

# Package ‘MM’

May 7, 2026

**Type** Package

**Title** The Multiplicative Multinomial Distribution

**Description** Various utilities for the Multiplicative Multinomial distribution.

**Version** 1.7-0

**Depends** R (>= 2.10.0)

**Imports** magic (>= 1.5-6), abind, quadform (>= 0.0-2), methods,  
partitions (>= 1.9-14), Oarray

**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>

**License** GPL-2

**LazyLoad** yes

**NeedsCompilation** no

**Repository** CRAN

**URL** <https://github.com/RobinHankin/MM>

**BugReports** <https://github.com/RobinHankin/MM/issues>

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MM-package	<i>The Multiplicative Multivariate distribution, and the Multivariate Multiplicative Binomial Distribution</i>
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## Description

Two generalizations of the Multiplicative Binomial distribution of Altham (1978).

## Details

The DESCRIPTION file:

```

Package:      MM
Type:        Package
Title:       The Multiplicative Multinomial Distribution
Description: Various utilities for the Multiplicative Multinomial distribution.
Version:     1.7-0
Depends:    R (>= 2.10.0)
Imports:    magic (>= 1.5-6), abind, quadform (>= 0.0-2), methods, partitions (>= 1.9-14), Oarray
Authors@R:  c(person(given = c("Robin", "K.", "S."), family = "Hankin", role = c("aut", "cre"), email = "hankin.rob@
Maintainer: Robin K. S. Hankin <hankin.rob@github.com>
License:    GPL-2
LazyLoad:   yes
NeedsCompilation: no
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URL:       https://github.com/RobinHankin/MM
BugReports: https://github.com/RobinHankin/MM/issues
Author:    Robin K. S. Hankin [aut, cre], P. M. E. Altham [aut]

```

Index of help topics:

Lindsey	The Poisson device of Lindsey and Mersch (1992).
MB	Multivariate multiplicative binomial distribution

MM	Various multiplicative multinomial probability utilities
MM-package	The Multiplicative Multivariate distribution, and the Multivariate Multiplicative Binomial Distribution
NormC	Normalizing constant for the multiplicative multinomial
[.paras	Extract or Replace parameters of a 'paras' object
danaher	Dataset due to Danaher
gunter	Convert from multiple multivariate observations to tabular form
multinomial	Multinomial function
optimizer	Maximum likelihood estimator for the MM
paras	Manipulate a paras object
pollen	Pollen data from Mosimann 1962
powell	Dataset due to Powell (1990)
rMM	Random samples from the multiplicative multinomial
skellam	Brassica Dataset due to Catcheside
suffstats	Sufficient statistics for the multiplicative multinomial
sweets	Synthetic dataset due to Hankin
voting	Synthetic dataset of voting behaviour due to Altham
wilson	Housing Dataset due to Wilson

**Author(s)**

Robin K. S. Hankin and P. M. E. Altham

Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>

**References**

P. M. E. Altham 1978. "Two Generalizations of the Binomial Distribution". *Applied Statistics* 27:162–167

P. M. E. Altham and Robin K. S. Hankin 2012. "Multivariate Generalizations of the Multiplicative Binomial Distribution: Introducing the MM Package", *Journal of Statistical Software*, 46(12), 1-23. [doi:10.18637/jss.v046.i12](https://doi.org/10.18637/jss.v046.i12)

**Examples**

```
data(voting)
Lindsey(voting, voting_tally)
```

```
jj <- paras(3)
rMM(10,4,jj)
```

---

 danaher

*Dataset due to Danaher*


---

### Description

Dataset due to Danaher; also an analysis *ab initio*

### Usage

```
data(danaher)
```

### Format

- `danaher` is a matrix (of class `Oarray`) that represents Danaher and Hardie's Table 1

### Details

Since bacon is often *eaten* with eggs, it is reasonable to expect that it is *purchased* with eggs.

Danaher and Hardie use a dataset obtained from a sample of 548 households over four consecutive store trips. They considered only grocery shopping trips with a total basket value of at least five dollars. For each household, they counted the total number of bacon purchases in their four eligible shopping trips, and the total number of egg purchases for the same trips.

Object `danaher` is a five-by-five matrix of class `Oarray` with entry  $(i, j)$  indicating the number of shoppers buying bacon on  $i$  occasions and eggs on  $j$  occasions (note the zero offset). Thus `danaher[1, 2]=16` indicates that 16 shoppers bought bacon on 1 occasion and eggs on 2 occasions.

### References

P. J. Danaher and B. G. S. Hardie 2005. "Bacon with your eggs? Applications of a new bivariate beta-binomial distribution". *The American Statistician*, 59(4):282

### See Also

[optimizer](#)

### Examples

```
data(danaher)
Lindsey_MB(danaher)

# Dataset from table 3 follows; see also the example at Lindsey.Rd
mags <-
c(2463, 35, 44, 14, 16, 7, 262, 20, 2, 2, 0, 0, 0, 2, 17, 2,
0, 2, 0, 0, 3, 8, 0, 0, 1, 0, 0, 4, 8, 0, 1, 1, 0, 0, 3, 3,
0, 0, 0, 0, 0, 1, 52, 2, 1, 0, 2, 0, 22)
dim(mags) <- c(7,7)
mags <- Oarray::as.Oarray(mags,offset=0)
dimnames(mags) <-
```

```
list(AA=as.character(0:6),Sig=as.character(0:6)) # messy kludge in Lindsey_MB()
summary(Lindsey_MB(mags))
```

---

Extract.paras                      *Extract or Replace parameters of a paras object*

---

### Description

Methods for "[" and "[<-", i.e., extraction or subsetting of paras objects.

### Arguments

x	Object of class paras
i	Elements to extract or replace
value	Replacement value

### Value

Always returns an object of class paras.

### Methods

- x[i]
- x[i] <- value
- x[i,j]
- x[i,j] <- value

### Note

These methods are included for completeness; it's not clear to me that they are likely to be used by anyone. It might be better to always use constructions like `x <- paras(4) ; p(x)[2] <- 0.1` instead; YMMV.

### Author(s)

Robin K. S. Hankin

### Examples

```
x <- paras(4)
x[2] <- 0.1
x[1,2] <- 0.12
x
```

---

`gunter`*Convert from multiple multivariate observations to tabular form*

---

**Description**

Convert from a matrix with rows corresponding to multivariate observations, to a tabular form listing every possible combination together with the number of times that combination was observed.

**Usage**

```
gunter(obs)
## S3 method for class 'gunter'
print(x, ...)
```

**Arguments**

<code>obs</code>	Argument. If a matrix, interpret each row as a multivariate observation (so the rowsums are constant). If an object of class MB, interpret appropriately; if an Oarray, coerce to an MB object
<code>x</code>	Object of class gunter to be printed by the print method
<code>...</code>	Further arguments, currently ignored

**Value**

For matrices and data frames, function `gunter()` returns an object of class `gunter`: a list of two elements, the first being a matrix (`'obs'`) with rows being possible observations, and the second (`'d'`) a vector with one entry for each row of matrix `obs`.

For MB objects and Oarray objects, function `gunter()` returns an object of class `gunter_MB`.

The print method returns its argument, invisibly, after printing it coerced to a list.

**Author(s)**

Bert Gunter, with tiny alterations by Robin Hankin

**Examples**

```
data(wilson)
gunter(non_met)

data(danaher)
gunter(danaher) # object of class gunter_MB
```

**Description**

Function `Lindsey()` returns a maximum likelihood fit of the multiplicative multinomial using the Poisson device of Lindsey and Mersch (1992), and in the context of the multiplicative multinomial by Altham and Lindsey (1998).

Function `Lindsey_MB()` returns a maximum likelihood fit for the multivariate multiplicative binomial, for the special case of a bivariate distribution. An example of coercing a table to the correct form for use with `Lindsey_MB()` is given in the examples section below. Also, see `danaher` for another example.

**Usage**

```
Lindsey(obs, n = NULL, give_fit = FALSE)
Lindsey_MB(a)
## S3 method for class 'Lindsey_output'
print(x, ...)
```

**Arguments**

<code>obs</code>	In <code>Lindsey()</code> , an integer matrix with each row corresponding to an observation. All row sums must match
<code>n</code>	Vector with elements corresponding to the rows of <code>obs</code> ; default of <code>NULL</code> corresponds to observing each row of <code>obs</code> once
<code>a</code>	In <code>Lindsey_MB()</code> , an object that is coerced to one of class <code>gunter_MB</code> . Typically, the user supplies an <code>Oarray</code> object or an <code>MB</code> object
<code>give_fit</code>	Boolean, with default <code>FALSE</code> meaning to return just the fit, coerced to an object of class <code>paras</code> and <code>TRUE</code> meaning to return a list with two elements, the first being a <code>paras</code> object and the second being the fit returned by <code>glm()</code>
<code>x</code>	In the <code>print</code> method, object of class <code>Lindsey_output</code>
<code>...</code>	In the <code>print</code> method, further arguments, currently ignored

**Details**

Uses the device first described by Lindsey in 1992; the ‘meat’ of which has R idiom

```
Off <- -rowSums(lfactorial(jj$tbl))
```

```
glm(jj$d ~ -1 + offset(Off) + (. )^2, data=data, family=poisson)
```

Function `Lindsey(..., give_fit=TRUE)` returns an object of class `Lindsey_output`, which has its own `print` method (which prints the summary of the fit rather than use the default method).

Function `Lindsey(..., give_fit=FALSE)` returns an object of class `paras`, which can then be passed on to functions such as `rMM()`, which take a `paras` object.

Function `Lindsey_MB()` returns an object of class `glm`.

**Author(s)**

P. M. E. Altham and Robin K. S. Hankin

**References**

- J. K. Lindsey and G. Mersch 1992. “Fitting and comparing probability distributions with log linear models”, *Computational Statistics and Data Analysis*, 13(4):373–384
- P. M. E. Altham and J. K. Lindsey, 1998. “Analysis of the human sex ratio using overdispersion models”, *Applied Statistics*, 47:149–157

**See Also**

[gunter](#), [danaher](#)

**Examples**

```
data(voting)
(o <- Lindsey(voting, voting_tally))
rMM(10,5,o)

data(danaher)
Lindsey_MB(danaher)

## Not run:  #(takes a long time)
data(pollen)
Lindsey(pollen)

## End(Not run)

# Example of Lindsey_MB() in use follows.

a <- matrix(c(63,40,26,7,69,42,19,5,48,21,16,2,33,11,9,1,21,8,9,0,
              7,8,1,0,5,3,1,0,9,2,0,0),byrow=TRUE,ncol=4)

# Alternatively, you can get this from the pscl package as follows:
# library(pscl); data(bioChemists)
# a <- table(subset(bioChemists, fem == 'Men' & art < 8))

dimnames(a) <- list(papers=0:7,children=0:3)
require(Oarray)
a <- as.Oarray(a,offset=0)
# thus a[3,1]==11 means that 11 subjects had 3 papers and 1 child

summary(Lindsey_MB(a))
```

**Description**

Various utilities to coerce and manipulate MB objects

**Usage**

```
MB(dep, m, pnames=character(0))
## S3 method for class 'MB'
as.array(x, ...)
## S4 method for signature 'MB'
getM(x)
## S3 method for class 'gunter_MB'
print(x, ...)
```

**Arguments**

dep	Primary argument to MB(). Typically a matrix with each row being an observation (see ‘details’ section below for an example). If an object of class Oarray, function MB() coerces to an MB object
m	Vector containing the relative sizes of the various marginal binomial distributions
x	Object of class MB to be converted to an Oarray object
...	Further arguments to as.array(), currently ignored
pnames	In function MB(), a character vector of names for the entries

**Details**

Function MB() returns an object of class MB. This is essentially a matrix with one row corresponding to a single observation; repeated rows indicate identical observations as shown below. Observational data is typically in this form. The idea is that the user can coerce to a gunter\_MB object, which is then analyzable by Lindsey().

The multivariate multiplicative binomial distribution is defined by

$$\prod_{i=1}^t \binom{m_i}{x_i z_i} p_i^{x_i} q_i^{z_i} \theta_i^{x_i z_i} \prod_{i < j} \phi_{ij}^{x_i x_j}$$

Thus if  $\theta = \phi = 1$  the system reduces to a product of independent binomial distributions with probability  $p_i$  and size  $m_i$  for  $i = 1, \dots, t$ .

There follows a short R transcript showing the MB class in use, with annotation.

The first step is to define an m vector:

```
R> m <- c(2,3,1)
```

This means that  $m_1 = 2, m_2 = 3, m_3 = 1$ . So  $m_1 = 2$  means that  $i = 1$  corresponds to a binomial distribution with size 2 [that is, the observation is in the set  $\{0, 1, 2\}$ ]; and  $m_2 = 3$  means that  $i = 2$  corresponds to a binomial with size 3 [ie the set  $\{0, 1, 2, 3\}$ ].

Now we need some observations:

```
R> a <- matrix(c(1,0,0, 1,0,0, 1,1,1, 2,3,1, 2,0,1),5,3,byrow=T)
```

```
R> a
      [,1] [,2] [,3]
[1,]    1    0    0
[2,]    1    0    0
[3,]    1    1    1
[4,]    2    3    1
[5,]    2    0    1
```

In matrix a, the first observation, viz  $c(1,0,0)$  is interpreted as  $x_1 = 1, x_2 = 0, x_3 = 0$ . Thus, because  $x_i + z_i = m_i$ , we have  $z_1 = 1, z_2 = 3, z_3 = 1$ . Now we can create an object of class MB, using function MB():

```
R> mx <- MB(a, m, letters[1:3])
```

The third argument gives names to the observations corresponding to the columns of a. The values of  $m_1, m_2, m_3$  may be extracted using getM():

```
R> getM(mx)
a b c
2 3 1
R>
```

The getM() function returns a named vector, with names given as the third argument to MB().

Now we illustrate the print method:

```
R> mx
      a na      b nb      c nc
[1,] 1 1      0 3      0 1
[2,] 1 1      0 3      0 1
[3,] 1 1      1 2      1 0
[4,] 2 0      3 0      1 0
[5,] 2 0      0 3      1 0
R>
```

See how the columns are in pairs: the first pair total 2 (because  $m_1 = 2$ ), the second pair total 3 (because  $m_2 = 3$ ), and the third pair total 1 (because  $m_3 = 1$ ). Each pair of columns has only a single degree of freedom, because  $m_i$  is known.

Also observe how the column names are in pairs. The print method puts these in place. Take the first two columns. These are named 'a' and 'na': this is intended to mean 'a' and 'not a'.

We can now coerce to a gunter\_MB:

```
R> (gx <- gunter(mx))
$tbl
  a b c
1  0 0 0
2  1 0 0
3  2 0 0
[snip]
24 2 3 1

$d
[1] 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 1

$m
a b c
2 3 1
```

Take the second line of the element `tbl` of `gx`, as an example. This reads `c(1,0,0)` corresponding to the observations of `a,b,c` respectively, and the second line of element `d` [“d” for “data”], viz 2, shows that this observation occurred twice (and in fact these were the first two lines of `a`).

Now we can coerce object `mx` to an array:

```
R> (ax <- as.array(mx))
, , c = 0

  b
a  0 1 2 3
  0 0 0 0 0
  1 0 0 2 0
  2 0 0 0 0

, , c = 1

  b
a  0 1 2 3
  0 0 1 0 0
  1 0 0 0 0
  2 1 1 0 0
>
```

(actually, `ax` is an `0array` object). The location of an element in `ax` corresponds to an observation of `abc`, and the entry corresponds to the number of times that observation was made. For example, `ax[1,2,0]=2` shows that `c(1,2,0)` occurred twice (the first two lines of `a`).

The Lindsey Poisson device is applicable: see `help(danaher)` for an application to the bivariate case and `help(Lindsey)` for an example where a table is created from scratch.

### Author(s)

Robin K. S. Hankin

**See Also**

[MM, Lindsey, danaher](#)

**Examples**

```
a <- matrix(c(1,0,0, 1,0,0, 1,1,1, 2,3,1, 2,0,1),5,3,byrow=TRUE)
m <- c(2,3,1)
mx <- MB(a, m, letters[1:3]) # mx is of class 'MB'; column headings
                             # mean "a" and "not a".
ax <- as.array(mx)
gx <- gunter(ax)
ax2 <- as.array(gx)

data(danaher)
summary(Lindsey_MB(danaher))
```

---

MM

*Various multiplicative multinomial probability utilities*


---

**Description**

Various multiplicative multinomial probability utilities for different types of observation

**Usage**

```
MM(y, n=NULL, paras)
MM_allsums(y, n=NULL, paras)
MM_diffsums(y, n=NULL, paras)
MM_allsums_A(y, paras)
MM_diffsums_A(y, paras)
MM_single(yrow, paras, givelog=FALSE)
MM_support(paras, ss)
```

**Arguments**

y	Observations: a matrix, each row is a single observation
yrow	A single observation corresponding to one row of matrix y
n	Integer vector with one element for each row of y. Default value of NULL means to interpret each row of y as being observed once
ss	Sufficient statistics, as returned by <code>suffstats()</code>
givelog	Boolean in <code>MM_single()</code> with TRUE meaning to return the log likelihood and default FALSE meaning to return the likelihood
paras	Object of class <code>paras</code>

## Details

Consider non-negative integers  $y_1, \dots, y_k$  with  $\sum y_i = y$ . Then suppose the frequency function of the distribution  $Y_1, \dots, Y_k$  is

$$C \cdot \binom{y}{y_1, \dots, y_k} \prod_{i=1}^k p_i^{y_i} \prod_{1 \leq i < j \leq k} \theta_{ij}^{y_i y_j}$$

where  $p_1, \dots, p_k \geq 0$ ,  $\sum p_i = 1$  correspond to probabilities; and  $\theta_{ij} > 0$  for  $1 \leq i < j \leq k$  are additional parameters.

Here  $C$  stands for a normalization constant:

$$C = C(p, \theta, Y) = \sum_{y_1 + \dots + y_k = y} \prod_{i=1}^k p_i^{y_i} \prod_{1 \leq i < j \leq k} \theta_{ij}^{y_i y_j}$$

which is evaluated numerically. This is computationally expensive.

The usual case is to use function `MM()`.

- Function `MM()` returns the log of the probability of a matrix of rows of independent multinomial observations. It is a wrapper for `MM_allsamesum()` and `MM_differsums()`. Recall that optional argument `n` specifies the number of times that each row is observed. Calls `NormC()`.
- Function `MM_allsamesum()` gives the log of the probability of observing a matrix where the rowsums are identical. Calls `NormC()`.
- Function `MM_differsums()` gives the log of the probability of observing a matrix where the rowsums are not necessarily identical. **Warning:** This function takes a long time to run. Calls `NormC()`, possibly many times.
- Functions `MM_allsamesum_A()` and `MM_differsums_A()` are analogous to functions `MM_allsamesum()` and `MM_differsums()` but interpret the matrix `y` as having rows corresponding to observations; each row is observed once, as in `data(pollen)`. Both call `NormC()`.
- Function `MM_single()` gives a likelihood function for a `paras` object with a single multinomial observation (that is, a single line of matrix `y`). Does not call `NormC()`.
- Function `MM_support()` gives the support (that is, the log-likelihood) of a `paras` object; argument `ss` is the sufficient statistic, as returned by `suffstats()`. Does not call `NormC()`.
- Function `dMM()` [documented more fully at `rMM.Rd`] gives the probability of a single multivariate observation (ie a single row of the matrix argument `y`). Calls `NormC()`.

## Author(s)

Robin K. S. Hankin

## Examples

```
data(voting)
```

```
data(voting)
p <- Lindsey(voting, voting_tally)
```

```
MM(voting,voting_tally,p) #No other value of 'p' gives a bigger value
```

---

 multinomial

*Multinomial function*


---

### Description

The multinomial function and its logarithm

### Usage

```
multinomial(x)
lmultinomial(x)
```

### Arguments

x                    Numeric vector

### Details

Function `multinomial()` returns

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1! n_2! \dots n_k!}$$

where  $\sum_i n_i = n$ , and function `lmultinomial()` returns the natural logarithm of this.

### Note

Uses logarithmic functions to avoid overflow.

### Author(s)

Robin K. S. Hankin

### Examples

```
x <- runif(10)
exp(lmultinomial(x)) - multinomial(x) #should be small
```

---

NormC	<i>Normalizing constant for the multiplicative multinomial</i>
-------	--

---

**Description**

Calculates the normalizing constant for the multiplicative multinomial using direct numerical summation

**Usage**

```
NormC(Y, paras, log = FALSE)
```

**Arguments**

Y	Total number of observations
paras	Object of class paras
log	Boolean, with default FALSE meaning to return the value, and TRUE meaning to return the natural logarithm

**Author(s)**

Robin K. S. Hankin

**Examples**

```
jj <- paras(3)
theta(jj) <- 2
NormC(5, jj)
```

---

optimizer	<i>Maximum likelihood estimator for the MM</i>
-----------	--

---

**Description**

Maximum likelihood estimator for the MM

**Usage**

```
optimizer(y, n = NULL, start = NULL, method = "nlm",
          printing = FALSE, give_fit=FALSE, ...)
optimizer_allsums(y, n = NULL, start = NULL, method = "nlm",
                  printing = FALSE, give_fit=FALSE, ...)
optimizer_diffsums(y, n = NULL, start = NULL, method = "nlm",
                   printing = FALSE, give_fit=FALSE, ...)
```

**Arguments**

<code>y</code>	Matrix with each row being a possible observation
<code>n</code>	Counts of observations corresponding to rows of <code>y</code>
<code>start</code>	Start value for optimization routine, taken to be an object of class <code>paras</code> . Default value of <code>NULL</code> means to start with <code>Lindsey(y, n)</code> , which theoretically should be the maximum likelihood estimate
<code>method</code>	String giving which optimization method to use. Default of <code>Nelder</code> means to use <code>optim()</code> with the Nelder-Mead method; the other supported option is <code>nlm</code>
<code>printing</code>	Boolean, with <code>TRUE</code> meaning to print information as the optimization progresses and default <code>FALSE</code> meaning to print nothing
<code>give_fit</code>	Boolean, with default <code>FALSE</code> meaning to return the maximum likelihood estimate in the form of a <code>paras</code> object, and <code>TRUE</code> meaning to return a two-element list, the first being the output of <code>nlm()</code> or <code>optim()</code> and the second being the MLE
<code>...</code>	Further arguments passed to the optimization routine. In particular, note that <code>hessian=TRUE</code> is useful in conjunction with <code>give_fit=TRUE</code>

**Details**

Function `optimizer()` is the user-friendly version: it is a wrapper for `optimizer_samesum()` and `optimizer_differsums()`; it dispatches according to whether the rowsums are identical or not.

These functions are slow because they need to evaluate `NormC()` repeatedly, which is expensive.

Function `optimizer_samesum()` nominally produces the same output as `Lindsey()`, but is more computationally intensive.

**Author(s)**

Robin K. S. Hankin

**See Also**

[Lindsey](#)

**Examples**

```
data(voting)
p1 <- Lindsey(voting,voting_tally)
p2 <- optimizer(voting,voting_tally,start=p1)

theta(p1) - theta(p2) # Should be zero

## Not run:
data(pollen)
p1 <- optimizer(pollen)
p2 <- Lindsey(pollen)
theta(p1) - theta(p2) # Isn't zero...numerical scruff...
```

```
## End(Not run)
```

---

```
paras
```

---

*Manipulate a paras object*

---

## Description

Various utilities to manipulate paras objects. Functions `pnames()` and `pnames<-()` operate on MB objects as expected.

## Usage

```
paras(x, p, theta, pnames = character(0))
p(x) <- value
theta(x) <- value
p(x)
theta(x)
pnames(x)
pnames(x) <- value
getVals(x)
## S4 method for signature 'paras'
length(x)
```

## Arguments

<code>x</code>	Object of class <code>paras</code>
<code>p</code>	In function <code>paras()</code> , a vector of the first $k - 1$ elements of the probabilities
<code>theta</code>	In function <code>paras()</code> , a $k$ by $k$ matrix with diagonal composed of ones
<code>pnames</code>	In function <code>paras()</code> , a character vector of names for the entries
<code>value</code>	Replacement value

## Details

A `paras` object contains the parameters needed to specify a multiplicative multinomial distribution.

Suppose `p` is an object of class `paras` object. Then `p` is a list of two elements. The first element, `p`, is a vector of length `length(p)` and the second is an upper-diagonal matrix square matrix of size `length(p)`. The vignette gives further details.

The functions documented here allow the user to inspect and change `paras` objects.

## Author(s)

Robin K. S. Hankin

## See Also

[MM](#), [MB](#)

### Examples

```
jj <- paras(5)
pnames(jj) <- letters[1:5]
p(jj) <- c(0.1, 0.1, 0.3, 0.1)
theta(jj) <- matrix(1:25,5,5)
pnames(jj) <- letters[1:5]
jj

# OK, we've defined jj, now use it with some other functions:
dMM(rep(1,5),jj)
MM_single(1:5,jj)
rMM(2,9,jj)
```

---

pollen

*Pollen data from Mosimann 1962*

---

### Description

Data from Mosimann 1962 detailing forest pollen counts

### Usage

```
data(pollen)
```

### Format

A matrix with four columns and 76 rows.

### Details

The rows each sum to 100; the values are counts of four different types of pollen. Each row corresponds to a different level in the core; the levels are in sequence with the first row being most recent and the last row being the oldest.

### References

J. E. Mosimann 1962. "On the compound multinomial distribution, the multivariate  $\beta$ -distribution, and correlations among proportions". *Biometrika*, volume 49, numbers 1 and 2, pp65-82.

### Examples

```
## Not run:
data(pollen)
Lindsey(pollen)

## End(Not run)
```

---

powell                      *Dataset due to Powell (1990)*

---

**Description**

Dataset due to Powell (1990)

**Usage**

```
data(powell)
```

**Format**

A frequency table of counts of association data.

**Source**

- W. Powell, M. Coleman and J. McNicol 1990 “The statistical analysis of potato culture data”. *Plant Cell, Tissue and Organ Culture* 23:159-164

**Examples**

```
data(powell)
Lindsey(powell, powell_counts)
```

---

rMM                      *Random samples from the multiplicative multinomial*

---

**Description**

Density, and random samples drawn from, the multiplicative multinomial

**Usage**

```
rMM(n, Y, paras, burnin = 4*Y, every = 4*Y, start = NULL)
dMM(Y, paras)
```

**Arguments**

n	Number of observations to make
Y	Sum of each observation (for example, 100 for the pollen dataset, 4 for voting)
paras	Parameters of the MM distribution; an object of class paras
every	Each row is recorded every every steps through the Markov chain. Thus every=10 means every tenth row is written to the returned matrix during MH process (and the other nine values are discarded)
burnin	Number of initial observations to ignore
start	Observation to start simulation, with default NULL corresponding to using a random start vector

**Details**

Function `rMM()` uses standard Metropolis-Hastings simulation.

Function `dMM()` is documented here for convenience; see `help(MM)` for related functionality.

**Value**

Returns a matrix with `n` rows and `length(paras)` columns. Each row is an observation.

**Author(s)**

Robin K. S. Hankin

**See Also**

[MM](#)

**Examples**

```
data(voting)
rMM(10,4,Lindsey(voting,voting_tally))

p <- paras(3)
theta(p) <- 2
dMM(1:3,p)
```

---

skellam

*Brassica Dataset due to Catcheside*

---

**Description**

Dataset due to Catcheside, used by Skellam (1948) and subsequently by Altham (1978).

**Usage**

```
data(skellam)
```

**Format**

A frequency table of counts of association data.

**Source**

- J. G. Skellam 1948. “A probability distribution derived from the binomial distribution by regarding the probability of success as variable between the sets of trials”. *Journal of the Royal Statistical Society, series B (Methodological)*. Volume 10, number 2, pp257-248.
- D. Catcheside 1937. *Cytologia, Fujii Jub. Vol.*

**Examples**

```
data(skellam)
Lindsey(skellam, skellam_counts)
```

suffstats

*Sufficient statistics for the multiplicative multinomial***Description**

Calculate, manipulate, and display sufficient statistics of the multiplicative multinomial. Functionality for analysing datasets, and distributions specified by their parameters is given; summary and print methods are also documented here.

**Usage**

```
suffstats(y, n = NULL)
expected_suffstats(L, Y)
## S3 method for class 'suffstats'
print(x, ...)
## S3 method for class 'suffstats'
summary(object, ...)
## S3 method for class 'summary.suffstats'
print(x, ...)
```

**Arguments**

<code>y, n</code>	In function <code>suffstats()</code> , argument <code>y</code> is a matrix with each row being a possible observation and <code>n</code> is counts of observations corresponding to rows of <code>y</code> with default <code>NULL</code> interpreted as each row of <code>y</code> being observed once. If <code>y</code> is an object of class <code>gunter</code> , this is interpreted sensibly
<code>L, Y</code>	In function <code>expected_suffstats()</code> , argument <code>L</code> is an object of class <code>Lindsey</code> [typically returned by function <code>Lindsey()</code> ], and <code>Y</code> is the known constant sum (ie the <code>rowSums()</code> of the observations)
<code>x, object</code>	An object of class <code>suffstats</code> or <code>summary.suffstats</code> , to be printed or summarized
<code>...</code>	Further arguments to the print or summary methods. Currently ignored

**Details**

Function `suffstats()` returns a list comprising a set of sufficient statistics for the observations `y, [n]`.

This function requires that the rowsums of `y` are all identical.

**Value**

Function `suffstats()` returns a list of four components:

**Y** Rowsums of  $y$

**nobs** Number of observations

**row\_sums** Column sums of  $y$ , counted with multiplicity

**cross\_prods** Matrix of summed squares

Function `summary.suffstats()` provides a summary of a `suffstats` object that is a list with two elements: `row_sums` and `cross_prods`, normalized with `nobs` and `Y` so that the values are comparable with that returned by `expected_suffstats()`. In particular, the sum of `row_sums` is the known sum  $y$ .

**Author(s)**

Robin Hankin and P. M. E. Altham

**Examples**

```
data(voting)
suffstats(voting, voting_tally)
```

```
data(wilson)
wilson <- gunter(non_met)
suffstats(wilson)
```

```
L <- Lindsey(wilson)
```

```
expected_suffstats(L,5)
summary(suffstats(wilson)) ## matches.
```

```
summary(suffstats(rMM(10,5,L))) # should be close.
```

---

sweets

*Synthetic dataset due to Hankin*

---

**Description**

Four objects:

- `sweets` is a  $2 \times 3 \times 21$  array
- `sweets_tally` is a length 37 vector
- `sweets_array` is a  $2 \times 3 \times 37$  vector
- `sweets_table` is a  $37 \times 6$  matrix

**Usage**

```
data(sweets)
```

**Details**

Object `sweets` is the raw dataset; objects `sweets_table` and `sweets_tally` are processed versions which are easier to analyze.

The father of a certain family brings home nine sweets of type `mm` and nine sweets of type `jb` each day for 21 days to his children, AMH, ZJH, and AGH.

The children share the sweets amongst themselves in such a way that each child receives exactly 6 sweets.

- Array `sweets` has dimension  $c(2, 3, 21)$ : 2 types of sweets, 3 children, and 21 days. Thus `sweets[, , 1]` shows that on the first day, AMH chose 0 sweets of type `mm` and 6 sweets of type `jb`; child ZJH chose 3 of each, and child AGH chose 6 sweets of type `mm` and 0 sweets of type `jb`.

Observe the constant marginal totals: the kids have the same overall number of sweets each, and there are a fixed number of each kind of sweet.

- Array `sweets_array` has dimension  $c(2, 3, 37)$ : 2 sweets, 3 children, and 37 possible ways of arranging a matrix with the specified marginal totals. This can be produced by `allboards()` of the **aylmer** package.
- `sweets_table` is a dataframe with six columns, one for each combination of child and sweet, and 37 rows, each row showing a permissible arrangement. All possibilities are present. The six entries of `sweets[, , 1]` correspond to the six elements of `sweets_table[1, ]`; the column names are mnemonics.
- `sweets_tally` shows how often each of the arrangements in `sweets_tally` was observed (that is, it's a table of the 21 observations in `sweets`)

**Source**

The Hankin family

**Examples**

```
data(sweets)

# show correspondence between sweets_table and sweets_tally:
cbind(sweets_table, sweets_tally)

# Sum the data, by sweet and child and test:
fisher.test(apply(sweets, 1:2, sum))
# Not significant!

# Now test for overdispersion.
# First set up the regressors:
```

```

jj1 <- apply(sweets_array,3,tcrossprod)
jj2 <- apply(sweets_array,3, crossprod)
dim(jj1) <- c(2,2,37)
dim(jj2) <- c(3,3,37)

theta_xy <- jj1[1,2,]
phi_ab <- jj2[1,2,]
phi_ac <- jj2[1,3,]
phi_bc <- jj2[2,3,]

# Now the offset:
Off <- apply(sweets_array,3,function(x){-sum(lfactorial(x))})

# Now the formula:
f <- formula(sweets_tally~ -1 + theta_xy + phi_ab + phi_ac + phi_bc)

# Now the Lindsey Poisson device:
out <- glm(formula=f, offset=Off, family=poisson)

summary(out)
# See how the residual deviance is comparable with the degrees of freedom

```

---

voting

*Synthetic dataset of voting behaviour due to Altham*


---

### Description

Synthetic dataset of voting behaviour due to Altham

### Usage

```
data(voting)
```

### Format

voting is a three-column matrix with each row being a configuration of voting in a household with four members, and three choices. Vector voting\_tally is a list of how many households voted, and Nvoting\_tally is a more extreme dataset of the same type, used to uncover bugs in Lindsey().

### Source

Supplied by P. M. E. Altham

### Examples

```
data(voting)
Lindsey(voting,voting_tally)
```

---

wilson

*Housing Dataset due to Wilson*

---

**Description**

Dataset due to Wilson

**Usage**

```
data(wilson)
```

**Format**

Two objects, `met_area` and `non_met`, which have three columns and either 17 or 18 rows. Each row corresponds to a neighborhood of five households, each of which votes for one of three choices: US, S, or VS. Each column corresponds to one of these choices. The rowsums are constant because there are exactly five households in each neighborhood.

**Source**

- J. R. Wilson 1989. “Chi-square tests for Overdispersion with Multiparameter Estimates”, *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 38(3):441–453
- S. S. Brier 1980. “Analysis of Contingency Tables Under Cluster Sampling”, *Biometrika* 67(3):591–596

**Examples**

```
data(wilson)  
Lindsey(non_met)
```

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