

# Package ‘Lmoments’

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**Title** L-Moments and Quantile Mixtures

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**Description** Contains functions to estimate L-moments and trimmed L-moments from the data. Also contains functions to estimate the parameters of the normal polynomial quantile mixture and the Cauchy polynomial quantile mixture from L-moments and trimmed L-moments.

**Imports** stats, Rcpp

**LinkingTo** Rcpp, RcppArmadillo

**License** GPL-2

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**NeedsCompilation** yes

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cauchypoly

*Cauchy-polynomial quantile mixture*

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**Description**

Density, distribution function, quantile function and random generation for the Cauchy-polynomial quantile mixture.

**Usage**

```
dcauchypoly(x,param)
pcauchypoly(x,param)
qcauchypoly(cp,param)
rcauchypoly(n,param)
cauchypoly_pdf(x,param)
cauchypoly_cdf(x,param)
cauchypoly_inv(cp,param)
cauchypoly_rnd(n,param)
```

**Arguments**

x	vector of quantiles
cp	vector of probabilities
n	number of observations
param	vector of parameters

**Details**

The length the parameter vector specifies the order of the polynomial in the quantile mixture. If  $k < \text{length}(\text{param})$  then  $\text{param}[1:(k-1)]$  contains the mixture coefficients of polynomials starting from the constant and  $\text{param}[k]$  is the mixture coefficient for Cauchy distribution. (Functions `cauchypoly_pdf`, `cauchypoly_cdf`, `cauchypoly_inv` and `cauchypoly_rnd` are aliases for compatibility with older versions of this package.)

**Value**

'dcauchypoly' gives the density, 'pcauchypoly' gives the cumulative distribution function, 'qcauchypoly' gives the quantile function, and 'rcauchypoly' generates random deviates.

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

**See Also**

[data2cauchypoly4](#) for the parameter estimation and [dnormpoly](#) for the normal-polynomial quantile mixture.

**Examples**

```
#Generates 500 random variables from the Cauchy-polynomial quantile mixture,
#calculates the trimmed L-moments,
#estimates parameters via trimmed L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params <- t1lmom2cauchypoly4(c(0,1,0.075,0.343));
x <- rcauchypoly(500,true_params);
t1lmom <- t1lmoments(x);
estim_params <- t1lmom2cauchypoly4(t1lmom);
plotpoints <- seq(-10,10,by=0.01);
histpoints <- c(seq(min(x)-1,-20,length.out=50),seq(-10,10,by=0.5),seq(20,max(x)+1,length.out=50));
hist(x, breaks=histpoints, freq=FALSE, xlim=c(-10,10));
lines(plotpoints, dcauchypoly(plotpoints,estim_params), col='red');
lines(plotpoints, dcauchypoly(plotpoints,true_params), col='blue');
```

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covnormpoly4

*Covariance matrix of the parameters of the normal-polynomial quantile mixture*

---

**Description**

Estimates covariance matrix of the four parameters of normal-polynomial quantile mixture

**Usage**

```
covnormpoly4(data)
```

**Arguments**

data            vector of observations

**Value**

covariance matrix of the four parameters of normal-polynomial quantile mixture

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

**See Also**

[Lmomcov](#) for covariance matrix of L-moments, [dnormpoly](#) for the normal-polynomial quantile mixture and [data2normpoly4](#) for the estimation of the normal-polynomial quantile mixture.

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`data2cauchypoly`*Estimation of the Cauchy-polynomial quantile mixture*

---

**Description**

Estimates the parameters of the Cauchy-polynomial quantile mixture from data or from trimmed L-moments

**Usage**

```
data2cauchypoly4(data)
t1lmom2cauchypoly4(t1lmom)
```

**Arguments**

<code>data</code>	vector
<code>t1lmom</code>	vector of trimmed L-moments

**Value**

vector containing the four parameters of the Cauchy-polynomial quantile mixture

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

**See Also**

[t1lmoments](#) for trimmed L-moments, [dcauchypoly](#) for the Cauchy-polynomial quantile mixture and [data2normpoly4](#) for the estimation of the normal-polynomial quantile mixture.

## Examples

```
#Generates 500 random variables from the Cauchy-polynomial quantile mixture,
#calculates the trimmed L-moments,
#estimates parameters via trimmed L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params <- t1lmom2cauchypoly4(c(0,1,0.075,0.343));
x <- rcauchypoly(500,true_params);
t1lmom <- t1lmoments(x);
estim_params <- t1lmom2cauchypoly4(t1lmom);
plotpoints <- seq(-10,10,by=0.01);
histpoints <- c(seq(min(x)-1,-20,length.out=50),seq(-10,10,by=0.5),seq(20,max(x)+1,length.out=50));
hist(x, breaks=histpoints, freq=FALSE, xlim=c(-10,10));
lines(plotpoints, dcauchypoly(plotpoints,estim_params,col='red'));
lines(plotpoints, dcauchypoly(plotpoints,true_params,col='blue'));
```

---

data2normpoly

*Estimation of normal-polynomial quantile mixture*

---

## Description

Estimates the parameters of normal-polynomial quantile mixture from data or from L-moments

## Usage

```
data2normpoly4(data)
lmom2normpoly4(lmom)
data2normpoly6(data)
lmom2normpoly6(lmom)
```

## Arguments

data	matrix or data frame
lmom	vector or matrix of L-moments

## Value

vector or matrix containing the four or six parameters of normal-polynomial quantile mixture

## Author(s)

Juha Karvanen <juha.karvanen@iki.fi>

## References

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

**See Also**

[dnormpoly](#) for L-moments, [dnormpoly](#) for the normal-polynomial quantile mixture and [data2cauchypoly4](#) for the estimation of Cauchy-polynomial quantile mixture.

**Examples**

```
#Generates a sample 500 observations from the normal-polynomial quantile mixture,
#calculates L-moments and their covariance matrix,
#estimates parameters via L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params <- lmom2normpoly4(c(0,1,0.2,0.05));
x <- rnormpoly(500,true_params);
lmoments <- Lmoments(x);
lmomcov <- Lmomcov(x);
estim_params <- lmom2normpoly4(lmoments);
hist(x,30,freq=FALSE);
plotpoints <- seq(min(x)-1,max(x)+1,by=0.01);
lines(plotpoints, dnormpoly(plotpoints,estim_params), col='red');
lines(plotpoints, dnormpoly(plotpoints,true_params), col='blue');
```

---

Lmoments

*L-moments*


---

**Description**

Calculates sample L-moments, L-coefficients and covariance matrix of L-moments.

**Usage**

```
Lmoments(data, rmax = 4, na.rm = FALSE, returnobject = FALSE, trim = c(0, 0))
Lcoefs(data, rmax = 4, na.rm = FALSE, trim = c(0, 0))
Lmomcov(data, rmax = 4, na.rm = FALSE)
Lmoments_calc(data, rmax = 4)
Lmomcov_calc(data, rmax = 4)
shiftedlegendre(rmax)
```

**Arguments**

data	matrix or data frame.
rmax	maximum order of L-moments.
na.rm	a logical value indicating whether 'NA' values should be removed before the computation proceeds.
returnobject	a logical value indicating whether a list object should be returned instead of an array of L-moments.
trim	c(0, 0) for ordinary L-moments and c(1, 1) for trimmed (t = 1) L-moments

**Value**

Lmoments returns an array of L-moments containing a row for each variable in data, or if returnobject=TRUE, a list containing

lambdas	an array of L-moments
ratios	an array of mean, L-scale and L-moment ratios
trim	the value of the parameter 'trim'
source	a string with value "Lmoments" or "t1lmoments".

Lcoefs returns an array of L-coefficients (mean, L-scale, L-skewness, L-kurtosis, ...) containing a row for each variable in data.

Lmomcov returns the covariance matrix of L-moments or a list of covariance matrices if the input has multiple columns. The numerical accuracy of the results decreases with increasing rmax. With  $rmax > 5$ , a warning is thrown, as the numerical accuracy of the results is likely less than  $\sqrt{.Machine\$double.eps}$ .

shiftedlegendre returns a matrix of the coefficients of the shifted Legendre polynomials up to a given order.

**Note**

Functions Lmoments and Lcoefs calculate trimmed L-moments if you specify `trim = c(1, 1)`. Lmoments\_calc and Lmomcov\_calc are internal C++ functions called by Lmoments and Lmomcov. The direct use of these functions is not recommended.

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>, Santeri Karppinen

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

Elamir, E. A., Seheult, A. H. 2004. Exact variance structure of sample L-moments, *Journal of Statistical Planning and Inference* **124** (2) 337–359.

Hosking, J. 1990. L-moments: Analysis and estimation distributions using linear combinations of order statistics, *Journal of Royal Statistical Society B* **52**, 105–124.

**See Also**

`t1lmoments` for trimmed L-moments, `dnormpoly`, `lmom2normpoly4` and `covnormpoly4` for the normal-polynomial quantile mixture and package **lmomco** for additional L-moment functions

**Examples**

```
#Generates a sample 500 observations from the normal-polynomial quantile mixture,
#calculates the L-moments and their covariance matrix,
#estimates parameters via L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params <- lmom2normpoly4(c(0,1,0.2,0.05));
x <- rnormpoly(500,true_params);
lmoments <- Lmoments(x);
lmomcov <- Lmomcov(x);
estim_params <- lmom2normpoly4(lmoments);
hist(x,30,freq=FALSE)
plotpoints <- seq(min(x)-1,max(x)+1,by=0.01);
lines(plotpoints, dnormpoly(plotpoints,estim_params),col='red');
lines(plotpoints, dnormpoly(plotpoints,true_params),col='blue');
```

normpoly

*Normal-polynomial quantile mixture***Description**

Density, distribution function, quantile function and random generation for the normal-polynomial quantile mixture.

**Usage**

```
dnormpoly(x,param)
pnormpoly(x,param)
qnormpoly(cp,param)
rnormpoly(n,param)
normpoly_pdf(x,param)
normpoly_cdf(x,param)
normpoly_inv(cp,param)
normpoly_rnd(n,param)
```

**Arguments**

x	vector of quantiles
cp	vector of probabilities
n	number of observations
param	vector of parameters

**Details**

The length the parameter vector specifies the order of the polynomial in the quantile mixture. If  $k < \text{length}(\text{param})$  then  $\text{param}[1:(k-1)]$  contains the mixture coefficients of polynomials starting from the constant and  $\text{param}[k]$  is the mixture coefficient for normal distribution. (Functions `normpoly_pdf`, `normpoly_cdf`, `normpoly_inv` and `normpoly_rnd` are aliases for compatibility with older versions of this package.)

**Value**

'dnormpoly' gives the density, 'pnormpoly' gives the cumulative distribution function, 'qnormpoly' gives the quantile function, and 'rnormpoly' generates random deviates.

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

**See Also**

[data2normpoly4](#) for the parameter estimation and [dcauchypoly](#) for the Cauchy-polynomial quantile mixture.

**Examples**

```
#Generates a sample 500 observations from the normal-polynomial quantile mixture,
#calculates L-moments and their covariance matrix,
#estimates parameters via L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params <- lmom2normpoly4(c(0,1,0.2,0.05));
x <- rnormpoly(500,true_params);
lmoments <- Lmoments(x);
lmomcov <- Lmomcov(x);
estim_params <- lmom2normpoly4(lmoments);
hist(x,30,freq=FALSE)
plotpoints <- seq(min(x)-1,max(x)+1,by=0.01);
lines(plotpoints, dnormpoly(plotpoints,estim_params),col='red');
lines(plotpoints, dnormpoly(plotpoints,true_params),col='blue');
```

---

t1lmoments

*Trimmed L-moments*


---

**Description**

Calculates sample trimmed L-moments with trimming parameter 1.

**Usage**

```
t1lmoments(data, rmax = 4)
t1lmoments_calc(data, rmax = 4)
```

**Arguments**

data               matrix or data frame.  
 rmax               maximum order of trimmed L-moments.

**Value**

array of trimmed L-moments (trimming parameter = 1) up to order 4 containing a row for each variable in data.

**Note**

Functions `link{Lmoments}` and `link{Lcoefs}` calculate trimmed L-moments if you specify `trim = c(1, 1)`. `t1lmoments_calc` is an internal C++ function called by `t1lmoments`. The direct use of this function is not recommended.

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>, Santeri Karppinen

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [https://users.jyu.fi/~jutakarv/papers/Karvanen\\_quantile\\_mixtures.pdf](https://users.jyu.fi/~jutakarv/papers/Karvanen_quantile_mixtures.pdf).

Elamir, E. A., Seheult, A. H. 2003. Trimmed L-moments, *Computational Statistics & Data Analysis* **43**, 299–314.

**See Also**

[Lmoments](#) for L-moments, and [dcauchypoly](#) and [t1lmom2cauchypoly4](#) for the Cauchy-polynomial quantile mixture

**Examples**

```
#Generates 500 random variables from the Cauchy-polynomial quantile mixture,
#calculates the trimmed L-moments,
#estimates parameters via trimmed L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params <- t1lmom2cauchypoly4(c(0,1,0.075,0.343));
x <- rcauchypoly(500,true_params);
t1lmom <- t1lmoments(x);
estim_params <- t1lmom2cauchypoly4(t1lmom);
plotpoints <- seq(-10,10,by=0.01);
histpoints <- c(seq(min(x)-1,-20,length.out=50),seq(-10,10,by=0.5),seq(20,max(x)+1,length.out=50));
hist(x,breaks=histpoints, freq=FALSE, xlim=c(-10,10));
lines(plotpoints, dcauchypoly(plotpoints, estim_params), col='red');
lines(plotpoints, dcauchypoly(plotpoints, true_params), col='blue');
```

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