

# Package ‘NPP’

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**Type** Package

**Title** Normalized Power Prior Bayesian Analysis

**Version** 0.7.0

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**Description** Posterior sampling in several commonly used distributions using normalized power prior as described in Duan, Ye and Smith (2006) <[doi:10.1002/env.752](https://doi.org/10.1002/env.752)> and Ibrahim et.al. (2015) <[doi:10.1002/sim.6728](https://doi.org/10.1002/sim.6728)>. Sampling of the power parameter is achieved via either independence Metropolis-Hastings or random walk Metropolis-Hastings based on transformation.

**Imports** MASS, mvtnorm, KernSmooth

**License** GPL (>= 2)

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## Contents

BerMNPP_MCMC1	2
BerMNPP_MCMC2	4
BerNPP_MCMC	6
BerOMNPP_MCMC1	8

BerOMNPP_MCMC2	10
IRTNPP	11
LaplacelogC	13
LMMNPP_MCMC1	14
LMMNPP_MCMC2	16
LMNPP_MCMC	19
LMOMNPP_MCMC1	22
LMOMNPP_MCMC2	24
logCdelta	26
logCknot	27
loglikBerD0	28
loglikNormD0	29
ModeDeltaBerNPP	30
ModeDeltaLMNPP	32
ModeDeltaMultinomialNPP	33
ModeDeltaNormalNPP	35
ModeDeltaPoisNPP	37
MultinomialNPP_MCMC	39
NormalNPP_MCMC	41
PHData	43
PoiMNPP_MCMC1	44
PoiMNPP_MCMC2	46
PoiOMNPP_MCMC1	47
PoiOMNPP_MCMC2	49
PoissonNPP_MCMC	50
SPDData	52
VaccineData	53
<b>Index</b>	<b>55</b>

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BerMNPP_MCMC1	<i>MCMC Sampling for Bernoulli Population with Multiple Historical Data using Normalized Power Prior</i>
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### Description

Incorporate multiple historical data sets for posterior sampling of a Bernoulli population using the normalized power prior. The Metropolis-Hastings algorithm, with either an independence proposal or a random walk proposal on the logit scale, is applied for the power parameter  $\delta$ . Gibbs sampling is utilized for the model parameter  $p$ .

### Usage

```
BerMNPP_MCMC1(n0, y0, n, y, prior_p, prior_delta_alpha,
               prior_delta_beta, prop_delta_alpha, prop_delta_beta,
               delta_ini, prop_delta, rw_delta, nsample, burnin, thin)
```

**Arguments**

<code>n0</code>	A non-negative integer vector representing the number of trials in historical data.
<code>y0</code>	A non-negative integer vector denoting the number of successes in historical data.
<code>n</code>	A non-negative integer indicating the number of trials in the current data.
<code>y</code>	A non-negative integer for the number of successes in the current data.
<code>prior_p</code>	a vector of the hyperparameters in the prior distribution $Beta(\alpha, \beta)$ for $p$ .
<code>prior_delta_alpha</code>	a vector of the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
<code>prior_delta_beta</code>	a vector of the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
<code>prop_delta_alpha</code>	a vector of the hyperparameter $\alpha$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
<code>prop_delta_beta</code>	a vector of the hyperparameter $\beta$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
<code>delta_ini</code>	the initial value of $\delta$ in MCMC sampling.
<code>prop_delta</code>	the class of proposal distribution for $\delta$ .
<code>rw_delta</code>	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if <code>prop_delta = 'RW'</code> .
<code>nsample</code>	specifies the number of posterior samples in the output.
<code>burnin</code>	the number of burn-ins. The output will only show MCMC samples after burnin.
<code>thin</code>	the thinning parameter in MCMC sampling.

**Details**

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ . The normalized power prior distribution is

$$\frac{\pi_0(\delta)\pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{\delta_k} d\theta}.$$

Here  $\pi_0(\delta)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\delta$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\delta_k$  is the corresponding power parameter.

**Value**

A list of class "NPP" comprising:

<code>acceptrate</code>	Acceptance rate in MCMC sampling for $\delta$ via the Metropolis-Hastings algorithm.
<code>p</code>	Posterior distribution of the model parameter $p$ .
<code>delta</code>	Posterior distribution of the power parameter $\delta$ .

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**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[BerMNPP\\_MCMC2](#); [BerOMNPP\\_MCMC1](#); [BerOMNPP\\_MCMC2](#)

**Examples**

```
BerMNPP_MCMC1(n0 = c(275, 287), y0 = c(92, 125), n = 39, y = 17,
  prior_p = c(1/2,1/2), prior_delta_alpha = c(1/2,1/2),
  prior_delta_beta = c(1/2,1/2),
  prop_delta_alpha = c(1,1)/2, prop_delta_beta = c(1,1)/2,
  delta_ini = NULL, prop_delta = "IND",
  nsample = 2000, burnin = 500, thin = 2)
```

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BerMNPP\_MCMC2

*MCMC Sampling for Bernoulli Population of multiple historical data using Normalized Power Prior*

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**Description**

Multiple historical data are combined individually. The NPP of multiple historical data is the product of the NPP of each historical data. Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter  $p$ , Gibbs sampling is used.

**Usage**

```
BerMNPP_MCMC2(n0, y0, n, y, prior_p, prior_delta_alpha, prior_delta_beta,
  prop_delta_alpha, prop_delta_beta, delta_ini, prop_delta,
  rw_delta, nsample, burnin, thin)
```

**Arguments**

`n0` a non-negative integer vector: number of trials in historical data.  
`y0` a non-negative integer vector: number of successes in historical data.  
`n` a non-negative integer: number of trials in the current data.

y	a non-negative integer: number of successes in the current data.
prior_p	a vector of the hyperparameters in the prior distribution $Beta(\alpha, \beta)$ for $p$ .
prior_delta_alpha	a vector of the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prior_delta_beta	a vector of the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prop_delta_alpha	a vector of the hyperparameter $\alpha$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prop_delta_beta	a vector of the hyperparameter $\beta$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
delta_ini	the initial value of $\delta$ in MCMC sampling.
prop_delta	the class of proposal distribution for $\delta$ .
rw_delta	the stepsize (variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if <code>prop_delta = 'RW'</code> .
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.

### Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ . The normalized power prior distribution is

$$\pi_0(\delta) \prod_{k=1}^K \frac{\pi_0(\theta) L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta) L(\theta|D_{0k})^{\delta_k} d\theta}.$$

Here  $\pi_0(\delta)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\delta$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\delta_k$  is the corresponding power parameter.

### Value

A list of class "NPP" with three elements:

acceptrate	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
p	posterior of the model parameter $p$ .
delta	posterior of the power parameter $\delta$ .

### Author(s)

Qiang Zhang <zqzjf0408@163.com>

## References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

## See Also

[BerMNPP\\_MCMC1](#); [BerOMNPP\\_MCMC1](#); [BerOMNPP\\_MCMC2](#)

## Examples

```
BerMNPP_MCMC2(n0 = c(275, 287), y0 = c(92, 125), n = 39, y = 17,
  prior_p=c(1/2,1/2), prior_delta_alpha=c(1/2,1/2),
  prior_delta_beta=c(1/2,1/2), prop_delta_alpha=c(1,1)/2,
  prop_delta_beta=c(1,1)/2, delta_ini=NULL, prop_delta="IND",
  nsample = 2000, burnin = 500, thin = 2)
```

---

BerNPP\_MCMC

*MCMC Sampling for Bernoulli Population using Normalized Power Prior*

---

## Description

Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter  $p$ , Gibbs sampling is used.

## Usage

```
BerNPP_MCMC(Data.Cur = c(100, 50), Data.Hist = c(100, 50),
  CompStat = list(n0 = NULL, y0 = NULL, n1 = NULL, y1 = NULL),
  prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'IND', rw.logit.delta = 0.1,
  ind.delta.alpha = 1, ind.delta.beta = 1, nsample = 5000,
  control.mcmc = list(delta.ini = NULL, burnin = 0, thin = 1))
```

## Arguments

Data.Cur	a non-negative integer vector of two elements: c(number of trials, number of successes) in the current data.
Data.Hist	a non-negative integer vector of two elements: c(number of trials, number of successes) in the historical data.

CompStat	<p>a list of four elements that represents the "compatibility(sufficient) statistics" for <math>p</math>. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in Bernoulli population providing CompStat is equivalent to provide the data summary as in Data.Cur and Data.Cur.</p> <p><math>n_0</math> is the number of trials in the historical data.  <math>y_0</math> is the number of successes in the historical data.  <math>n_1</math> is the number of trials in the current data.  <math>y_1</math> is the number of successes in the current data.</p>
prior	<p>a list of the hyperparameters in the prior for both <math>p</math> and <math>\delta</math>.</p> <p>p.alpha is the hyperparameter <math>\alpha</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>p</math>.  p.beta is the hyperparameter <math>\beta</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>p</math>.  delta.alpha is the hyperparameter <math>\alpha</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>\delta</math>.  delta.beta is the hyperparameter <math>\beta</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>\delta</math>.</p>
MCMCmethod	sampling method for $\delta$ in MCMC. It can be either 'IND' for independence proposal; or 'RW' for random walk proposal on logit scale.
rw.logit.delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if MCMCmethod = 'RW' .
ind.delta.alpha	specifies the first parameter $\alpha$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
ind.delta.beta	specifies the first parameter $\beta$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
nsample	specifies the number of posterior samples in the output.
control.mcmc	<p>a list of three elements used in posterior sampling.</p> <p>delta.ini is the initial value of <math>\delta</math> in MCMC sampling.  burnin is the number of burn-ins. The output will only show MCMC samples after burnin.  thin is the thinning parameter in MCMC sampling.</p>

### Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ , and the deviance information criteria.

### Value

A list of class "NPP" with four elements:

p	posterior of the model parameter $p$ .
delta	posterior of the power parameter $\delta$ .
acceptance	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
DIC	the deviance information criteria for model diagnostics.

**Author(s)**

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**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[MultinomialNPP\\_MCMC](#); [NormalNPP\\_MCMC](#); [PoissonNPP\\_MCMC](#)

**Examples**

```
BerNPP_MCMC(Data.Cur = c(493, 473), Data.Hist = c(680, 669),
  prior = list(p.alpha = 0.5, p.beta = 0.5, delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'RW', rw.logit.delta = 1, nsample = 5000,
  control.mcmc = list(delta.ini = NULL, burnin = 2000, thin = 5))
```

---

BerOMNPP\_MCMC1

*MCMC Sampling for Bernoulli Population of multiple ordered historical data using Normalized Power Prior*

---

**Description**

Multiple ordered historical data are incorporated together. Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter  $\gamma$ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter  $p$ , Gibbs sampling is used.

**Usage**

```
BerOMNPP_MCMC1(n0, y0, n, y, prior_gamma, prior_p, gamma_ind_prop,
  gamma_ini, nsample, burnin, thin, adjust = FALSE)
```

**Arguments**

- |                          |   |
|--------------------------|---|
| <code>n0</code>          | a non-negative integer vector: number of trials in historical data.   |
| <code>y0</code>          | a non-negative integer vector: number of successes in historical data.  |
| <code>n</code>           | a non-negative integer: number of trials in the current data.   |
| <code>y</code>           | a non-negative integer: number of successes in the current data.  |
| <code>prior_gamma</code> | a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ . |
| <code>prior_p</code>     | a vector of the hyperparameters in the prior distribution $Beta(\alpha, \beta)$ for $p$ .                                 |

gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
gamma_ini	the initial value of $\gamma$ in MCMC sampling.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.
adjust	Logical, indicating whether or not to adjust the parameters of the proposal distribution.

### Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\gamma$ . The normalized power prior distribution is given by:

$$\frac{\pi_0(\gamma)\pi_0(\theta) \prod_{k=1}^K L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta) \prod_{k=1}^K L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\theta}$$

Here,  $\pi_0(\gamma)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\gamma$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\sum_{i=1}^k \gamma_i$  is the corresponding power parameter.

### Value

A list of class "NPP" with three elements:

acceptrate	the acceptance rate in MCMC sampling for $\gamma$ using Metropolis-Hastings algorithm.
p	posterior of the model parameter $p$ .
delta	posterior of the power parameter $\delta$ . It is equal to the cumulative sum of $\gamma$ .

### Author(s)

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### References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

### See Also

[BerMNPP\\_MCMC1](#), [BerMNPP\\_MCMC2](#), [BerOMNPP\\_MCMC2](#)

### Examples

```
BerOMNPP_MCMC1(n0 = c(275, 287), y0 = c(92, 125), n = 39, y = 17, prior_gamma=c(1,1,1)/3,
prior_p=c(1/2,1/2), gamma_ind_prop=rep(1,3)/2, gamma_ini=NULL,
nsample = 2000, burnin = 500, thin = 2, adjust = FALSE)
```

BerOMNPP\_MCMC2

*MCMC Sampling for Bernoulli Population of multiple ordered historical data using Normalized Power Prior***Description**

Multiple ordered historical data are combined individually. Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter  $\gamma$ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter  $p$ , Gibbs sampling is used.

**Usage**

```
BerOMNPP_MCMC2(n0, y0, n, y, prior_gamma, prior_p, gamma_ind_prop, gamma_ini,
               nsample, burnin, thin, adjust = FALSE)
```

**Arguments**

<code>n0</code>	a vector of non-negative integers: numbers of trials in historical data.
<code>y0</code>	a vector of non-negative integers: numbers of successes in historical data.
<code>n</code>	a non-negative integer: number of trials in the current data.
<code>y</code>	a non-negative integer: number of successes in the current data.
<code>prior_gamma</code>	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
<code>prior_p</code>	a vector of the hyperparameters in the prior distribution $Beta(\alpha, \beta)$ for $p$ .
<code>gamma_ind_prop</code>	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
<code>gamma_ini</code>	the initial value of $\gamma$ in MCMC sampling.
<code>nsample</code>	specifies the number of posterior samples in the output.
<code>burnin</code>	the number of burn-ins. The output will only show MCMC samples after burn-in.
<code>thin</code>	the thinning parameter in MCMC sampling.
<code>adjust</code>	Whether or not to adjust the parameters of the proposal distribution.

**Details**

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\gamma$ . The normalized power prior distribution is

$$\pi_0(\gamma) \prod_{k=1}^K \frac{\pi_0(\theta) L(\theta | D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta) L(\theta | D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\theta}.$$

Here  $\pi_0(\gamma)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\gamma$  and  $\theta$ , respectively.  $L(\theta | D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\sum_{i=1}^k \gamma_i$  is the corresponding power parameter.

**Value**

A list of class "NPP" with three elements:

acceptrate	the acceptance rate in MCMC sampling for $\gamma$ using Metropolis-Hastings algorithm.
p	posterior of the model parameter $p$ .
delta	posterior of the power parameter $\delta$ . It is equal to the cumulative sum of $\gamma$

**Author(s)**

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**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[BerMNPP\\_MCMC1](#); [BerMNPP\\_MCMC2](#); [BerOMNPP\\_MCMC1](#)

**Examples**

```
BerOMNPP_MCMC2(n0 = c(275, 287), y0 = c(92, 125), n = 39, y = 17, prior_gamma=c(1,1,1)/3,
  prior_p=c(1/2,1/2), gamma_ind_prop=rep(1,3)/2, gamma_ini=NULL,
  nsample = 2000, burnin = 500, thin = 2, adjust = FALSE)
```

---

IRTNPP	<i>MCMC Sampling for IRT Model Ability Parameters using Normalized Power Prior</i>
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---

**Description**

Conduct posterior sampling for IRT model ability parameters with normalized power prior.

For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used.

For the model parameters  $\beta$ , a Metropolis-Hastings algorithm with either normal proposal, or uniform proposal is used.

**Usage**

```
IRTNPP(y, dseq, prior_mu, prior_sd, MCsize, disa, difa1, difa2,
  cut, prior_beta, prior_delta, disb, difb1, difb2,
  prop_delta, rw_delta, rw_n_beta, rw_u_beta, ind_delta,
  prop_beta, n_sample, burnin, thin)
```

**Arguments**

<code>y</code>	a vector that contains historical data and current data, where the first half consists of historical data and the second half consists of current data.
<code>dseq</code>	numeric vector or scalar between 0 and 1. The value of $\delta$ .
<code>prior_mu</code>	the prior mean of each ability parameter $\beta$ .
<code>prior_sd</code>	the prior standard deviation of each ability parameter $\beta$ .
<code>Mysize</code>	positive integer. Sample size of importance sampling.
<code>disa</code>	a matrix of item discriminability parameters in historical data.
<code>difa1</code>	a vector of the first difficulty parameter of items in historical data.
<code>difa2</code>	a vector of the second difficulty parameter of items in historical data.
<code>cut</code>	critical value between 0 and 1. If $\delta$ is less than or equal to this value, select the initial prior of $\beta$ as the proposed distribution of importance sampling; Otherwise, select the posterior distribution with historical data of $\beta$ as the proposed distribution.
<code>prior_beta</code>	list. Parameters of normal prior for $\beta$ .
<code>prior_delta</code>	list. Parameters of beta prior for $\delta$ .
<code>disb</code>	a matrix of item discriminability parameters in current data.
<code>difb1</code>	a vector of the first difficulty parameter of items in current data.
<code>difb2</code>	a vector of the second difficulty parameter of items in current data.
<code>prop_delta</code>	character. The class of proposal distribution for $\delta$ .
<code>rw_delta</code>	numeric. The stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if <code>prop_delta = 'RW'</code> .
<code>prop_beta</code>	character. The class of proposal distribution for $\beta$ .
<code>rw_n_beta</code>	numeric vector. Standard deviation of proposed distribution of for $\beta$ .
<code>rw_u_beta</code>	numeric. <code>rw_u_beta*2</code> is the interval length of uniform distribution.
<code>ind_delta</code>	numeric vector. Two parameters when the proposed distribution of $\delta$ is beta distribution.
<code>n_sample</code>	positive integer. Specifies the number of posterior samples in the output.
<code>burnin</code>	positive integer. The output will only show MCMC samples after burnin.
<code>thin</code>	positive integer. The thinning parameter in MCMC sampling.

**Details**

This function needs three additional R packages: KernSmooth, msm, mvtnorm.

This function needs two additional R functions: makePositiveDefinite, Metro\_Hastings.

The outputs include the posterior estimates of the ability parameters of the IRT model and power parameter, as well as the acceptance rates in sampling  $\delta$  and  $\beta$ .

**Value**

A vector consisting of 5 parts: the acceptance rate in MCMC sampling for  $\beta$  and  $\delta$  using Metropolis-Hastings algorithm, the posterior mean of  $\beta$  and  $\delta$ , the posterior standard deviation of  $\beta$  and  $\delta$ , the posterior median of  $\beta$  and  $\delta$ , and the posterior mode of power parameter  $\delta$ .

**Author(s)**

Qiang Zhang <zqzjf0408@163.com>

**References**

Chalmers, R.P. (2012). mirt: A multidimensional item response theory package for the R environment. *Journal of Statistical Software* 48:1–29.

Matteucci, M., Veldkamp, B. (2015). The approach of power priors for ability estimation in IRT models. *Qual Quant* 49:917–926.

Han, Z., Zhang, Q., Wang, M., Ye, K., Chen, M.H. (2023). On efficient posterior inference in normalized power prior Bayesian analysis. *Biometrical Journal* 65:2200194.

---

LaplaceLogC

*A Function to Calculate  $\log C(\delta)$  Based on Laplace Approximation*


---

**Description**

The function assumes that the prior of the model parameters is very flat that had very minor impact on the shape of the power prior (posterior based on the D0).

**Usage**

```
LaplaceLogC(delta, loglikmle, detHessian, ntheta)
```

**Arguments**

delta	the power parameter between 0 and 1. The function returns $\log C(\delta)$
loglikmle	a scalar; the loglikelihood of the historical data evaluated at the maximum likelihood estimates based on the historical data
detHessian	determinant of the Hessian matrix evaluated at the loglikelihood function with respect to the maximum likelihood estimates based on the historical data
ntheta	an positive integer indicating number of parameters in the model

**Value**

$\log C(\delta)$  based on the Laplace approximation. Can be used for the posterior sampling in the normalized power prior.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

## References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

## See Also

[logCknot](#)

---

LMMNPP\_MCMC1

*MCMC Sampling for Linear Regression Model of multiple historical data using Normalized Power Prior*

---

## Description

Multiple historical data are incorporated together. Conduct posterior sampling for Linear Regression Model with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameters  $(\beta, \sigma^2)$ , Gibbs sampling is used.

## Usage

```
LMMNPP_MCMC1(D0, X, Y, a0, b, mu0, R, delta_ini, prop_delta,
              prior_delta_alpha, prior_delta_beta, prop_delta_alpha,
              prop_delta_beta, rw_delta, nsample, burnin, thin)
```

## Arguments

- |                   |  |
|-------------------|--|
| D0                | a list of $k$ elements representing $k$ historical data, where the $i^{th}$ element corresponds to the $i^{th}$ historical data named as “D0i”.                            |
| X                 | a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations. |
| Y                 | a vector of individual level of the response y in the current data.  |
| a0                | a positive shape parameter for inverse-gamma prior on model parameter $\sigma^2$ .   |
| b                 | a positive scale parameter for inverse-gamma prior on model parameter $\sigma^2$ .   |
| mu0               | a vector of the mean for prior $\beta \sigma^2$ .  |
| R                 | a inverse matrix of the covariance matrix for prior $\beta \sigma^2$ .   |
| delta_ini         | the initial value of $\delta$ in MCMC sampling.  |
| prop_delta        | the class of proposal distribution for $\delta$ .  |
| prior_delta_alpha | a vector of the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .  |

prior_delta_beta	a vector of the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prop_delta_alpha	a vector of the hyperparameter $\alpha$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prop_delta_beta	a vector of the hyperparameter $\beta$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
rw_delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if prop_delta = 'RW'.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.

### Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling  $\delta$ . Let  $\theta=(\beta, \sigma^2)$ , the normalized power prior distribution is

$$\frac{\pi_0(\delta)\pi_0(\theta) \prod_{k=1}^K L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta) \prod_{k=1}^K L(\theta|D_{0k})^{\delta_k} d\theta}$$

Here  $\pi_0(\delta)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\delta$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\delta_k$  is the corresponding power parameter.

### Value

A list of class "NPP" with four elements:

acceptrate	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
beta	posterior of the model parameter $\beta$ in vector or matrix form.
sigma	posterior of the model parameter $\sigma^2$ .
delta	posterior of the power parameter $\delta$ .

### Author(s)

Qiang Zhang <zqzjf0408@163.com>

### References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[LMMNPP\\_MCMC2](#); [LMOMNPP\\_MCMC1](#); [LMOMNPP\\_MCMC2](#)

**Examples**

```
## Not run:
set.seed(1234)
sigseq0 = 1

n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01%*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigseq0))
D01 = cbind(X01, Y01)

n02 = 70
theta02 = c(0, 2, 3)
X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigseq0))
D02 = cbind(X02, Y02)

n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigseq0))
D03 = cbind(X03, Y03)

D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)

n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigseq0))

LMMNPP_MCMC1(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)),
             delta_ini=NULL, prior_delta_alpha=c(1,1,1), prior_delta_beta=c(1,1,1),
             prop_delta_alpha=c(1,1,1), prop_delta_beta=c(1,1,1),
             prop_delta="RW", rw_delta=0.9, nsample=5000, burnin=1000, thin=3)

## End(Not run)
```

---

LMMNPP\_MCMC2

*MCMC Sampling for Linear Regression Model of multiple historical data using Normalized Power Prior*

---

**Description**

Multiple historical data are combined individually. The NPP of multiple historical data is the product of the NPP of each historical data. Conduct posterior sampling for Linear Regression Model

with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameters  $(\beta, \sigma^2)$ , Gibbs sampling is used.

### Usage

```
LMMNPP_MCMC2(D0, X, Y, a0, b, mu0, R, delta_ini, prop_delta,
              prior_delta_alpha, prior_delta_beta, prop_delta_alpha,
              prop_delta_beta, rw_delta, nsample, burnin, thin)
```

### Arguments

D0	a list of $k$ elements representing $k$ historical data, where the $i^{th}$ element corresponds to the $i^{th}$ historical data named as “D0i”.
X	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
Y	a vector of individual level of the response y in the current data.
a0	a positive shape parameter for inverse-gamma prior on model parameter $\sigma^2$ .
b	a positive scale parameter for inverse-gamma prior on model parameter $\sigma^2$ .
mu0	a vector of the mean for prior $\beta \sigma^2$ .
R	a inverse matrix of the covariance matrix for prior $\beta \sigma^2$ .
delta_ini	the initial value of $\delta$ in MCMC sampling.
prop_delta	the class of proposal distribution for $\delta$ .
prior_delta_alpha	a vector of the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prior_delta_beta	a vector of the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prop_delta_alpha	a vector of the hyperparameter $\alpha$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prop_delta_beta	a vector of the hyperparameter $\beta$ in the proposal distribution $Beta(\alpha, \beta)$ for each $\delta$ .
rw_delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if prop_delta = 'RW'.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.

## Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling  $\delta$ . Let  $\theta=(\beta, \sigma^2)$ , the normalized power prior distribution is

$$\pi_0(\delta) \prod_{k=1}^K \frac{\pi_0(\theta)L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta)L(\theta|D_{0k})^{\delta_k} d\theta}.$$

Here  $\pi_0(\delta)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\delta$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\delta_k$  is the corresponding power parameter.

## Value

A list of class "NPP" with four elements:

acceptrate	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
beta	posterior of the model parameter $\beta$ in vector or matrix form.
sigma	posterior of the model parameter $\sigma^2$ .
delta	posterior of the power parameter $\delta$ .

## Author(s)

Qiang Zhang <zqzjf0408@163.com>

## References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

## See Also

[LMMNPP\\_MCMC1](#); [LMOMNPP\\_MCMC1](#); [LMOMNPP\\_MCMC2](#)

## Examples

```
## Not run:
set.seed(1234)
sigsq0 = 1

n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)

n02 = 70
theta02 = c(0, 2, 3)
```

```

X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)

n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)

D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)

n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))

LMNPP_MCMC2(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)),
            delta_ini=NULL, prior_delta_alpha=c(1,1,1), prior_delta_beta=c(1,1,1),
            prop_delta_alpha=c(1,1,1), prop_delta_beta=c(1,1,1),
            prop_delta="RW", rw_delta=0.9, nsample=5000, burnin=1000, thin=5)

## End(Not run)

```

LMNPP\_MCMC

*MCMC Sampling for Normal Linear Model using Normalized Power Prior*

## Description

Conduct posterior sampling for normal linear model with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the regression parameter  $\beta$  and  $\sigma^2$ , Gibbs sampling is used.

## Usage

```

LMNPP_MCMC(y.Cur, y.Hist, x.Cur = NULL, x.Hist = NULL,
           prior = list(a = 1.5, b = 0, mu0 = 0,
                       Rinv = matrix(1, nrow = 1), delta.alpha = 1, delta.beta = 1),
           MCMCmethod = 'IND', rw.logit.delta = 0.1,
           ind.delta.alpha = 1, ind.delta.beta = 1, nsample = 5000,
           control.mcmc = list(delta.ini = NULL, burnin = 0, thin = 1))

```

## Arguments

`y.Cur` a vector of individual level of the response  $y$  in current data.  
`y.Hist` a vector of individual level of the response  $y$  in historical data.

x.Cur	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
x.Hist	a vector or matrix or data frame of covariate observed in the historical data. If more than 1 covariate available, the number of rows is equal to the number of observations.
prior	a list of the hyperparameters in the prior for model parameters $(\beta, \sigma^2)$ and $\delta$ . The form of the prior for model parameter $(\beta, \sigma^2)$ is in the section "Details". a a positive hyperparameter for prior on model parameters. It is the power $a$ in formula $(1/\sigma^2)^a$ ; See details. b equals 0 if a flat prior is used for $\beta$ . Equals 1 if a normal prior is used for $\beta$ ; See details. mu0 a vector of the mean for prior $\beta \sigma^2$ . Only applicable if b = 1. Rinv inverse of the matrix $R$ . The covariance matrix of the prior for $\beta \sigma^2$ is $\sigma^2 R^{-1}$ . delta.alpha is the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ . delta.beta is the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .
MCMCmethod	sampling method for $\delta$ in MCMC. It can be either 'IND' for independence proposal; or 'RW' for random walk proposal on logit scale.
rw.logit.delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if MCMCmethod = 'RW' .
ind.delta.alpha	specifies the first parameter $\alpha$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
ind.delta.beta	specifies the first parameter $\beta$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
nsample	specifies the number of posterior samples in the output.
control.mcmc	a list of three elements used in posterior sampling. delta.ini is the initial value of $\delta$ in MCMC sampling. burnin is the number of burn-ins. The output will only show MCMC samples after burnin. thin is the thinning parameter in MCMC sampling.

### Details

If  $b = 1$ , prior for  $(\beta, \sigma)$  is  $(1/\sigma^2)^a * N(mu0, \sigma^2 R^{-1})$ , which includes the g-prior. If  $b = 0$ , prior for  $(\beta, \sigma)$  is  $(1/\sigma^2)^a$ . The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate when sampling  $\delta$ , and the deviance information criteria.

### Value

A list of class "NPP" with five elements:

beta posterior of the model parameter  $\beta$  in vector or matrix form.

sigmasq	posterior of the model parameter $\sigma^2$ .
delta	posterior of the power parameter $\delta$ .
acceptance	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
DIC	the deviance information criteria for model diagnostics.

### Author(s)

Zifei Han <hanzifei1@gmail.com>

### References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics 4: Proceedings of the Fourth Valencia International Meeting, Bernardo, J.M, Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.*

Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A* 186:453-461.

### See Also

[BerNPP\\_MCMC](#); [MultinomialNPP\\_MCMC](#); [PoissonNPP\\_MCMC](#); [NormalNPP\\_MCMC](#)

### Examples

```
set.seed(123)
x1 = runif(100, min = 0, max = 10)
x0 = runif(100, min = 0, max = 1)
y1 = 10+ 2*x1 + rnorm(100, mean = 0, sd = 1)
y0 = 10+ 1.5*x0 + rnorm(100, mean = 0, sd = 1)

RegPost = LMNPP_MCMC(y.Cur = y1, y.Hist = y0, x.Cur = x1, x.Hist = x0,
  prior = list(a = 1.5, b = 0, mu0 = c(0, 0),
    Rinv = diag(100, nrow = 2),
    delta.alpha = 1, delta.beta = 1), MCMCmethod = 'IND',
  ind.delta.alpha= 1, ind.delta.beta= 1, nsample = 5000,
  control.mcmc = list(delta.ini = NULL,
    burnin = 2000, thin = 2))
```

---

LMOMNPP_MCMC1	<i>MCMC Sampling for Linear Regression Model of multiple historical data using Ordered Normalized Power Prior</i>
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---

## Description

Multiple historical data are incorporated together. Conduct posterior sampling for Linear Regression Model with ordered normalized power prior. For the power parameter  $\gamma$ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameters  $(\beta, \sigma^2)$ , Gibbs sampling is used.

## Usage

```
LMOMNPP_MCMC1(D0, X, Y, a0, b, mu0, R, gamma_ini, prior_gamma,
               gamma_ind_prop, nsample, burnin, thin, adjust)
```

## Arguments

D0	a list of $k$ elements representing $k$ historical data, where the $i^{th}$ element corresponds to the $i^{th}$ historical data named as “D0i”.
X	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
Y	a vector of individual level of the response y in the current data.
a0	a positive shape parameter for inverse-gamma prior on model parameter $\sigma^2$ .
b	a positive scale parameter for inverse-gamma prior on model parameter $\sigma^2$ .
mu0	a vector of the mean for prior $\beta \sigma^2$ .
R	a inverse matrix of the covariance matrix for prior $\beta \sigma^2$ .
gamma_ini	the initial value of $\gamma$ in MCMC sampling.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.
adjust	Whether or not to adjust the parameters of the proposal distribution.

## Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling  $\gamma$ . Let  $\theta=(\beta, \sigma^2)$ , the normalized power prior distribution is

$$\frac{\pi_0(\gamma)\pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\theta}.$$

Here  $\pi_0(\gamma)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\gamma$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\sum_{i=1}^k \gamma_i$  is the corresponding power parameter.

## Value

A list of class "NPP" with four elements:

acceptrate	the acceptance rate in MCMC sampling for $\gamma$ using Metropolis-Hastings algorithm.
beta	posterior of the model parameter $\beta$ in vector or matrix form.
sigma	posterior of the model parameter $\sigma^2$ .
delta	posterior of the power parameter $\delta$ .

## Author(s)

Qiang Zhang <zqzjf0408@163.com>

## References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

## See Also

[LMMNPP\\_MCMC1](#); [LMMNPP\\_MCMC2](#); [LMOMNPP\\_MCMC2](#)

## Examples

```
## Not run:
set.seed(1234)
sigsq0 = 1

n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)

n02 = 70
theta02 = c(0, 2, 3)
```

```

X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)

n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)

D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)

n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))

LMOMNPP_MCMC1(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)),
              gamma_ini=NULL, prior_gamma=rep(1/4,4), gamma_ind_prop=rep(1/4,4),
              nsample=5000, burnin=1000, thin=5, adjust=FALSE)

## End(Not run)

```

LMOMNPP\_MCMC2

*MCMC Sampling for Linear Regression Model of multiple historical data using Ordered Normalized Power Prior*

## Description

Multiple historical data are combined individually. The NPP of multiple historical data is the product of the NPP of each historical data. Conduct posterior sampling for Linear Regression Model with ordered normalized power prior. For the power parameter  $\gamma$ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameters  $(\beta, \sigma^2)$ , Gibbs sampling is used.

## Usage

```
LMOMNPP_MCMC2(D0, X, Y, a0, b, mu0, R, gamma_ini, prior_gamma,
              gamma_ind_prop, nsample, burnin, thin, adjust)
```

## Arguments

- D0 a list of  $k$  elements representing  $k$  historical data, where the  $i^{th}$  element corresponds to the  $i^{th}$  historical data named as “D0i”.
- X a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
- Y a vector of individual level of the response y in the current data.

a0	a positive shape parameter for inverse-gamma prior on model parameter $\sigma^2$ .
b	a positive scale parameter for inverse-gamma prior on model parameter $\sigma^2$ .
mu0	a vector of the mean for prior $\beta \sigma^2$ .
R	a inverse matrix of the covariance matrix for prior $\beta \sigma^2$ .
gamma_ini	the initial value of $\gamma$ in MCMC sampling.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.
adjust	Whether or not to adjust the parameters of the proposal distribution.

### Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling  $\gamma$ . Let  $\theta=(\beta, \sigma^2)$ , the normalized power prior distribution is

$$\pi_0(\gamma) \prod_{k=1}^K \frac{\pi_0(\theta)L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta)L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\theta}.$$

Here  $\pi_0(\gamma)$  and  $\pi_0(\theta)$  are the initial prior distributions of  $\gamma$  and  $\theta$ , respectively.  $L(\theta|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\sum_{i=1}^k \gamma_i$  is the corresponding power parameter.

### Value

A list of class "NPP" with four elements:

acceptrate	the acceptance rate in MCMC sampling for $\gamma$ using Metropolis-Hastings algorithm.
beta	posterior of the model parameter $\beta$ in vector or matrix form.
sigma	posterior of the model parameter $\sigma^2$ .
delta	posterior of the power parameter $\delta$ .

### Author(s)

Qiang Zhang <zqzjf0408@163.com>

### References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[LMMNPP\\_MCMC1](#); [LMMNPP\\_MCMC2](#); [LMOMNPP\\_MCMC1](#)

**Examples**

```
## Not run:
set.seed(1234)
sigsq0 = 1

n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01%*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)

n02 = 70
theta02 = c(0, 2, 3)
X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)

n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)

D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)

n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))

LMOMNPP_MCMC1(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)),
              gamma_ini=NULL, prior_gamma=rep(1/4,4), gamma_ind_prop=rep(1/4,4),
              nsample=5000, burnin=1000, thin=5, adjust=FALSE)

## End(Not run)
```

---

logCdelta

*A Function to Interpolate  $\log C(\delta)$  Based on Its Values on Selected Knots*

---

**Description**

The function returns the interpolated value (a scalar) of  $\log C(\delta)$  based on its results on selected knots, given input vector of  $\delta$ .

**Usage**

```
logCdelta(delta, deltaknot, lCknot)
```

**Arguments**

delta            a scalar of the input value of  $\delta$ .

deltaknot        a vector of the knots for  $\delta$ . It should be selected before conduct the sampling.

lCknot           a vector of the values  $\log C(\delta)$  on selected knots, coming from the function logCknot.

**Value**

A sequence of the values,  $\log C(\delta)$  on selected knots.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[loglikNormD0](#); [loglikBerD0](#); [logCknot](#)

---

logCknot

*A Function to Calculate  $\log C(\delta)$  on Selected Knots*

---

**Description**

The function returns a sequence of the values,  $\log C(\delta)$  on selected knots, given input vector of  $\delta$ .

**Usage**

```
logCknot(deltaknot, llikf0)
```

**Arguments**

deltaknot        a vector of the knots for  $\delta$ . It should be selected before conduct the sampling.

llikf0           a matrix of the log-likelihoods of class "npp".

**Value**

A sequence of the values,  $\log C(\delta)$  on selected knots.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[loglikNormD0](#); [loglikBerD0](#); [logCdelta](#)

---

loglikBerD0

*A Function to Calculate Log-likelihood of the Historical Data, Given Matrix-valued Parameters, for Bernoulli Population*

---

**Description**

The function returns a matrix of class "npp", each element is a log-likelihood of the historical data. It is an intermediate step to calculate the "normalizing constant"  $C(\delta)$  in the normalized power prior, for the purpose of providing a flexible implementation. Users can specify their own likelihood function of the same class following this structure.

**Usage**

```
loglikBerD0(D0, thetalist, ntheta = 1)
```

**Arguments**

D0	a vector of each observation(binary) in historical data.
thetalist	a list of parameter values. The number of elements is equal to ntheta. Each element is a matrix. The sample should come from the posterior of the powered likelihood for historical data, with each column corresponds to a distinct value of the power parameter $\delta$ (the corresponding power parameter increases from left to right). The number of rows is the number of Monte Carlo samples for each $\delta$ fixed. The number of columns is the number of selected knots (number of distinct $\delta$ ).
ntheta	a positive integer indicating number of parameters to be estimated in the model. Default is 1 for Bernoulli.

**Value**

A numeric matrix of log-likelihood, for the historical data given the matrix(or array)-valued parameters.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[loglikNormD0](#); [logCknot](#); [logCdelta](#)

---

loglikNormD0

*A Function to Calculate Log-likelihood of the Historical Data, Given Array-valued Parameters, for Normal Population*

---

**Description**

The function returns a matrix of class "npp", each element is a log-likelihood of the historical data. It is an intermediate step to calculate the "normalizing constant"  $C(\delta)$  in the normalized power prior, for the purpose of providing a flexible implementation. Users can specify their own likelihood function of the same class following this structure.

**Usage**

```
loglikNormD0(D0, thetalist, ntheta = 2)
```

**Arguments**

D0	a vector of each observation in historical data.
thetalist	a list of parameter values. The number of elements is equal to ntheta. Each element is a matrix. The sample should come from the posterior of the powered likelihood for historical data, with each column corresponds to a distinct value of the power parameter $\delta$ (the corresponding power parameter increases from left to right). The number of rows is the number of Monte Carlo samples for each $\delta$ fixed. The number of columns is the number of selected knots (number of distinct $\delta$ ).
ntheta	a positive integer indicating number of parameters to be estimated in the model.

**Value**

A numeric matrix of log-likelihood, for the historical data given the matrix(or array)-valued parameters.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[loglikBerD0](#); [logCknot](#); [logCdelta](#)

---

ModeDeltaBerNPP

*Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Bernoulli Population*

---

**Description**

The function returns the posterior mode of the power parameter  $\delta$  in Bernoulli population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

**Usage**

```
ModeDeltaBerNPP(Data.Cur, Data.Hist,
  CompStat = list(n0 = NULL, y0 = NULL, n1 = NULL, y1 = NULL),
  npoints = 1000,
  prior = list(p.alpha = 1, p.beta = 1,
    delta.alpha = 1, delta.beta = 1))
```

**Arguments**

- Data.Cur      a non-negative integer vector of two elements: c(number of success, number of failure) in the current data.
- Data.Hist      a non-negative integer vector of two elements: c(number of success, number of failure) in the historical data.

CompStat	<p>a list of four elements that represents the "compatibility(sufficient) statistics" for <math>p</math>. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in Bernoulli population providing CompStat is equivalent to provide the data summary as in Data.Cur and Data.Cur.</p> <p><math>n_0</math> is the number of trials in the historical data.  <math>y_0</math> is the number of successes in the historical data.  <math>n_1</math> is the number of trials in the current data.  <math>y_1</math> is the number of successes in the current data.</p>
npoints	is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.
prior	<p>a list of the hyperparameters in the prior for both <math>p</math> and <math>\delta</math>.</p> <p>p.alpha is the hyperparameter <math>\alpha</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>p</math>.  p.beta is the hyperparameter <math>\beta</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>p</math>.  delta.alpha is the hyperparameter <math>\alpha</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>\delta</math>.  delta.beta is the hyperparameter <math>\beta</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>\delta</math>.</p>

### Details

See example.

### Value

A numeric value between 0 and 1.

### Author(s)

Zifei Han <hanzifei1@gmail.com>

### References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

### See Also

[ModeDeltaNormalNPP](#); [ModeDeltaPoisNPP](#); [ModeDeltaMultinomialNPP](#)

### Examples

```
ModeDeltaBerNPP(Data.Cur = c(100, 40), Data.Hist = c(100, 40), npoints = 1000,
                prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1))
```

```
ModeDeltaBerNPP(Data.Cur = c(100, 40), Data.Hist = c(100, 35), npoints = 1000,
```

```
prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1))
ModeDeltaBerNPP(Data.Cur = c(100, 40), Data.Hist = c(100, 50), npoints = 1000,
prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1))
```

---

ModeDeltaLMNPP	<i>Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Normal Linear Model</i>
----------------	--

---

### Description

The function returns the posterior mode of the power parameter  $\delta$  in normal linear model. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

### Usage

```
ModeDeltaLMNPP(y.Cur, y.Hist, x.Cur = NULL, x.Hist = NULL, npoints = 1000,
prior = list(a = 1.5, b = 0, mu0 = 0, Rinv = matrix(1, nrow = 1),
delta.alpha = 1, delta.beta = 1))
```

### Arguments

y.Cur	a vector of individual level of the response y in current data.
y.Hist	a vector of individual level of the response y in historical data.
x.Cur	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
x.Hist	a vector or matrix or data frame of covariate observed in the historical data. If more than 1 covariate available, the number of rows is equal to the number of observations.
npoints	is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.
prior	a list of the hyperparameters in the prior for model parameters $(\beta, \sigma^2)$ and $\delta$ . The form of the prior for model parameter $(\beta, \sigma^2)$ is in the section "Details". a a positive hyperparameter for prior on model parameters. It is the power $a$ in formula $(1/\sigma^2)^a$ ; See details. b equals 0 if a flat prior is used for $\beta$ . Equals 1 if a normal prior is used for $\beta$ ; See details. mu0 a vector of the mean for prior $\beta \sigma^2$ . Only applicable if b = 1. Rinv inverse of the matrix $R$ . The covariance matrix of the prior for $\beta \sigma^2$ is $\sigma^2 R^{-1}$ . delta.alpha is the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ . delta.beta is the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .

**Details**

If  $b = 1$ , prior for  $(\beta, \sigma)$  is  $(1/\sigma^2)^a * N(\mu_0, \sigma^2 R^{-1})$ , which includes the g-prior. If  $b = 0$ , prior for  $(\beta, \sigma)$  is  $(1/\sigma^2)^a$ . The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate when sampling  $\delta$ , and the deviance information criteria.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics 4: Proceedings of the Fourth Valencia International Meeting*, Bernardo, J.M., Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.

Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A* 186:453-461.

**See Also**

[ModeDeltaBerNPP](#); [ModeDeltaNormalNPP](#); [ModeDeltaMultinomialNPP](#); [ModeDeltaNormalNPP](#)

---

ModeDeltaMultinomialNPP

*Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Multinomial Population*

---

**Description**

The function returns the posterior mode of the power parameter  $\delta$  in multinomial population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

**Usage**

```
ModeDeltaMultinomialNPP(Data.Cur, Data.Hist, CompStat = list(n0 = NULL, n1 = NULL),
  npoints = 1000, prior = list(theta.dir.alpha = c(0.5, 0.5, 0.5),
    delta.alpha = 1, delta.beta = 1))
```

**Arguments**

Data.Cur	a non-negative integer vector of $K$ elements: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ) in the current data.
Data.Hist	a non-negative integer vector of $K$ elements: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ) in the historical data.
CompStat	a list of two elements that represents the "compatibility(sufficient) statistics" for $\theta$ . Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in multinomial case providing CompStat is equivalent to provide the data summary as in Data.Cur and Data.Cur. n0 is a non-negative integer vector of $K$ elements for compatible statistics in historical data: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ). n1 is a non-negative integer vector of $K$ elements for compatible statistics in current data: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ).
npoints	is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.
prior	a list of the hyperparameters in the prior for both $p$ and $\delta$ . theta.dir is a vector of $K$ elements of the hyperparameter $\alpha$ in the prior distribution $Dir(\alpha[1], \alpha[2], \dots, \alpha[K])$ for $\theta$ . delta.alpha a scalar, the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ . delta.beta a scalar, the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .

**Details**

See example.

**Value**

A numeric value between 0 and 1.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[ModeDeltaBerNPP](#); [ModeDeltaNormalNPP](#); [ModeDeltaPoisNPP](#)

**Examples**

```
ModeDeltaMultinomialNPP(CompStat = list(n0 = c(25,25,25,25), n1 = c(25,25,25,25)),
  prior = list(theta.dir.alpha = c(0.5, 0.5, 0.5, 0.5),
    delta.alpha = 1, delta.beta = 1))
```

```
ModeDeltaMultinomialNPP(CompStat = list(n0 = c(22,25,28,25), n1 = c(25,22,25,28)),
  prior = list(theta.dir.alpha = c(0.5, 0.5, 0.5, 0.5),
    delta.alpha = 1, delta.beta = 1))
```

```
ModeDeltaMultinomialNPP(CompStat = list(n0 = c(15,25,30,30), n1 = c(25,25,25,25)),
  prior = list(theta.dir.alpha = c(0.5, 0.5, 0.5, 0.5),
    delta.alpha = 1, delta.beta = 1))
```

---

ModeDeltaNormalNPP	<i>Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Normal Population</i>
--------------------	--

---

**Description**

The function returns the posterior mode of the power parameter  $\delta$  in multinomial population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

**Usage**

```
ModeDeltaNormalNPP(Data.Cur, Data.Hist,
  CompStat = list(n0 = NULL, mean0 = NULL, var0 = NULL,
    n1 = NULL, mean1 = NULL, var1 = NULL),
  npoints = 1000,
  prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1))
```

**Arguments**

Data.Cur	a vector of individual level current data.
Data.Hist	a vector of individual level historical data.
CompStat	a list of six elements(scalar) that represents the "compatibility(sufficient) statistics" for model parameters. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored.
	n0 is the sample size of historical data.
	mean0 is the sample mean of the historical data.
	var0 is the sample variance of the historical data.

	n1 is the sample size of current data.
	mean1 is the sample mean of the current data.
	var1 is the sample variance of the current data.
npoints	is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.
prior	a list of the hyperparameters in the prior for both $(\mu, \sigma^2)$ and $\delta$ . The form of the prior for model parameter $(\mu, \sigma^2)$ is $(1/\sigma^2)^a$ . When $a = 1$ it corresponds to the reference prior, and when $a = 1.5$ it corresponds to the Jeffrey's prior. a is the power $a$ in formula $(1/\sigma^2)^a$ , the prior for $(\mu, \sigma^2)$ jointly. delta.alpha is the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ . delta.beta is the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .

**Details**

See example.

**Value**

A numeric value between 0 and 1.

**Author(s)**

Zifei Han <hanzifei1@gmail.com>

**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.
- Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics 4: Proceedings of the Fourth Valencia International Meeting*, Bernardo, J.M, Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.
- Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A* 186:453-461.

**See Also**

[ModeDeltaBerNPP](#); [ModeDeltaMultinomialNPP](#); [ModeDeltaPoisNPP](#)

**Examples**

```
ModeDeltaNormalNPP(CompStat = list(n0 = 50, mean0 = 0, var0 = 1,
                                   n1 = 50, mean1 = 0, var1 = 1), npoints = 1000,
                  prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1))
```

```

ModeDeltaNormalNPP(CompStat = list(n0 = 50, mean0 = 0, var0 = 1,
                                   n1 = 40, mean1 = 0.2, var1 = 1), npoints = 1000,
                  prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1))

ModeDeltaNormalNPP(CompStat = list(n0 = 50, mean0 = 0, var0 = 1,
                                   n1 = 40, mean1 = 0.6, var1 = 1), npoints = 1000,
                  prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1))

```

---

ModeDeltaPoisNPP	<i>Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Poisson Population</i>
------------------	---

---

### Description

The function returns the posterior mode of the power parameter  $\delta$  in multinomial population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

### Usage

```

ModeDeltaPoisNPP(Data.Cur, Data.Hist,
                 CompStat = list(n0 = NULL, mean0 = NULL, n1 = NULL, mean1 = NULL),
                 npoints = 1000, prior = list(lambda.shape = 1/2,
                                             lambda.scale = 100, delta.alpha = 1, delta.beta = 1))

```

### Arguments

Data.Cur	a non-negative integer vector of each observed current data.
Data.Hist	a non-negative integer vector of each observed historical data.
CompStat	a list of four elements that represents the "compatibility(sufficient) statistics" for $\lambda$ . Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. n0 is the number of observations in the historical data. mean0 is the sample mean of the historical data. n1 is the number of observations in the current data. mean1 is the sample mean of the current data.
npoints	is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.
prior	a list of the hyperparameters in the prior for both $\lambda$ and $\delta$ . A Gamma distribution is used as the prior of $\lambda$ , and a Beta distribution is used as the prior of $\delta$ . lambda.shape is the shape (hyper)parameter in the prior distribution $Gamma(shape, scale)$ for $\lambda$ . lambda.scale is the scale (hyper)parameter in the prior distribution $Gamma(shape, scale)$ for $\lambda$ .

delta.alpha is the hyperparameter  $\alpha$  in the prior distribution  $Beta(\alpha, \beta)$  for  $\delta$ .

delta.beta is the hyperparameter  $\beta$  in the prior distribution  $Beta(\alpha, \beta)$  for  $\delta$ .

### Details

See example.

### Value

A numeric value between 0 and 1.

### Author(s)

Zifei Han <hanzifei1@gmail.com>

### References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

### See Also

[ModeDeltaBerNPP](#); [ModeDeltaNormalNPP](#); [ModeDeltaMultinomialNPP](#)

### Examples

```
ModeDeltaPoisNPP(CompStat = list(n0 = 50, mean0 = 10, n1 = 50, mean1 = 10), npoints = 1000,
  prior = list(lambda.shape = 1/2, lambda.scale = 100,
    delta.alpha = 1, delta.beta = 1))
```

```
ModeDeltaPoisNPP(CompStat = list(n0 = 50, mean0 = 10, n1 = 50, mean1 = 9.5), npoints = 1000,
  prior = list(lambda.shape = 1/2, lambda.scale = 100,
    delta.alpha = 1, delta.beta = 1))
```

```
ModeDeltaPoisNPP(CompStat = list(n0 = 50, mean0 = 10, n1 = 50, mean1 = 9), npoints = 1000,
  prior = list(lambda.shape = 1/2, lambda.scale = 100,
    delta.alpha = 1, delta.beta = 1))
```

---

MultinomialNPP\_MCMC    *MCMC Sampling for Multinomial Population using Normalized Power Prior*

---

### Description

Conduct posterior sampling for multinomial population with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter vector  $\theta$ , Gibbs sampling is used. Assume the prior for model parameter  $\theta$  comes from a Dirichlet distribution.

### Usage

```
MultinomialNPP_MCMC(Data.Cur = c(10, 10, 10), Data.Hist = c(10, 10, 10),
  CompStat = list(n0 = NULL, n1 = NULL),
  prior = list(theta.dir = c(0.5, 0.5, 0.5),
    delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'IND', rw.logit.delta = 0.1,
  ind.delta.alpha = 1, ind.delta.beta = 1, nsample = 5000,
  control.mcmc = list(delta.ini = NULL, burnin = 0, thin = 1))
```

### Arguments

Data.Cur	a non-negative integer vector of $K$ elements: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ) in the current data.
Data.Hist	a non-negative integer vector of $K$ elements: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ) in the historical data.
CompStat	a list of two elements that represents the "compatibility(sufficient) statistics" for $\theta$ . Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in multinomial case providing CompStat is equivalent to provide the data summary as in Data.Cur and Data.Cur. n0 is a non-negative integer vector of $K$ elements for compatible statistics in historical data: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ). n1 is a non-negative integer vector of $K$ elements for compatible statistics in current data: c(number of success in group 1, number of success in group 2, ..., number of success in group $K$ ).
prior	a list of the hyperparameters in the prior for both $p$ and $\delta$ . theta.dir is a vector of $K$ elements of the hyperparameter $\alpha$ in the prior distribution $Dir(\alpha[1], \alpha[2], \dots, \alpha[K])$ for $\theta$ . delta.alpha a scalar, the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .

	delta.beta a scalar, the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .
MCMCmethod	sampling method for $\delta$ in MCMC. It can be either 'IND' for independence proposal; or 'RW' for random walk proposal on logit scale.
rw.logit.delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if MCMCmethod = 'RW' .
ind.delta.alpha	specifies the first parameter $\alpha$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
ind.delta.beta	specifies the first parameter $\beta$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
nsample	specifies the number of posterior samples in the output.
control.mcmc	a list of three elements used in posterior sampling. delta.ini is the initial value of $\delta$ in MCMC sampling. burnin is the number of burn-ins. The output will only show MCMC samples after burnin. thin is the thinning parameter in MCMC sampling.

### Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ , and the deviance information criteria.

### Value

A list of class "NPP" with four elements:

p	posterior of the model parameter $\theta$ .
delta	posterior of the power parameter $\delta$ .
acceptance	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
DIC	the deviance information criteria for model diagnostics.

### Author(s)

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### References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

### See Also

[BerNPP\\_MCMC](#); [NormalNPP\\_MCMC](#); [PoissonNPP\\_MCMC](#)

**Examples**

```
MultinomialNPP_MCMC(Data.Cur = c(3,11,3,669), Data.Hist = c(9,20,9,473),
  prior = list(theta.dir = c(1,1,1,1),
  delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'IND', rw.logit.delta = 0.1,
  ind.delta.alpha = 1, ind.delta.beta = 1, nsample = 10000,
  control.mcmc = list(delta.ini = NULL,
  burnin = 2000, thin = 5))
```

---

NormalNPP_MCMC	<i>MCMC Sampling for Normal Population using Normalized Power Prior</i>
----------------	---

---

**Description**

Conduct posterior sampling for normal population with normalized power prior. The initial prior  $\pi(\mu|\sigma^2)$  is a flat prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter  $\mu$  and  $\sigma^2$ , Gibbs sampling is used.

**Usage**

```
NormalNPP_MCMC(Data.Cur, Data.Hist,
  CompStat = list(n0 = NULL, mean0 = NULL, var0 = NULL,
  n1 = NULL, mean1 = NULL, var1 = NULL),
  prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'IND', rw.logit.delta = 0.1,
  ind.delta.alpha = 1, ind.delta.beta = 1, nsample = 5000,
  control.mcmc = list(delta.ini = NULL, burnin = 0, thin = 1))
```

**Arguments**

Data.Cur	a vector of individual level current data.
Data.Hist	a vector of individual level historical data.
CompStat	a list of six elements (scalar) that represents the "compatibility (sufficient) statistics" for model parameters. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. n0 is the sample size of historical data. mean0 is the sample mean of the historical data. var0 is the sample variance of the historical data. n1 is the sample size of current data. mean1 is the sample mean of the current data. var1 is the sample variance of the current data.

prior	<p>a list of the hyperparameters in the prior for both <math>(\mu, \sigma^2)</math> and <math>\delta</math>. The form of the prior for model parameter <math>(\mu, \sigma^2)</math> is <math>(1/\sigma^2)^a</math>. When <math>a = 1</math> it corresponds to the reference prior, and when <math>a = 1.5</math> it corresponds to the Jeffrey's prior.</p> <p><math>a</math> is the power <math>a</math> in formula <math>(1/\sigma^2)^a</math>, the prior for <math>(\mu, \sigma^2)</math> jointly.</p> <p>delta.alpha is the hyperparameter <math>\alpha</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>\delta</math>.</p> <p>delta.beta is the hyperparameter <math>\beta</math> in the prior distribution <math>Beta(\alpha, \beta)</math> for <math>\delta</math>.</p>
MCMCmethod	sampling method for $\delta$ in MCMC. It can be either 'IND' for independence proposal; or 'RW' for random walk proposal on logit scale.
rw.logit.delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if MCMCmethod = 'RW' .
ind.delta.alpha	specifies the first parameter $\alpha$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
ind.delta.beta	specifies the first parameter $\beta$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
nsample	specifies the number of posterior samples in the output.
control.mcmc	<p>a list of three elements used in posterior sampling.</p> <p>delta.ini is the initial value of <math>\delta</math> in MCMC sampling.</p> <p>burnin is the number of burn-ins. The output will only show MCMC samples after burnin.</p> <p>thin is the thinning parameter in MCMC sampling.</p>

### Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ , and the deviance information criteria.

### Value

A list of class "NPP" with five elements:

mu	posterior of the model parameter $\mu$ .
sigmasq	posterior of the model parameter $\sigma^2$ .
delta	posterior of the power parameter $\delta$ .
acceptance	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
DIC	the deviance information criteria for model diagnostics.

### Author(s)

Zifei Han <hanzifei1@gmail.com>

## References

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.
- Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics 4: Proceedings of the Fourth Valencia International Meeting*, Bernardo, J.M., Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.
- Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A* 186:453-461.

## See Also

[BerNPP\\_MCMC](#); [MultinomialNPP\\_MCMC](#); [PoissonNPP\\_MCMC](#);

## Examples

```
set.seed(1234)
NormalData0 <- rnorm(n = 100, mean= 20, sd = 1)

set.seed(12345)
NormalData1 <- rnorm(n = 50, mean= 30, sd = 1)

NormalNPP_MCMC(Data.Cur = NormalData1, Data.Hist = NormalData0,
  CompStat = list(n0 = 100, mean0 = 10, var0 = 1,
    n1 = 100, mean1 = 10, var1 = 1),
  prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'RW', rw.logit.delta = 1,
  ind.delta.alpha= 1, ind.delta.beta= 1, nsample = 10000,
  control.mcmc = list(delta.ini = NULL, burnin = 0, thin = 1))
```

---

PHData

*PH Data on four sites in Virginia*

---

## Description

The dataset is used to assess if there is site impairment. The site impairment is defined as whether the pH values at a site indicate that the site violates a (lower) standard of 6.0 more than 10% of the time.

## Usage

```
data("PHData")
```

**Format**

A data frame with 325 observations on the following 3 variables.

Station the site number, labeled as 1 to 4

Data.Time indicator of historical data (coded as 0) or current data (coded as 1)

PH value of PH on the site

**Examples**

```
data(PHData)
```

---

PoiMNPP\_MCMC1

*MCMC Sampling for Poisson Population using Normalized Power Prior with Multiple Historical Data*

---

**Description**

This function incorporates multiple sets of historical data for posterior sampling in a Poisson population using a normalized power prior. The power parameter  $\delta$  uses a Metropolis-Hastings algorithm, which can be either an independence proposal or a random walk proposal on its logit scale. For the model parameter  $\lambda$ , Gibbs sampling is employed.

**Usage**

```
PoiMNPP_MCMC1(n0, n, prior_lambda, prop_delta, prior_delta_alpha,
               prior_delta_beta, rw_delta, delta_ini, nsample, burnin, thin)
```

**Arguments**

n0	A vector of natural numbers: number of successes in historical data.
n	A natural number: number of successes in the current data.
prior_lambda	A vector of hyperparameters for the prior distribution $Gamma(\alpha, \beta)$ of $\lambda$ .
prop_delta	The class of proposal distribution for $\delta$ .
prior_delta_alpha	A vector of hyperparameter $\alpha$ for the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prior_delta_beta	A vector of hyperparameter $\beta$ for the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
rw_delta	The stepsize (variance of the normal distribution) for the random walk proposal of logit $\delta$ . This is only applicable if prop_delta = 'RW'.
delta_ini	The initial value for $\delta$ in MCMC sampling.
nsample	Specifies the number of posterior samples in the output.
burnin	The number of burn-ins. Only the MCMC samples after this burn-in will be shown in the output.
thin	The thinning parameter used in MCMC sampling.

**Details**

The function returns posteriors for both the model and power parameters, as well as the acceptance rate for sampling  $\delta$ . The normalized power prior distribution is given by:

$$\frac{\pi_0(\delta)\pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{\delta_k}}{\int \pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{\delta_k} d\lambda}$$

Here,  $\pi_0(\delta)$  and  $\pi_0(\lambda)$  are the initial prior distributions for  $\delta$  and  $\lambda$ , respectively.  $L(\lambda|D_{0k})$  is the likelihood function based on historical data  $D_{0k}$ , with  $\delta_k$  being its corresponding power parameter.

**Value**

A list of class "NPP" comprising:

acceptrate	The acceptance rate in MCMC sampling for $\delta$ using the Metropolis-Hastings algorithm.
lambda	Posterior samples of the model parameter $\lambda$ .
delta	Posterior samples of the power parameter $\delta$ .

**Author(s)**

Qiang Zhang <zqzjf0408@163.com>

**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y., and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K., and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[PoiMNPP\\_MCMC2](#), [PoiOMNPP\\_MCMC1](#), [PoiOMNPP\\_MCMC2](#)

**Examples**

```
PoiMNPP_MCMC1(n0 = c(0, 3, 5), n = 3, prior_lambda = c(1, 1/10), prop_delta = "IND",
  prior_delta_alpha = c(1, 1, 1), prior_delta_beta = c(1, 1, 1),
  rw_delta = 0.1, delta_ini = NULL, nsample = 2000, burnin = 500, thin = 2)
```

---

PoiMNPP_MCMC2	<i>MCMC Sampling for Poisson Population of multiple historical data using Normalized Power Prior</i>
---------------	--

---

### Description

Multiple historical data are combined individually. Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter  $\lambda$ , Gibbs sampling is used.

### Usage

```
PoiMNPP_MCMC2(n0, n, prior_lambda, prop_delta, prior_delta_alpha,
               prior_delta_beta, rw_delta, delta_ini, nsample, burnin, thin)
```

### Arguments

n0	a natural number vector : number of successes in historical data.
n	a natural number : number of successes in the current data.
prior_lambda	a vector of the hyperparameters in the prior distribution $Gamma(\alpha, \beta)$ for $\lambda$ .
prop_delta	the class of proposal distribution for $\delta$ .
prior_delta_alpha	a vector of the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
prior_delta_beta	a vector of the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for each $\delta$ .
rw_delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if prop_delta = 'RW'.
delta_ini	the initial value of $\delta$ in MCMC sampling.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.

### Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ . The normalized power prior distribution is

$$\pi_0(\delta) \prod_{k=1}^K \frac{\pi_0(\lambda) L(\lambda | D_{0k})^{\delta_k}}{\int \pi_0(\lambda) L(\lambda | D_{0k})^{\delta_k} d\lambda}.$$

Here  $\pi_0(\delta)$  and  $\pi_0(\lambda)$  are the initial prior distributions of  $\delta$  and  $\lambda$ , respectively.  $L(\lambda | D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\delta_k$  is the corresponding power parameter.

**Value**

A list of class "NPP" with three elements:

acceptrate	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
lambda	posterior of the model parameter $\lambda$ .
delta	posterior of the power parameter $\delta$ .

**Author(s)**

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**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[PoiMNPP\\_MCMC1](#); [PoiOMNPP\\_MCMC1](#); [PoiOMNPP\\_MCMC2](#)

**Examples**

```
PoiMNPP_MCMC2(n0=c(0,3,5),n=3,prior_lambda=c(1,1/10),prop_delta="IND",
  prior_delta_alpha=c(1,1,1), prior_delta_beta=c(1,1,1), rw_delta=0.1,
  delta_ini=NULL, nsample = 2000, burnin = 500, thin = 2)
```

---

PoiOMNPP_MCMC1	<i>MCMC Sampling for Poisson Population of multiple ordered historical data using Normalized Power Prior</i>
----------------	--

---

**Description**

Multiple ordered historical data are incorporated together. Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter  $\gamma$ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter  $\lambda$ , Gibbs sampling is used.

**Usage**

```
PoiOMNPP_MCMC1(n0,n,prior_gamma,prior_lambda, gamma_ind_prop,
  gamma_ini,nsample,burnin,thin)
```

**Arguments**

n0	a natural number vector : number of successes in historical data.
n	a natural number : number of successes in the current data.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
prior_lambda	a vector of the hyperparameters in the prior distribution $Gamma(\alpha, \beta)$ for $\lambda$ .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
gamma_ini	the initial value of $\gamma$ in MCMC sampling.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.

**Details**

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\gamma$ . The normalized power prior distribution is

$$\frac{\pi_0(\gamma)\pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\lambda}.$$

Here  $\pi_0(\gamma)$  and  $\pi_0(\lambda)$  are the initial prior distributions of  $\gamma$  and  $\lambda$ , respectively.  $L(\lambda|D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\sum_{i=1}^k \gamma_i$  is the corresponding power parameter.

**Value**

A list of class "NPP" with three elements:

acceptrate	the acceptance rate in MCMC sampling for $\gamma$ using Metropolis-Hastings algorithm.
lambda	posterior of the model parameter $\lambda$ .
delta	posterior of the power parameter $\delta$ . It is equal to the cumulative sum of $\gamma$

**Author(s)**

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**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[PoiMNPP\\_MCMC1](#); [PoiMNPP\\_MCMC2](#); [PoiOMNPP\\_MCMC2](#)

**Examples**

```
PoiOMNPP_MCMC1(n0=c(0,3,5),n=3,prior_gamma=c(1/2,1/2,1/2,1/2), prior_lambda=c(1,1/10),
  gamma_ind_prop=rep(1,4),gamma_ini=NULL, nsample = 2000, burnin = 500, thin = 2)
```

---

PoiOMNPP_MCMC2	<i>MCMC Sampling for Poisson Population of multiple ordered historical data using Normalized Power Prior</i>
----------------	--

---

**Description**

Multiple ordered historical data are combined individually. Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter  $\gamma$ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter  $\lambda$ , Gibbs sampling is used.

**Usage**

```
PoiOMNPP_MCMC2(n0,n,prior_gamma,prior_lambda, gamma_ind_prop,gamma_ini,
  nsample, burnin, thin)
```

**Arguments**

n0	a natural number vector : number of successes in historical data.
n	a natural number : number of successes in the current data.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
prior_lambda	a vector of the hyperparameters in the prior distribution $Gamma(\alpha, \beta)$ for $\lambda$ .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_K)$ for $\gamma$ .
gamma_ini	the initial value of $\gamma$ in MCMC sampling.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.

**Details**

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\gamma$ . The normalized power prior distribution is

$$\pi_0(\gamma) \prod_{k=1}^K \frac{\pi_0(\lambda) L(\lambda | D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\lambda) L(\lambda | D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\lambda}.$$

Here  $\pi_0(\gamma)$  and  $\pi_0(\lambda)$  are the initial prior distributions of  $\gamma$  and  $\lambda$ , respectively.  $L(\lambda | D_{0k})$  is the likelihood function of historical data  $D_{0k}$ , and  $\sum_{i=1}^k \gamma_i$  is the corresponding power parameter.

**Value**

A list of class "NPP" with three elements:

acceptrate      the acceptance rate in MCMC sampling for  $\gamma$  using Metropolis-Hastings algorithm.

lambda            posterior of the model parameter  $\lambda$ .

delta             posterior of the power parameter  $\delta$ . It is equal to the cumulative sum of  $\gamma$

**Author(s)**

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**References**

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[PoiMNPP\\_MCMC1](#); [PoiMNPP\\_MCMC2](#); [PoiOMNPP\\_MCMC1](#)

**Examples**

```
PoiOMNPP_MCMC2(n0=c(0,3,5),n=3,prior_gamma=c(1/2,1/2,1/2,1/2),
  prior_lambda=c(1,1/10), gamma_ind_prop=rep(1,4),
  gamma_ini=NULL, nsample = 2000, burnin = 500, thin = 2)
```

---

PoissonNPP_MCMC	<i>MCMC Sampling for Bernoulli Population using Normalized Power Prior</i>
-----------------	--

---

**Description**

Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter  $\delta$ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter  $\lambda$ , Gibbs sampling is used.

**Usage**

```
PoissonNPP_MCMC(Data.Cur, Data.Hist,
  CompStat = list(n0 = NULL, mean0 = NULL, n1 = NULL, mean1 = NULL),
  prior = list(lambda.shape = 1/2, lambda.scale = 100,
    delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'IND', rw.logit.delta = 0.1,
  ind.delta.alpha= 1, ind.delta.beta= 1, nsample = 5000,
  control.mcmc = list(delta.ini = NULL, burnin = 0, thin = 1))
```

**Arguments**

Data.Cur	a non-negative integer vector of each observed current data.
Data.Hist	a non-negative integer vector of each observed historical data.
CompStat	a list of four elements that represents the "compatibility(sufficient) statistics" for $\lambda$ . Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. n0 is the number of observations in the historical data. mean0 is the sample mean of the historical data. n1 is the number of observations in the current data. mean1 is the sample mean of the current data.
prior	a list of the hyperparameters in the prior for both $\lambda$ and $\delta$ . A Gamma distribution is used as the prior of $\lambda$ , and a Beta distribution is used as the prior of $\delta$ . lambda.shape is the shape (hyper)parameter in the prior distribution $Gamma(shape, scale)$ for $\lambda$ . lambda.scale is the scale (hyper)parameter in the prior distribution $Gamma(shape, scale)$ for $\lambda$ . delta.alpha is the hyperparameter $\alpha$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ . delta.beta is the hyperparameter $\beta$ in the prior distribution $Beta(\alpha, \beta)$ for $\delta$ .
MCMCmethod	sampling method for $\delta$ in MCMC. It can be either 'IND' for independence proposal; or 'RW' for random walk proposal on logit scale.
rw.logit.delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit $\delta$ . Only applicable if MCMCmethod = 'RW'.
ind.delta.alpha	specifies the first parameter $\alpha$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
ind.delta.beta	specifies the first parameter $\beta$ when independent proposal $Beta(\alpha, \beta)$ for $\delta$ is used. Only applicable if MCMCmethod = 'IND'
nsample	specifies the number of posterior samples in the output.
control.mcmc	a list of three elements used in posterior sampling. delta.ini is the initial value of $\delta$ in MCMC sampling. burnin is the number of burn-ins. The output will only show MCMC samples after burnin. thin is the thinning parameter in MCMC sampling.

**Details**

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling  $\delta$ , and the deviance information criteria.

**Value**

A list of class "NPP" with four elements:

lambda                    posterior of the model parameter  $\lambda$ .

delta	posterior of the power parameter $\delta$ .
acceptance	the acceptance rate in MCMC sampling for $\delta$ using Metropolis-Hastings algorithm.
DIC	the deviance information criteria for model diagnostics.

**Author(s)**

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**References**

- Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.
- Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

**See Also**

[MultinomialNPP\\_MCMC](#); [NormalNPP\\_MCMC](#); [BernNPP\\_MCMC](#);

**Examples**

```
set.seed(1234)
DataHist <- rpois(n = 100, lambda = 49)
set.seed(12345)
DataCur <- rpois(n = 100, lambda = 49)

PoissonNPP_MCMC(Data.Cur = DataCur, Data.Hist = DataHist,
  CompStat = list(n0 = 20, mean0 = 10, n1 = 30, mean1 = 11),
  prior = list(lambda.shape = 1/2, lambda.scale = 100,
    delta.alpha = 1, delta.beta = 1),
  MCMCmethod = 'RW', rw.logit.delta = 1,
  ind.delta.alpha = 1, ind.delta.beta = 1, nsample = 10000,
  control.mcmc = list(delta.ini = NULL, burnin = 2000, thin = 1))
```

---

SPDData

*Dataset for Diagnostic Test (PartoSure Test, Medical Device) Evaluation for Spontaneous Preterm Delivery*

---

**Description**

The diagnostic test was developed to aid in rapidly assess the risk of spontaneous preterm delivery within 7 days from the time of diagnosis in pre-pregnant women with signs and symptoms. The same diagnostic test was used for two populations in US and EU respectively. The number of counts in the four cells (True positive, false positive, false negative, true negative) was recorded.

**Usage**

```
data("SPDData")
```

**Format**

A data frame with 2 observations on the following 5 variables.

Data.Region region where the diagnostic test was conducted  
 TPDP number of subjects with tested positive and the disease status positive (true positive)  
 TPDN number of subjects with tested positive but the disease status negative (false positive)  
 TNDP number of subjects with tested negative and the disease status positive (false negative)  
 TNDN number of subjects with tested negative and the disease status negative (true negative)

**Source**

[https://www.accessdata.fda.gov/cdrh\\_docs/pdf16/P160052C.pdf](https://www.accessdata.fda.gov/cdrh_docs/pdf16/P160052C.pdf)

**Examples**

```
data(SPDData)
```

---

VaccineData	<i>Dataset of a Vaccine Trial for RotaTeq and Multiple Historical Trials for Control Group</i>
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---

**Description**

The study was designed to investigate the concomitant use of RotaTeq(Test Vaccine) and some routine pediatric vaccines between 2001-2005. The dataset includes four historical control trials. The purpose of the study is to borrow the historical controls for the non-inferiority trial. The interest is in the response rate to the routine vaccines.

**Usage**

```
data("VaccineData")
```

**Format**

A data frame with 6 observations on the following 7 variables.

Data.Time indicator of historical data (coded as 0) or current data (coded as 1).  
 StudyID character to distinguish different studies.  
 Group indicator of control group (coded as 0) or treatment group (coded as 1).  
 Start.Year start year of the trial  
 End.Year end year of the trial  
 N total number of patients enrolled and dosed in the group  
 y total number of patients respond to the vaccine

**References**

Liu, G.F. (2018). A Dynamic Power Prior for Borrowing Historical Data in Noninferiority Trials with Binary Endpoint. *Pharmaceutical Statistics* 17:61-73.

**Examples**

```
data(VaccineData)
```

# Index

- \* **Bernoulli**
  - VaccineData, 53
- \* **multinomial**
  - SPDData, 52
- \* **multiple historical data normalized power prior**
  - BerMNPP\_MCMC1, 2
  - BerMNPP\_MCMC2, 4
  - BerOMNPP\_MCMC1, 8
  - BerOMNPP\_MCMC2, 10
  - LMMNPP\_MCMC1, 14
  - LMMNPP\_MCMC2, 16
  - LMOMNPP\_MCMC1, 22
  - LMOMNPP\_MCMC2, 24
  - PoiMNPP\_MCMC2, 46
  - PoiOMNPP\_MCMC1, 47
  - PoiOMNPP\_MCMC2, 49
- \* **multiple historical data**
  - PoiMNPP\_MCMC1, 44
- \* **normalized power prior**
  - BerNPP\_MCMC, 6
  - LaplaceLogC, 13
  - LMNPP\_MCMC, 19
  - logCdelta, 26
  - logCknot, 27
  - loglikBerD0, 28
  - loglikNormD0, 29
  - ModeDeltaBerNPP, 30
  - ModeDeltaLMNPP, 32
  - ModeDeltaMultinomialNPP, 33
  - ModeDeltaNormalNPP, 35
  - ModeDeltaPoisNPP, 37
  - MultinomialNPP\_MCMC, 39
  - NormalNPP\_MCMC, 41
  - PoiMNPP\_MCMC1, 44
  - PoissonNPP\_MCMC, 50
- \* **normal**
  - PHData, 43
- BerMNPP\_MCMC1, 2, 6, 9, 11
- BerMNPP\_MCMC2, 4, 4, 9, 11
- BerNPP\_MCMC, 6, 21, 40, 43, 52
- BerOMNPP\_MCMC1, 4, 6, 8, 11
- BerOMNPP\_MCMC2, 4, 6, 9, 10
- IRTNPP, 11
- LaplaceLogC, 13
- LMMNPP\_MCMC1, 14, 18, 23, 26
- LMMNPP\_MCMC2, 16, 16, 23, 26
- LMNPP\_MCMC, 19
- LMOMNPP\_MCMC1, 16, 18, 22, 26
- LMOMNPP\_MCMC2, 16, 18, 23, 24
- logCdelta, 26, 28–30
- logCknot, 14, 27, 27, 29, 30
- loglikBerD0, 27, 28, 28, 30
- loglikNormD0, 27–29, 29
- ModeDeltaBerNPP, 30, 33, 35, 36, 38
- ModeDeltaLMNPP, 32
- ModeDeltaMultinomialNPP, 31, 33, 33, 36, 38
- ModeDeltaNormalNPP, 31, 33, 35, 35, 38
- ModeDeltaPoisNPP, 31, 35, 36, 37
- MultinomialNPP\_MCMC, 8, 21, 39, 43, 52
- NormalNPP\_MCMC, 8, 21, 40, 41, 52
- PHData, 43
- PoiMNPP\_MCMC1, 44, 47, 48, 50
- PoiMNPP\_MCMC2, 45, 46, 48, 50
- PoiOMNPP\_MCMC1, 45, 47, 47, 50
- PoiOMNPP\_MCMC2, 45, 47, 48, 49
- PoissonNPP\_MCMC, 8, 21, 40, 43, 50
- SPDData, 52
- VaccineData, 53