

Package ‘LSE’

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Type Package

Title Constrained Least Squares and Generalized QR Factorization

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Author Sergio Andrés Cabrera Miranda <<https://orcid.org/0000-0002-8126-8521>>, Juan Gabriel Triana Laverde <<https://orcid.org/0000-0003-2991-6082>>

Maintainer Sergio Andrés Cabrera Miranda <sergio05acm@gmail.com>

Description The solution of equality constrained least squares problem (LSE) is given through four analytics methods (Generalized QR Factorization, Lagrange Multipliers, Direct Elimination and Null Space method). We expose the orthogonal decomposition called Generalized QR Factorization (GQR) and also RQ factorization. Finally some codes for the solution of LSE applied in quaternions.

URL <https://github.com/sergio05acm/LSE>

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Antiquaternion	<i>Antiquaternion</i>
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Description

A quaternion matrix obtained by the Quaternion function, can be transformed into a matrix, which contains as many quaternions as rows obtained, each column represent an imaginary axis (1,i,j,k).

Usage

Antiquaternion(x)

Arguments

x Quaternion matrix object.

Details

This function shows in each row a quaternion, and in each column an axis in the order a+bi+cj+dk.

Value

Numeric matrix.

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

See Also

See Also as [Quaternion](#)

Examples

Antiquaternion(Quaternion(1,0,1,0))

Direct Elimination *Direct Elimination for LSE problem.*

Description

Direct Elimination allows to give an analytic solution for equality constrained least squares problem (LSE). Requires MASS and pracma library.

Usage

```
Dir_Elimination(A,C,b,d)
```

Arguments

A	Design matrix, m rows and n columns.
C	Constraint matrix, p rows and n columns.
b	Response vector for A, $Ax=b$, m rows and 1 column.
d	Response vector for C, $Cx=d$, p rows and 1 column.

Details

Direct Elimination method gives a numerical vector as the solution of a least squares problem ($Ax=b$), when impose some restrictions (additional information, extramuestral information or a priori information) that lead to another linear equality system ($Cx=d$). See significance constraint ($x=0$) or inclusion restriction ($x+y=1$), etc.

Value

Numerical vector for a LSE problem.

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

Lawson, C. L., & Hanson, R. J. (1974). Linear least squares with linear inequality constraints. Solving least squares problems, 158-173.

Van Benthem, M. H., Keenan, M. R., & Haaland, D. M. (2002). Application of equality constraints on variables during alternating least squares procedures. *Journal of Chemometrics: A Journal of the Chemometrics Society*, 16(12), 613-622.

Examples

```
A = matrix(runif(50,-1,1),10,5)
C = matrix(runif(20,-1,1),4,5)
b = matrix(runif(10,-1,1),10,1)
d = matrix(runif(4,-1,1),4,1)

Dir_Elimination(A,C,b,d)
```

GQR

Generalized QR Factorization

Description

This code provides a simultaneous orthogonal factorization for two matrices A and B. This code requires pragma library.

Usage

```
GQR(x,y)
```

Arguments

x	Numerical matrix with m rows and n columns.
y	Numerical matrix with p rows and n columns.

Details

Given two matrices, with the same number of rows, this algorithm provides a single factorization, such that $A=QR$ and $(Q^T)B=WS$.

Value

Q	Orthogonal matrix for A
R	Trapezoidal matrix for A
W	Orthogonal matrix for $(Q^T)B$
S	Trapezoidal matrix for $(Q^T)B$

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

Cabrera Miranda, S. A., & Triana Laverde, J. G. (2021). El problema de los mínimos cuadrados con restricciones de igualdad mediante la factorización QR generalizada. *Selecciones Matemáticas*, 8(02), 437-443. (English Article).

Anderson, E., Bai, Z., & Dongarra, J. (1992). Generalized QR factorization and its applications. *Linear Algebra and its Applications*, 162, 243-271.

Examples

```
A = matrix(c(1,1,1,1,3,1,1,-1,1,1,1,1),4,3,byrow=TRUE)
C = matrix(c(1,1,1,1,1,-1),2,3,byrow=TRUE)
GQR(t(A),t(C))
```

Lagrange

*Lagrange multipliers for LSE problem.***Description**

Lagrange multipliers allows to give a analytic solution for equality constrained least squares problem (LSE).

Usage

```
Lagrange(A,C,b,d)
```

Arguments

A	Design matrix, m rows and n columns.
C	Constraint matrix, p rows and n columns.
b	Response vector for A, $Ax=b$, m rows and 1 column.
d	Response vector for C, $Cx=d$, p rows and 1 column.

Details

The Lagrange multipliers method gives a numerical vector as the solution of a least squares problem ($Ax=b$) through unification the model and their restrictions in one function, the restrictions impose in the model (additional information, extramuestral information or a priori information) lead to another linear equality system ($Cx=d$). See significance constraint ($x=0$) or inclusion restriction ($x+y=1$), etc.

Value

Numerical vector for a LSE problem.

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

Rao, C. R., Toutenburg, H., Shalabh, H. C., & Schomaker, M. (2008). Linear models and generalizations. Least Squares and Alternatives (3rd edition) Springer, Berlin Heidelberg New York.

Theil, H. (1971). Principles of econometrics (No. 04; HB139, T44.).

Examples

```
A = matrix(runif(50,-1,1),10,5)
C = matrix(runif(20,-1,1),4,5)
b = matrix(runif(10,-1,1),10,1)
d = matrix(runif(4,-1,1),4,1)
```

```
Lagrange(A,C,b,d)
```

LSE

LSE package

Description

Solve the equality constrained least squares problem for real value and for quaternions, also allows to use the Generalized QR factorization for two matrices simultaneously.

Author(s)

Sergio Andrés Cabrera Miranda <sergio05acm@gmail.com>

LSE_GQR

LSE and GQR Factorization

Description

This code provides the solution of equality constrained least squares problem through Generalized QR Factorization. Require MASS package.

Usage

```
LSE_GQR(A,C,b,d)
```

Arguments

A	Design matrix, m rows and n columns.
C	Constraint matrix, p rows and n columns.
b	Response vector for A, $Ax=b$, m rows and 1 column.
d	Response vector for C, $Cx=d$, p rows and 1 column.

Details

This algorithm provides the solution of the equality constrained least squares problem through Generalized QR factorization. This algorithm requires the same number of columns for matrices A and C.

Value

Numerical vector for a LSE problem.

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

Anderson, E., Bai, Z., & Dongarra, J. (1992). Generalized QR factorization and its applications. *Linear Algebra and its Applications*, 162, 243-271.

Examples

```
A = matrix(c(1,2,3,4,5,6),3,2,byrow = TRUE)
C = matrix(c(1,1),1,2,byrow=TRUE)
b = matrix(c(7,1,3),3,1,byrow=TRUE)
d = matrix(c(1),1,1,byrow=TRUE)
```

```
LSE_GQR(A,C,b,d) #You can verify that x+y=1 satisfies the constraint.
```

Nullspace

Nullspace method for LSE problem.

Description

Null Space method allows to give an analytic solution for equality constrained least squares problem (LSE). Requires pracma library.

Usage

```
Nullspace(A,C,b,d)
```

Arguments

A	Design matrix, m rows and n columns.
C	Constraint matrix, p rows and n columns.
b	Response vector for A, $Ax=b$, m rows and 1 column.
d	Response vector for C, $Cx=d$, p rows and 1 column.

Details

Null Space method gives a numerical vector as the solution of a least squares problem ($Ax=b$), using an unconstrained problem equivalent to the LSE proposed, this method can be applied when impose some restrictions (additional information, extramuestral information or a priori information) that lead to another linear equality system ($Cx=d$). See significance constraint ($x=0$) or inclusion restriction ($x+y=1$), etc.

Value

Numerical vector for a LSE problem.

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

- Lawson, C. L., & Hanson, R. J. (1974). Linear least squares with linear inequality constraints. Solving least squares problems, 158-173.
- Van Benthem, M. H., Keenan, M. R., & Haaland, D. M. (2002). Application of equality constraints on variables during alternating least squares procedures. *Journal of Chemometrics: A Journal of the Chemometrics Society*, 16(12), 613-622.

Examples

```
A = matrix(runif(50,-1,1),10,5)
C = matrix(runif(20,-1,1),4,5)
b = matrix(runif(10,-1,1),10,1)
d = matrix(runif(4,-1,1),4,1)

Nullspace(A,C,b,d)
```

Quaternion

Quaternion transformation

Description

A quaternion $q=a+bi+cj+dk$ can be transformed into a real value matrix $M(4 \times 4)$.

Usage

Quaternion(a,b,c,d)

Arguments

- | | |
|---|--------------------------------------|
| a | Real value coefficient. |
| b | Coefficient of the imaginary i-axis. |
| c | Coefficient of the imaginary j-axis. |
| d | Coefficient of the imaginary k-axis. |

Value

Real value matrix to represent a quaternion.

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

Al-Zhour, Z. (2019). Some new linear representations of matrix quaternions with some applications. Journal of King Saud University-Science, 31(1), 42-47.

Examples

```
Quaternion(1,0,1,0)

rbind(cbind(Quaternion(-1,0,1,0),Quaternion(0,0,0,-1)),
      cbind(Quaternion(0,1,0,0),Quaternion(1,0,1,0))
      )
```

RQ

RQ Factorization of a matrix

Description

RQ factorization allows to develop an orthogonal transformation in a matrix through Householder reflections. Requires pracma package.

Usage

```
RQ(y)
```

Arguments

y Numeric matrix or vector.

Details

RQ factorization make a orthogonal transformation at the rows of the matrix, beginning in the last one, and finishing with the first one row.

Value

Q Orthogonal matrix for x
R Triangular matrix for x

Author(s)

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

Examples

```
A = matrix(runif(12,0,5),4,3,byrow=TRUE)
RQ(A)
```

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