

Package ‘MarkowitzR’

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Title Statistical Significance of the Markowitz Portfolio

BugReports <https://github.com/shabbychef/MarkowitzR/issues>

Description A collection of tools for analyzing significance of Markowitz portfolios, using the delta method on the second moment matrix, <[doi:10.48550/arXiv.1312.0557](https://doi.org/10.48550/arXiv.1312.0557)>.

Depends R (>= 3.0.2)

Imports matrixcalc, gtools

Suggests sandwich, SharpeR, testthat, formatR, knitr

URL <https://github.com/shabbychef/MarkowitzR>

VignetteBuilder knitr

Collate 'MarkowitzR.r' 'portinf.r' 'utils.r' 'vcov.r'

RoxygenNote 7.1.1

NeedsCompilation no

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Repository CRAN

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itheta_vcov *Compute variance covariance of Inverse 'Unified' Second Moment*

Description

Computes the variance covariance matrix of the inverse unified second moment matrix.

Usage

```
itheta_vcov(X,vcov.func=vcov,fit.intercept=TRUE)
```

Arguments

<code>X</code>	an $n \times p$ matrix of observed returns.
<code>vcov.func</code>	a function which takes an object of class <code>lm</code> , and computes a variance-covariance matrix. If equal to the string "normal", we assume multivariate normal returns.
<code>fit.intercept</code>	a boolean controlling whether we add a column of ones to the data, or fit the raw uncentered second moment. For now, must be true when assuming normal returns.

Details

Given p -vector x with mean μ and covariance, Σ , let y be x with a one prepended. Then let $\Theta = E(yy^T)$, the uncentered second moment matrix. The inverse of Θ contains the (negative) Markowitz portfolio and the precision matrix.

Given n contemporaneous observations of p -vectors, stacked as rows in the $n \times p$ matrix X , this function estimates the mean and the asymptotic variance-covariance matrix of Θ^{-1} .

One may use the default method for computing covariance, via the `vcov` function, or via a 'fancy' estimator, like `sandwich:vcovHAC`, `sandwich:vcovHC`, *etc.*

Value

a list containing the following components:

<code>mu</code>	a $q = (p + 1)(p + 2)/2$ vector of 1 + squared maximum Sharpe, the negative Markowitz portfolio, then the vech'd precision matrix of the sample data
<code>Ohat</code>	the $q \times q$ estimated variance covariance matrix.
<code>n</code>	the number of rows in X .
<code>pp</code>	the number of assets plus <code>as.numeric(fit.intercept)</code> .

Note

By flipping the sign of X , the inverse of Θ contains the *positive* Markowitz portfolio and the precision matrix on X . Performing this transform before passing the data to this function should be considered idiomatic.

A more general form of this function exists as `mp_vcov`.

Replaces similar functionality from SharpeR package, but with modified API.

Author(s)

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References

Pav, S. E. "Asymptotic Distribution of the Markowitz Portfolio." 2013 <https://arxiv.org/abs/1312.0557>

Pav, S. E. "Portfolio Inference with this One Weird Trick." R in Finance, 2014 <http://past.rinfinance.com/agenda/2014/talk/StevenPav.pdf>

See Also

[theta_vcov](#), [mp_vcov](#).

Examples

```
X <- matrix(rnorm(1000*3),ncol=3)
# putting in -X is idiomatic:
ism <- itheta_vcov(-X)
iSigmas.n <- itheta_vcov(-X,vcov.func="normal")
iSigmas.n <- itheta_vcov(-X,fit.intercept=FALSE)
# compute the marginal Wald test statistics:
qidx <- 2:ism$pp
wald.stats <- ism$mu[qidx] / sqrt(diag(ism$Ohat[qidx,qidx]))

# make it fat tailed:
X <- matrix(rt(1000*3,df=5),ncol=3)
ism <- itheta_vcov(X)
qidx <- 2:ism$pp
wald.stats <- ism$mu[qidx] / sqrt(diag(ism$Ohat[qidx,qidx]))

if (require(sandwich)) {
  ism <- itheta_vcov(X,vcov.func=vcovHC)
  qidx <- 2:ism$pp
  wald.stats <- ism$mu[qidx] / sqrt(diag(ism$Ohat[qidx,qidx]))
}

# add some autocorrelation to X
Xf <- filter(X,c(0.2),"recursive")
colnames(Xf) <- colnames(X)
ism <- itheta_vcov(Xf)
qidx <- 2:ism$pp
wald.stats <- ism$mu[qidx] / sqrt(diag(ism$Ohat[qidx,qidx]))

if (require(sandwich)) {
  ism <- itheta_vcov(Xf,vcov.func=vcovHAC)
  qidx <- 2:ism$pp
  wald.stats <- ism$mu[qidx] / sqrt(diag(ism$Ohat[qidx,qidx]))
}
```

Description

Inference on the Markowitz portfolio.

Markowitz Portfolio

Suppose x is a p -vector of returns of some assets with expected value μ and covariance Σ . The *Markowitz Portfolio* is the portfolio $w = \Sigma^{-1}\mu$. Scale multiples of this portfolio solve various portfolio optimization problems, among them

$$\operatorname{argmax}_{w: w^{\top} \Sigma w \leq R^2} \frac{\mu^{\top} w - r_0}{\sqrt{w^{\top} \Sigma w}}$$

This packages supports various statistical tests around the elements of the Markowitz Portfolio, and its Sharpe ratio, including the possibility of hedging, and scalar conditional heteroskedasticity and conditional expectation.

Legal Mumbo Jumbo

MarkowitzR is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU Lesser General Public License for more details.

Note

This package is maintained as a hobby.

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References

- Pav, S. E. "Asymptotic Distribution of the Markowitz Portfolio." 2013 <https://arxiv.org/abs/1312.0557>
- Pav, S. E. "Portfolio Inference with this One Weird Trick." R in Finance, 2014 <http://past.rinfinance.com/agenda/2014/talk/StevenPav.pdf>
- Britten-Jones, Mark. "The Sampling Error in Estimates of Mean-Variance Efficient Portfolio Weights." The Journal of Finance 54, no. 2 (1999): 655–671. <https://www.jstor.org/stable/2697722>
- Bodnar, Taras and Okhrin, Yarema. "On the Product of Inverse Wishart and Normal Distributions with Applications to Discriminant Analysis and Portfolio Theory." Scandinavian Journal of Statistics 38, no. 2 (2011): 311–331. [doi:10.1111/j.14679469.2011.00729.x](https://doi.org/10.1111/j.14679469.2011.00729.x)

Markowitz, Harry. "Portfolio Selection." The Journal of Finance 7, no. 1 (1952): 77–91. <https://www.jstor.org/stable/2975974>

Brandt, Michael W. "Portfolio Choice Problems." Handbook of Financial Econometrics 1 (2009): 269–336. <https://scholars.duke.edu/publication/964964>

See Also

Useful links:

- <https://github.com/shabbychef/MarkowitzR>
- Report bugs at <https://github.com/shabbychef/MarkowitzR/issues>

MarkowitzR-NEWS

News for package 'MarkowitzR':

Description

News for package 'MarkowitzR'

Changes in **MarkowitzR** Version 1.0.3 (2023-08-20)

- fix package documentation.

Changes in **MarkowitzR** Version 1.0.2 (2020-01-07)

- emergency fix for broken tests under atlas on CRAN.

Changes in **MarkowitzR** Version 1.0.1 (2018-05-25)

- move figures around for README on CRAN.

Changes in **MarkowitzR** Version 0.9900 (2016-09-15)

- yet again, conform to CRAN rules.

Changes in **MarkowitzR** Version 0.1502 (2015-01-26)

- conform to CRAN rules.

Changes in **MarkowitzR** Version 0.1403 (2014-06-01)

- fix bug preventing multi-row hedging or constraint matrices.

MarkowitzR Initial Version 0.1402 (2014-02-14)

- first CRAN release.

mp_vcov

*Estimate Markowitz Portfolio***Description**

Estimates the Markowitz Portfolio or Markowitz Coefficient subject to subspace and hedging constraints, and heteroskedasticity.

Usage

```
mp_vcov(X, feat=NULL, vcov.func=vcov, fit.intercept=TRUE, weights=NULL, Jmat=NULL, Gmat=NULL)
```

Arguments

<code>X</code>	an $n \times p$ matrix of observed returns.
<code>feat</code>	an $n \times f$ matrix of observed features. defaults to none, in which case <code>fit.intercept</code> must be TRUE. If <code>fit.intercept</code> is true, ones will be prepended to the features.
<code>vcov.func</code>	a function which takes an object of class <code>lm</code> , and computes a variance-covariance matrix. If equal to the string "normal", we assume multivariate normal returns.
<code>fit.intercept</code>	a boolean controlling whether we add a column of ones to the data, or fit the raw uncentered second moment. For now, must be true when assuming normal returns.
<code>weights</code>	an optional n vector of the weights. The returns and features will be multiplied by the weights. Weights should be inverse volatility estimates. Defaults to homoskedasticity.
<code>Jmat</code>	an optional $p_j \times p$ matrix of the subspace in which we constrain portfolios. Defaults essentially to the $p \times p$ identity matrix.
<code>Gmat</code>	an optional $p_g \times p$ matrix of the subspace to which we constrain portfolios to have zero covariance. The rowspace of <code>Gmat</code> must be spanned by the rowspace of <code>Jmat</code> . Defaults essentially to the $0 \times p$ empty matrix.

Details

Suppose that the expectation of p -vector x is linear in the f -vector f , but the covariance of x is stationary and independent of f . The 'Markowitz Coefficient' is the $p \times f$ matrix W such that, conditional on observing f , the portfolio Wf maximizes Sharpe. When f is the constant 1, the Markowitz Coefficient is the traditional Markowitz Portfolio.

Given n observations of the returns and features, given as matrices X, F , this code computes the Markowitz Coefficient along with the variance-covariance matrix of the Coefficient and the precision matrix. One may give optional weights, which are inverse conditional volatility. One may also give optional matrix J, G which define subspace and hedging constraints. Briefly, they constrain the portfolio optimization problem to portfolios in the row space of J and with zero covariance with the rows of G . It must be the case that the rows of J span the rows of G . J defaults to the $p \times p$ identity matrix, and G defaults to a null matrix.

One may use the default method for computing covariance, via the `vcov` function, or via a 'fancy' estimator, like `sandwich:vcovHAC`, `sandwich:vcovHC`, *etc.*

Value

a list containing the following components:

mu	Letting $r = f + p + fit.intercept$, this is a $q = (r)(r + 1)/2$ vector...
Ohat	The $q \times q$ estimated variance covariance matrix of mu.
W	The estimated Markowitz coefficient, a $p \times (fit.intercept + f)$ matrix. The first column corresponds to the intercept term if it is fit. Note that for convenience this function performs the sign flip, which is not performed on mu.
What	The estimated variance covariance matrix of $vech(W)$. Letting $s = p(fit.intercept + f)$, this is a $s \times s$ matrix.
widxs	The indices into mu giving W, and into Ohat giving What.
n	The number of rows in X.
ff	The number of features plus $as.numeric(fit.intercept)$.
p	The number of assets.

Note

Should also modify to include the theta estimates.

Author(s)

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References

Pav, S. E. "Asymptotic Distribution of the Markowitz Portfolio." 2013 <https://arxiv.org/abs/1312.0557>

Pav, S. E. "Portfolio Inference with this One Weird Trick." R in Finance, 2014 <http://past.rinfinance.com/agenda/2014/talk/StevenPav.pdf>

See Also

[theta_vcov](#), [itheta_vcov](#)

Examples

```
set.seed(1001)
X <- matrix(rnorm(1000*3),ncol=3)
ism <- mp_vcov(X,fit.intercept=TRUE)
walds <- ism$W / sqrt(diag(ism$What))
print(t(walds))
# subspace constraint
Jmat <- matrix(rnorm(6),ncol=3)
ism <- mp_vcov(X,fit.intercept=TRUE,Jmat=Jmat)
walds <- ism$W / sqrt(diag(ism$What))
print(t(walds))
# hedging constraint
Gmat <- matrix(1,nrow=1,ncol=3)
```

```

ism <- mp_vcov(X,fit.intercept=TRUE,Gmat=Gmat)
walds <- ism$W / sqrt(diag(ism$What))

# now conditional expectation:

# generate data with given W, Sigma
Xgen <- function(W,Sigma,Feat) {
  Btrue <- Sigma %**% W
  Xmean <- Feat %**% t(Btrue)
  Shalf <- chol(Sigma)
  X <- Xmean + matrix(rnorm(prod(dim(Xmean))),ncol=dim(Xmean)[2]) %**% Shalf
}

n.feat <- 2
n.ret <- 8
n.obs <- 10000
set.seed(101)
Feat <- matrix(rnorm(n.obs * n.feat),ncol=n.feat)
Wtrue <- 10 * matrix(rnorm(n.feat * n.ret),ncol=n.feat)
Sigma <- cov(matrix(rnorm(100*n.ret),ncol=n.ret))
Sigma <- Sigma + diag(seq(from=1,to=3,length.out=n.ret))
X <- Xgen(Wtrue,Sigma,Feat)
ism <- mp_vcov(X,feat=Feat,fit.intercept=TRUE)
Wcomp <- cbind(0,Wtrue)
errs <- ism$W - Wcomp
dim(errs) <- c(length(errs),1)
Zerr <- solve(t(chol(ism$What)),errs)

```

theta_vcov

Compute variance covariance of 'Unified' Second Moment

Description

Computes the variance covariance matrix of sample mean and second moment.

Usage

```
theta_vcov(X,vcov.func=vcov,fit.intercept=TRUE)
```

Arguments

X	an $n \times p$ matrix of observed returns.
vcov.func	a function which takes an object of class <code>lm</code> , and computes a variance-covariance matrix. If equal to the string "normal", we assume multivariate normal returns.
fit.intercept	a boolean controlling whether we add a column of ones to the data, or fit the raw uncentered second moment. For now, must be true when assuming normal returns.

Details

Given p -vector x , the 'unified' sample is the $(p+1)(p+2)/2$ vector of 1, x , and $\text{vech}(xx^\top)$ stacked on top of each other. Given n contemporaneous observations of p -vectors, stacked as rows in the $n \times p$ matrix X , this function computes the mean and the variance-covariance matrix of the 'unified' sample.

One may use the default method for computing covariance, via the `vcov` function, or via a 'fancy' estimator, like `sandwich:vcovHAC`, `sandwich:vcovHC`, *etc.*

Value

a list containing the following components:

<code>mu</code>	a $q = (p + 1)(p + 2)/2$ vector of 1, then the mean, then the vech'd second moment of the sample data.
<code>Ohat</code>	the $q \times q$ estimated variance covariance matrix. When <code>fit.intercept</code> is true, the left column and top row are all zeros.
<code>n</code>	the number of rows in X .
<code>pp</code>	the number of assets plus <code>as.numeric(fit.intercept)</code> .

Note

Replaces similar functionality from SharpeR package, but with modified API.

Author(s)

Steven E. Pav <shabbychef@gmail.com>

References

- Pav, S. E. "Asymptotic Distribution of the Markowitz Portfolio." 2013 <https://arxiv.org/abs/1312.0557>
- Pav, S. E. "Portfolio Inference with this One Weird Trick." R in Finance, 2014 <http://past.rinfinance.com/agenda/2014/talk/StevenPav.pdf>

See Also

[itheta_vcov](#).

Examples

```
X <- matrix(rnorm(1000*3), ncol=3)
Sigmas <- theta_vcov(X)
Sigmas.n <- theta_vcov(X,vcov.func="normal")
Sigmas.n <- theta_vcov(X,fit.intercept=FALSE)

# make it fat tailed:
X <- matrix(rt(1000*3,df=5), ncol=3)
Sigmas <- theta_vcov(X)
```

```
if (require(sandwich)) {  
  Sigmas <- theta_vcov(X,vcov.func=vcovHC)  
}  
  
# add some autocorrelation to X  
Xf <- filter(X,c(0.2),"recursive")  
colnames(Xf) <- colnames(X)  
Sigmas <- theta_vcov(Xf)  
  
if (require(sandwich)) {  
  Sigmas <- theta_vcov(Xf,vcov.func=vcovHAC)  
}
```

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