust = 1,
  recall = F,
  recallParams = c(1/3, 1/3)
)
```

**Arguments**

xheaped	heaped values from which to estimate density of x
rounds	rounding values, numeric vector of length $\geq 1$
burnin	burn-in sample size
samples	sampling iteration size
setBias	if TRUE a rounding Bias parameter is estimated. For values above 0.5, the respondents are more prone to round down, while for values $< 0.5$ they are more likely to round up
weights	optional numeric vector of sampling weights
bw	bandwidth selector method, defaults to "nrd0" see <code>density</code> for more options
boundary	TRUE for positive only data (no positive density for negative values)
unequal	if TRUE a probit model is fitted for the rounding probabilities with $\log(\text{true value})$ as regressor
random	if TRUE a random effect probit model is fitted for rounding probabilities
adjust	as in <code>density</code> , the user can multiply the bandwidth by a certain factor such that $\text{bw} = \text{adjust} * \text{bw}$
recall	if TRUE a recall error is introduced to the heaping model
recallParams	recall error model parameters expression( $\nu$ ) and expression( $\eta$ ). Default is $c(1/3, 1/3)$

**Value**

The function returns a list object with the following objects (besides all input objects):

meanPostDensity	Vector of Mean Posterior Density
gridx	Vector Grid on which density is evaluated
resultDensity	Matrix with Estimated Density for each iteration
resultRR	Matrix with rounding probability threshold values for each iteration (on probit scale)
resultBias	Vector with estimated Bias parameter for each iteration
resultBeta	Vector with estimated Beta parameter for each iteration
resultX	Matrix of true latent values X estimates

**Examples**

```
#Simple Rounding -----
xtrue=rnorm(3000)
xrounded=round(xtrue)
est <- dheaping(xrounded,rounds=1,burnin=20,samples=50)
plot(est,trueX=xtrue)

#####
####Heaping
#####

#Real Data Example -----
# Student learning hours per week
data(students)
xheaped <- as.numeric(na.omit(students$StudyHrs))
## Not run: est <- dheaping(xheaped,rounds=c(1,2,5,10), boundary=TRUE, unequal=TRUE,burnin=20,samples=50)
plot(est)
summary(est)
## End(Not run)

#Simulate Data -----
Sim1 <- createSim.Kernelheaping(n=500, distribution="norm",rounds=c(1,10,100),
thresholds=c(-0.5244005, 0.5244005), sd=100)
## Not run: est <- dheaping(Sim1$xheaped,rounds=Sim1$rounds)
plot(est,trueX=Sim1$x)
## End(Not run)

#Biased rounding
Sim2 <- createSim.Kernelheaping(n=500, distribution="gamma",rounds=c(1,2,5,10),
thresholds=c(-1.2815516, -0.6744898, 0.3853205),downbias=0.2,
shape=4,scale=8,offset=45)
## Not run: est <- dheaping(Sim2$xheaped, rounds=Sim2$rounds, setBias=T, bw="SJ")
plot(est, trueX=Sim2$x)
summary(est)
tracePlots(est)
## End(Not run)
```

```

Sim3 <- createSim.Kernelheaping(n=500, distribution="gamma", rounds=c(1,2,5,10),
thresholds=c(1.84, 2.64, 3.05), downbias=0.75, Beta=-0.5, shape=4, scale=8)
## Not run: est <- dheaping(Sim3$xheaped, rounds=Sim3$rounds, boundary=TRUE, unequal=TRUE, setBias=T)
plot(est, trueX=Sim3$x)
## End(Not run)

```

---

dshape3dProp

*3d Kernel density estimation for data classified in polygons or shapes*


---

## Description

3d Kernel density estimation for data classified in polygons or shapes

## Usage

```

dshape3dProp(
  data,
  burnin = 2,
  samples = 5,
  shapefile,
  gridsize = 200,
  boundary = FALSE,
  deleteShapes = NULL,
  fastWeights = TRUE,
  numChains = 1,
  numThreads = 1
)

```

## Arguments

data	data.frame with 5 columns: x-coordinate, y-coordinate (i.e. center of polygon) and number of observations in area for partial population and number of observations for complete observations and third variable (numeric).
burnin	burn-in sample size
samples	sampling iteration size
shapefile	shapefile with number of polygons equal to <code>nrow(data) / length(unique(data[,5]))</code>
gridsize	number of evaluation grid points
boundary	boundary corrected kernel density estimate?
deleteShapes	shapefile containing areas without observations
fastWeights	if TRUE weights for boundary estimation are only computed for first 10 percent of samples to speed up computation
numChains	number of chains of SEM algorithm
numThreads	number of threads to be used (only applicable if more than one chains)

---

dshapebivr	<i>Bivariate Kernel density estimation for data classified in polygons or shapes</i>
------------	--

---

## Description

Bivariate Kernel density estimation for data classified in polygons or shapes

## Usage

```
dshapebivr(
  data,
  burnin = 2,
  samples = 5,
  adaptive = FALSE,
  shapefile,
  gridsize = 200,
  boundary = FALSE,
  deleteShapes = NULL,
  fastWeights = TRUE,
  numChains = 1,
  numThreads = 1
)
```

## Arguments

data	data.frame with 3 columns: x-coordinate, y-coordinate (i.e. center of polygon) and number of observations in area.
burnin	burn-in sample size
samples	sampling iteration size
adaptive	TRUE for adaptive kernel density estimation
shapefile	shapefile with number of polygons equal to nrow(data)
gridsize	number of evaluation grid points
boundary	boundary corrected kernel density estimate?
deleteShapes	shapefile containing areas without observations
fastWeights	if TRUE weights for boundary estimation are only computed for first 10 percent of samples to speed up computation
numChains	number of chains of SEM algorithm
numThreads	number of threads to be used (only applicable if more than one chains)

**Value**

The function returns a list object with the following objects (besides all input objects):

Mestimates	kde object containing the corrected density estimate
gridx	Vector Grid of x-coordinates on which density is evaluated
gridy	Vector Grid of y-coordinates on which density is evaluated
resultDensity	Matrix with Estimated Density for each iteration
resultX	Matrix of true latent values X estimates

**Examples**

```
## Not run:
library(maptools)

# Read Shapefile of Berlin Urban Planning Areas (download available from:
# https://www.statistik-berlin-brandenburg.de/opendata/RBS_OD_LOR_2015_12.zip)
Berlin <- rgdal::readOGR("X:/SomeDir/RBS_OD_LOR_2015_12.shp") #(von daten.berlin.de)

# Get Dataset of Berlin Population (download available from:
# https://www.statistik-berlin-brandenburg.de/opendata/EWR201512E_Matrix.csv)
data <- read.csv2("X:/SomeDir/EWR201512E_Matrix.csv")

# Form Dataset for Estimation Process
dataIn <- cbind(t(sapply(1:length(Berlin@polygons),
  function(x) Berlin@polygons[[x]]@labpt)), data$E_E65U80)

#Estimate Bivariate Density
Est <- dshapebivr(data = dataIn, burnin = 5, samples = 10, adaptive = FALSE,
  shapefile = Berlin, gridsize = 325, boundary = TRUE)
## End(Not run)

# Plot Density over Area:
## Not run: breaks <- seq(1E-16,max(Est$Mestimates$estimate),length.out = 20)
image.plot(x=Est$Mestimates$eval.points[[1]],y=Est$Mestimates$eval.points[[2]],
  z=Est$Mestimates$estimate, asp=1, breaks = breaks,
  col = colorRampPalette(brewer.pal(9,"YlOrRd"))(length(breaks)-1))
plot(Berlin, add=TRUE)
## End(Not run)
```

---

dshapebivrProp

*Bivariate Kernel density estimation for data classified in polygons or shapes*


---

**Description**

Bivariate Kernel density estimation for data classified in polygons or shapes

**Usage**

```
dshapebivrProp(
  data,
  burnin = 2,
  samples = 5,
  adaptive = FALSE,
  shapefile,
  gridsize = 200,
  boundary = FALSE,
  deleteShapes = NULL,
  fastWeights = TRUE,
  numChains = 1,
  numThreads = 1
)
```

**Arguments**

data	data.frame with 4 columns: x-coordinate, y-coordinate (i.e. center of polygon) and number of observations in area for partial population and number of observations for complete observations.
burnin	burn-in sample size
samples	sampling iteration size
adaptive	TRUE for adaptive kernel density estimation
shapefile	shapefile with number of polygons equal to nrow(data)
gridsize	number of evaluation grid points
boundary	boundary corrected kernel density estimate?
deleteShapes	shapefile containing areas without observations
fastWeights	if TRUE weights for boundary estimation are only computed for first 10 percent of samples to speed up computation
numChains	number of chains of SEM algorithm
numThreads	number of threads to be used (only applicable if more than one chains)

**Examples**

```
## Not run:
library(maptools)

# Read Shapefile of Berlin Urban Planning Areas (download available from:
# https://www.statistik-berlin-brandenburg.de/opendata/RBS_OD_LOR_2015_12.zip)
Berlin <- rgdal::readOGR("X:/SomeDir/RBS_OD_LOR_2015_12.shp") #(von daten.berlin.de)

# Get Dataset of Berlin Population (download available from:
# https://www.statistik-berlin-brandenburg.de/opendata/EWR201512E_Matrix.csv)
data <- read.csv2("X:/SomeDir/EWR201512E_Matrix.csv")

# Form Dataset for Estimation Process
```

```

dataIn <- cbind(t(sapply(1:length(Berlin@polygons),
function(x) Berlin@polygons[[x]]@labpt)), data$E_E65U80, data$E_E)

#Estimate Bivariate Proportions (may take some minutes)
PropEst <- dshapebivrProp(data = dataIn, burnin = 5, samples = 20, adaptive = FALSE,
shapefile = Berlin, gridsize=325, numChains = 16, numThreads = 4)
## End(Not run)

# Plot Proportions over Area:
## Not run:
breaks <- seq(0,0.4,by=0.025)
image.plot(x=PropEst$Mestimates$eval.points[[1]],y=PropEst$Mestimates$eval.points[[2]],
           z=PropEst$proportion+1E-96, asp=1, breaks = breaks,
           col = colorRampPalette(brewer.pal(9,"YlOrRd"))(length(breaks)-1))
plot(Berlin, add=TRUE)
## End(Not run)

```

---

Kernelheaping

*Kernel Density Estimation for Heaped Data*


---

### Description

In self-reported or anonymized data the user often encounters heaped data, i.e. data which are rounded (to a possibly different degree of coarseness). While this is mostly a minor problem in parametric density estimation the bias can be very large for non-parametric methods such as kernel density estimation. This package implements a partly Bayesian algorithm treating the true unknown values as additional parameters and estimates the rounding parameters to give a corrected kernel density estimate. It supports various standard bandwidth selection methods. Varying rounding probabilities (depending on the true value) and asymmetric rounding is estimable as well. Additionally, bivariate non-parametric density estimation for rounded data is supported.

### Details

The most important function is [dheaping](#). See the help and the attached examples on how to use the package.

---

plot.bivrounding

*Plot Kernel density estimate of heaped data naively and corrected by partly bayesian model*


---

### Description

Plot Kernel density estimate of heaped data naively and corrected by partly bayesian model

### Usage

```

## S3 method for class 'bivrounding'
plot(x, trueX = NULL, ...)

```

**Arguments**

x	bivrounding object produced by dbivr function
trueX	optional, if true values X are known (in simulations, for example) the 'Oracle' density estimate is added as well
...	additional arguments given to standard plot function

**Value**

plot with Kernel density estimates (Naive, Corrected and True (if provided))

---

plot.Kernelheaping	<i>Plot Kernel density estimate of heaped data naively and corrected by partly bayesian model</i>
--------------------	---

---

**Description**

Plot Kernel density estimate of heaped data naively and corrected by partly bayesian model

**Usage**

```
## S3 method for class 'Kernelheaping'
plot(x, trueX = NULL, ...)
```

**Arguments**

x	Kernelheaping object produced by dheaping function
trueX	optional, if true values X are known (in simulations, for example) the 'Oracle' density estimate is added as well
...	additional arguments given to standard plot function

**Value**

plot with Kernel density estimates (Naive, Corrected and True (if provided))

---

sim.Kernelheaping      *Simulation of heaping correction method*

---

### Description

Simulation of heaping correction method

### Usage

```
sim.Kernelheaping(
  simRuns,
  n,
  distribution,
  rounds,
  thresholds,
  downbias = 0.5,
  setBias = FALSE,
  Beta = 0,
  unequal = FALSE,
  burnin = 5,
  samples = 10,
  bw = "nrd0",
  offset = 0,
  boundary = FALSE,
  adjust = 1,
  ...
)
```

### Arguments

simRuns	number of simulations runs
n	sample size
distribution	name of the distribution where random sampling is available, e.g. "norm"
rounds	rounding values, numeric vector of length $\geq 1$
thresholds	rounding thresholds
downbias	Bias parameter used in the simulation
setBias	if TRUE a rounding Bias parameter is estimated. For values above 0.5, the respondents are more prone to round down, while for values $< 0.5$ they are more likely to round up
Beta	Parameter of the probit model for rounding probabilities used in simulation
unequal	if TRUE a probit model is fitted for the rounding probabilities with $\log(\text{true value})$ as regressor
burnin	burn-in sample size
samples	sampling iteration size

bw	bandwidth selector method, defaults to "nrd0" see density for more options
offset	location shift parameter used simulation in simulation
boundary	TRUE for positive only data (no positive density for negative values)
adjust	as in density, the user can multiply the bandwidth by a certain factor such that bw=adjust*bw
...	additional attributes handed over to createSim.Kernelheaping

**Value**

List of estimation results

**Examples**

```
## Not run: Sims1 <- sim.Kernelheaping(simRuns=2, n=500, distribution="norm",
rounds=c(1,10,100), thresholds=c(0.3,0.4,0.3), sd=100)
## End(Not run)
```

---

simSummary.Kernelheaping  
*Simulation Summary*

---

**Description**

Simulation Summary

**Usage**

```
simSummary.Kernelheaping(sim, coverage = 0.9)
```

**Arguments**

sim	Simulation object returned from sim.Kernelheaping
coverage	probability for computing coverage intervals

**Value**

list with summary statistics

---

 students

*Student0405*


---

**Description**

Data collected during 2004 and 2005 from students in statistics classes at a large state university in the northeastern United States.

**Source**

<http://mathfaculty.fullerton.edu/mori/Math120/Data/readme>

**References**

Utts, J. M., & Heckard, R. F. (2011). Mind on statistics. Cengage Learning.

---

 summary.Kernelheaping *Prints some descriptive statistics (means and quantiles) for the estimated rounding, bias and acceleration (beta) parameters*


---

**Description**

Prints some descriptive statistics (means and quantiles) for the estimated rounding, bias and acceleration (beta) parameters

**Usage**

```
## S3 method for class 'Kernelheaping'
summary(object, ...)
```

**Arguments**

object	Kernelheaping object produced by dheaping function
...	unused

**Value**

Prints summary statistics

---

toOtherShape	<i>Transfer observations to other shape</i>
--------------	---

---

**Description**

Transfer observations to other shape

**Usage**

```
toOtherShape(Mestimates, shapefile)
```

**Arguments**

Mestimates	Estimation object created by functions dshapebivr and dbivr
shapefile	The new shapefile for which the observations shall be transferred to

**Value**

The function returns the count, sd and 90

---

tracePlots	<i>Plots some trace plots for the rounding, bias and acceleration (beta) parameters</i>
------------	---

---

**Description**

Plots some trace plots for the rounding, bias and acceleration (beta) parameters

**Usage**

```
tracePlots(x, ...)
```

**Arguments**

x	Kernelheaping object produced by dheaping function
...	additional arguments given to standard plot function

**Value**

Prints summary statistics

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