Package ‘MLEce’

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Type Package

Title Asymptotic Efficient Closed-Form Estimators for Multivariate Distributions

Version 2.1.0

Description Asymptotic efficient closed-form estimators (MLEces) are provided in this package for three multivariate distributions(gamma, Weibull and Dirichlet) whose maximum likelihood estimators (MLEs) are not in closed forms. Closed-form estimators are strong consistent, and have the similar asymptotic normal distribution like MLEs. But the calculation of MLEces are much faster than the corresponding MLEs. Further details and explanations of MLEces can be found in.
Jang, et al. (2023) <doi:10.1111/stan.12299>.
Kim, et al. (2023) <doi:10.1080/03610926.2023.2179880>.

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Contents

`benchMLEce` *Performing closed-form estimators against other methods*

Description

Performing closed-form estimators against other methods

Usage

```
benchMLEce(data, distname, methods)
```

Arguments

`data`	a numeric matrix.
`distname`	a character indicating which distribution to be fitted. `BiGam` stands for the bivariate gamma distribution, `BiWei` stands for the bivariate Weibull distribution, and `Dirichlet` stands for the Dirichlet distribution.
`methods`	a vector of methods: two characters among `"MLEce"` (efficient closed-form estimator), `"MLE"`, `"MME"` and `"CME"`. `MLEce` stands for efficient closed-form estimators, , `"MLE"` (maximum likelihood estimator), `"MME"` (method of moments estimator) and `"CME"` (correlation based method estimator for the bivariate Weibull distribution).

Value

A matrix with estimate and time in seconds per method for assigned distributions.

Examples

```
#bivariate gamma distribution
data_BiGam= rBiGam(100, c(1,4,5))
benchMLEce(data_BiGam, distname="BiGam", methods=c("MLEce","MME"))
#bivariate Weibull distribution
data_BiWei <- rBiWei(n=50, c(4,3,3,4,0.6))
benchMLEce(data_BiWei, distname="BiWei", methods=c("MLE","CME"))
#multivariate Dirichlet distribution
data_Diri <- LaplacesDemon::rdirichlet(80, c(3,4,1,3,4))
benchMLEce(data_Diri, distname="Dirichlet", methods=c("MLEce","MLE"))
```

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coef.MLEce — *Getting estimated values of efficient closed-form estimators*

Description

Getting estimated values of efficient closed-form estimators

Usage

```
## S3 method for class 'MLEce'
coef(object, digits = max(3, getOption("digits") - 3), ...)
```

Arguments

`object`	an object of class `"MLEce"` made by the function `MLEce`.
`digits`	a numeric number of significant digits.
`...`	not used, but exists because of the compatibility.

Value

a numeric vector or a list, containing assigned distribution and estimated values, is given.

Examples

```
data_BiGam = rBiGam(100, c(1,4,5))
res_BiGam = MLEce(data_BiGam, "BiGam")
coef(res_BiGam)
data_BiWei = rBiWei(n=50, c(4,3,3,4,0.6))
est_BiWei <-MLEce(data_BiWei, "BiWei")
coef(est_BiWei)
data_Diri <- LaplacesDemon::rdirichlet(n=60, c(3,1,2,4))
est_Diri <- MLEce(data_Diri, "Dirichlet")
coef(est_Diri)
```

confCI — *Getting confidence intervals for efficient closed-form estimators*

Description

Getting confidence intervals for efficient closed-form estimators

Usage

```
confCI(object, bootsize = 1000, level = 0.95)
```

Arguments

object	an object of class "MLEce" made by the function MLEce.
bootsize	a numeric value for the steps in the bootstrap method; default value is 1,000.
level	a numeric value between 0 and 1 for controlling the significance level of confidence interval; default value is 0.95.

Details

The confidence interval is obtained by bootstrap method for the estimated parameters in the assigned distribution.

Value

a numeric a list is given, containing assigned distribution, confidence intervals and alpha which is equal to one minus the significance level.

Examples

```
data(flood)
est_BiGam <- MLEce(flood, "BiGam")
confCI(est_BiGam)
datt = rBiWei(n=50, c(4,3,3,4,0.6))
est_BiWei <-MLEce(datt, "BiWei")
confCI(est_BiWei)
data_Diri <- LaplacesDemon::rdirichlet(n=60, c(3,1,2,4))
est_Diri <- MLEce(data_Diri, "Dirichlet")
confCI(est_Diri)
```

`flood` *The flood events data of the Madawaska basin.*

Description

The data is a subset of the flood events data of the Madawaska basin which is located in the province of Qebec, Canada. Daily streamflow data from 1919 to 1995 are available from HYDAT CD (1998) and Yue (2001).

Usage

```
data(flood, package = "MLEce")
```

Format

A dataframe with 2 variables and 77 observations as follows:

Vnorm daily average flood volume

Q flood peak

References

Environment Canada (1998), HYDAT CD-ROM Version 98-1.05.8: Surface water and sediment data.

Yue. S. (2001) A bivariate gamma distribution for use in multivariate flood frequency analysis. *Hydrological Processes*, 15, 1033–1045.

`fossil_pollen` *The counts data of the frequency of occurrence of different kinds of fossil pollen grains.*

Description

The data is about the counts of the frequency of occurrence of different kinds of fossil pollen grains and is available in Mosimann (1962).

Usage

```
data(fossil_pollen, package = "MLEce")
```

Format

A dataframe with 73 observations and 4 variables: pinus, abies, quercus and alnus pollens.

References

Mosimann, J. E. (1962) On the compound multinomial distribution, the multivariate beta distribution, and correlations among proportions. *Biometrika*, 49, 65-82.

gof *Goodness-of-fit test for the efficient closed-form estimators*

Description

Goodness-of-fit test for the efficient closed-form estimators

Usage

```
gof(x, digits = max(3, getOption("digits") - 3), ...)

## S3 method for class 'gof'
print(x, digits = max(3, getOption("digits") - 3), ...)
```

Arguments

`x`	an object of class "MLEce" made by the function `MLEce`.
`digits`	a numeric number of significant digits.
`...`	additional arguments affecting the goodness-of-fit test.

Details

Generalized Cramer-von Mises test (chiu and Liu, 2009) is applied to do the goodness-of-fit test for multivariate distributions. For the bivariate gamma and Dirichlet distributions, the L2-symmetric discrepancy (SD2) statistics are applied. But the L2-centred discrepancy (CD2) statistics are applied in the bivariate Weibull distribution.

References

chiu, S. N. and Liu, K. I. (2009) Generalized Cramer-Von Mises goodness-of-fit tests for multivariate distributions. *Computational Statistics and Data Analysis*, 53, 3817-3834.

Examples

```
data_BiGam <-  rBiGam(100, c(1,4,5))
res_BiGam  <- MLEce(data_BiGam, "BiGam")
gof(res_BiGam)
datt = rBiWei(n=50, c(4,3,3,4,0.6))
est_BiWei <-MLEce(datt, "BiWei")
gof(est_BiWei)
data_Diri <- LaplacesDemon::rdirichlet(n=60, c(3,1,2,4))
est_Diri <- MLEce(data_Diri, "Dirichlet")
gof(est_Diri)
```

`MLEce` *Calculating MLEces for three different distributions*

Description

The closed-form estimators (MLEces) are calculated for three distributions: bivariate gamma, bivariate Weibull and multivariate Dirichlet.

Usage

```
MLEce(data, distname)
```

Arguments

`data`	a numeric matrix.
`distname`	a character indicating which distribution to be fitted. `BiGam` stands for the bivariate gamma distribution, `BiWei` stands for the bivariate Weibull distribution, and `Dirichlet` stands for the multivariate Dirichlet distribution.

Details

Based on root n-consistent estimators, the closed-form estimators (MLEces) are calculated for the parameters in bivariate gamma, bivariate Weibull and multivariate Dirichlet distributions whose maximum likelihood estimators (MLEs) are not in closed forms. The MLEces are strong consistent and asymptotic normally like the corresponding MLEs, but their calculation are much faster than MLEs. For the bivariate gamma and multivariate Dirichlet distribution, their root n-consistent estimators are the corresponding method of moments estimators (MMEs). The correlation-based estimators (CMEs) are applied as root n-consistent estimators in the bivariate Weibull distribution.

Value

`MLEce` returns an object of class `"MLEce"`. The object class `"MLEce"` is a list containing the following components.

`distribution`	a character string of a distribution assuming that data set comes from and the data was fitted to.
`estimation`	the estimated values of parameters in assigned distribution.

References

Kim, H.-M., Jang, Y.-H., Arnold, B. C. and Zhao, J. (2023) New efficient estimators for the Weibull distribution. *Communications in Statistics - Theory and Methods*, 1-26.

Jang, Y.-H., Zhao, J., Kim, H.-M., Yu, K., Kwon, S.and Kim, S. (2023) New closed-form efficient estimator for the multivariate gamma distribution. *Statistica Neerlandica*, 1–18.

Chang, J. H., Lee, S. K. and Kim, H.-M. (2023) An asymptotically efficient closed–form estimator for the multivariate Dirichlet distribution. submitted.

Examples

```
#bivariate gamma distribution
data_BiGam <-  rBiGam(100, c(1,4,5))
res_BiGam  <- MLEce(data_BiGam, "BiGam")
print(res_BiGam)
data(flood)
est_BiGam <- MLEce(flood, "BiGam")
print(est_BiGam)
#bivariate Weibull distribution
data_BiWei <- rBiWei(n=30, c(4,3,3,4,0.6))
res_BiWei <- MLEce(data_BiWei, "BiWei")
print(res_BiWei)
#real data example
data(airquality)
air_data <- airquality[ ,3:4]
air_data[ ,2] <- air_data[ ,2]*0.1
est_BiWei <- MLEce(air_data, "BiWei")
print(est_BiWei)
#Dirichlet distribution
data_Diri <- LaplacesDemon::rdirichlet(n=60, c(1,2,3))
res_Diri <- MLEce(data_Diri, "Dirichlet")
print(res_Diri)
data(fossil_pollen)
#real data example
fossil_data <- fossil_pollen/rowSums(fossil_pollen)
eps <- 1e-10
fossil_data <- (fossil_data +eps)/(1+2*eps)
est_fossil <- MLEce(fossil_data, "Dirichlet")
print(est_fossil)
```

---

plot.MLEce *Providing some plots for effective closed-form estimators*

Description

plot method for a class "MLEce".

Usage

```
## S3 method for class 'MLEce'
plot(
  x,
  which = c(1, 2, 3, 4),
  ask = prod(par("mfcol")) < length(which) && dev.interactive(),
  ...
)
```

Arguments

`x`	an object of class "MLEce" made by the function `MLEce`.
`which`	if a subset of the plots is required, specify a subset of 1:4.
`ask`	logical; if TRUE, the user is asked before each plot.
`...`	not used, but exists because of the compatibility.

Details

The boxplot for given data is presented first with `which=1`. For `which=2`, a contour line is drawn by the probability density function of the estimated parameter based on effective closed-form estimators. In the counter plot, the x-axis is the first column of data and the y-axis is the second column of data. For `which=3`, a marginally fitted probability density plot is given for the first column of input data. And a fitted line is added for the efficient closed-form estimator. For `which=4`, is a marginally fitted probability density plot is given like the former one for the second column of input data. Note that, marginally fitted probability density plots in `which=3` and `which=4` present comparisons between efficient closed form estimators (MLEces) and correlation based method estimators (CMEs) for the bivariate Weibull distribution. Note that this `plot` commend is limited at the bivariate distributions.

Examples

```
data(flood)
est_BiGam <- MLEce(flood, "BiGam")
plot(est_BiGam, c(3))
air_data <- airquality[ ,3:4]
air_data[ ,2] <- air_data[ ,2]*0.1
est_BiWei <- MLEce(air_data, "BiWei")
plot(est_BiWei)
data(fossil_pollen)
fossil_data <- cbind(fossil_pollen[,1]/100,rowSums(fossil_pollen[,-1]/100))
est_fossil <- MLEce(fossil_data, "Dirichlet")
plot(est_fossil,c(2))
```

---

rBiGam — *Generating random data for the bivariate gamma distribution with parameters.*

Description

Generating random data for the bivariate gamma distribution with parameters.

Usage

```
rBiGam(n, paras)
```

Arguments

`n`	number of observations.
`paras`	parameters of bivariate gamma distribution (shape1, shape2, scale).

Details

Random generation for the bivariate gamma distribution is presented. The specific generation formulas can be found in Jang, et al. (2020).

Value

`rBiGam` generates random deviates. The length of generated data is determined by `"n"`.

References

Jang, Y.-H., Zhao, J., Kim, H.-M., Yu, K., Kwon, S.and Kim, S. (2023) New closed-form efficient estimator for the multivariate gamma distribution. *Statistica Neerlandica*, 1–18.

Examples

```
datt = rBiGam(n=50, c(4,3,3))
```

`rBiWei` *Generating random data for the bivariate Weibull distribution.*

Description

Generating random data for the bivariate Weibull distribution.

Usage

```
rBiWei(n, paras)
```

Arguments

`n`	number of observations.
`paras`	parameters of bivariate Weibull distribution (alpha1, beta1, alpha2, beta2, delta).

Details

`rBiWei` generates random number data for bivariate Weibull distribution.

Value

`rBiWei` generates random deviates.The length of generated data is determined by `"n"`

Examples

```
datt = rBiWei(n=50, c(4,3,3,4,0.6))
```

summary.MLEce *Summarizing effective closed-form estimation function*

Description

summary method for a class "MLEce".

Usage

```
## S3 method for class 'MLEce'
summary(object, ...)

## S3 method for class 'summary.MLEce'
print(x, digits = max(3, getOption("digits") - 3), ...)
```

Arguments

object	an object of class "MLEce" made by the function MLEce.
...	not used, but exists because of the compatibility.
x	an object of class "summary.MLEce".
digits	a numeric number of significant digits.

Value

summary presents information about effective closed-form estimators calculated by MLEce containing the following components.

Distribution	the distribution assigned to fit the data to.
Quantile	a numeric vector describing the data set with min, 1st quantile, median, 3rd quantile, and max values.
Correlation	correlation coefficient between two vectors of the data
Estimation	estimated values of parameters, standard error and confidence intervals are given.

Examples

```
#bivariate gamma distribution
data(flood)
est_res1 <- MLEce(flood, "BiGam")
summary(est_res1)
#bivariate Weibull distribution
datt = rBiWei(n=50, c(2,3,3,4,0.4))
est_res2 <-MLEce(datt, "BiWei")
summary(est_res2)
#Dirichilet distribution
data(fossil_pollen)
fossil_data <- fossil_pollen/rowSums(fossil_pollen)
eps <- 1e-10
```

```
fossil_data <- (fossil_data +eps)/(1+2*eps)
est_res3 <- MLEce(fossil_data, "Dirichlet")
summary(est_res3)
```

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