

# Package ‘Pareto’

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**Type** Package

**Title** The Pareto, Piecewise Pareto and Generalized Pareto Distribution

**Version** 2.4.5

**Description** Utilities for the Pareto, piecewise Pareto and generalized Pareto distribution that are useful for reinsurance pricing. In particular, the package provides a non-trivial algorithm that can be used to match the expected losses of a tower of reinsurance layers with a layer-independent collective risk model. The theoretical background of the matching algorithm and most other methods are described in Ulrich Riegel (2018) <[doi:10.1007/s13385-018-0177-3](https://doi.org/10.1007/s13385-018-0177-3)>.

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dGenPareto	<i>Density of the generalized Pareto Distribution</i>
------------	---

---

### Description

Calculates the density function of the generalized Pareto distribution

### Usage

```
dGenPareto(x, t, alpha_ini, alpha_tail, truncation = NULL)
```

### Arguments

x	Numeric. The function evaluates the density at x.
t	Numeric. Threshold of the Pareto distribution.
alpha_ini	Numeric. Initial Pareto alpha.
alpha_tail	Numeric. Tail Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the generalized Pareto distribution is truncated at truncation.

### Value

Density function of the Pareto distribution with parameters t, alpha\_ini and alpha\_tail evaluated at x

### Examples

```
x <- 0:10 * 1000
dGenPareto(x, 1000, 1, 3)
dGenPareto(x, 1000, 1, 3, truncation = 5000)
```

---

dPareto                      *Density of the Pareto Distribution*

---

**Description**

Calculates the density function of the Pareto distribution

**Usage**

```
dPareto(x, t, alpha, truncation = NULL)
```

**Arguments**

x	Numeric. The function evaluates the density at x.
t	Numeric. Threshold of the Pareto distribution.
alpha	Numeric. Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

Density function of the Pareto distribution with parameters t and alpha evaluated at x

**Examples**

```
x <- 0:10 * 1000
dPareto(x, 1000, 2)
dPareto(x, 1000, 2, truncation = 5000)
```

---

dPiecewisePareto                      *Density of the Piecewise Pareto Distribution*

---

**Description**

Calculates the density function of the piecewise Pareto distribution

**Usage**

```
dPiecewisePareto(x, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

x	Numeric. The function evaluates the density at x.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

**Value**

Density function of the piecewise Pareto distribution with parameter vectors t and alpha evaluated at x

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
x <- 0:10 * 1000
dPiecewisePareto(x, t, alpha)
dPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "lp")
dPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

Example1\_AP

*Example data: Attachment Points*

---

**Description**

Example data: Attachment Points

**Usage**

```
Example1_AP
```

**Format**

An object of class numeric of length 5.

---

 Example1\_EL

*Example data: Expected Losses*


---

**Description**

Example data: Expected Losses

**Usage**

Example1\_EL

**Format**

An object of class `numeric` of length 5.

---

Excess\_Frequency

*Expected Frequency in Excess of a Threshold*


---

**Description**

Calculates the expected frequency in excess of a threshold for a collective model

**Usage**

```
Excess_Frequency(CollectiveModel, x = 0)
```

**Arguments**

`CollectiveModel`

A collective model object. Currently only `PPP_Models` are handled.

`x`

Numeric. Threshold.

**Value**

The expected frequency in excess of `x` for the given `CollectiveModel`

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Excess_Frequency(PPPM, c(-Inf, 0, 1000, 2000, 3000, Inf))
```

---

`Excess_Frequency.PGP_Model`*Expected Frequency in Excess of a Threshold*

---

**Description**

Calculates the expected frequency in excess of a threshold for a PGP\_model

**Usage**

```
## S3 method for class 'PGP_Model'  
Excess_Frequency(CollectiveModel, x = 0)
```

**Arguments**

CollectiveModel	PGP_Model object.
x	Numeric. Threshold.

**Value**

The expected frequency in excess of x for the given CollectiveModel

**Examples**

```
PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)  
PGPM  
Excess_Frequency(PGPM, c(-Inf, 0, 1000, 2000, 3000, Inf))
```

---

`Excess_Frequency.PPP_Model`*Expected Frequency in Excess of a Threshold*

---

**Description**

Calculates the expected frequency in excess of a threshold for a PPP\_model

**Usage**

```
## S3 method for class 'PPP_Model'  
Excess_Frequency(CollectiveModel, x = 0)
```

**Arguments**

CollectiveModel  
 PPP\_Model object.  
 x  
 Numeric. Threshold.

**Value**

The expected frequency in excess of x for the given CollectiveModel

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Excess_Frequency(PPPM, c(-Inf, 0, 1000, 2000, 3000, Inf))
```

---

 Fit\_PML\_Curve

*Fits a Collective Model to a PML Curve*


---

**Description**

Fits a PPP\_Model that matches the values of a PML curve

**Usage**

```
Fit_PML_Curve(
  return_periods,
  amounts,
  tail_alpha = 2,
  truncation = NULL,
  truncation_type = "lp",
  dispersion = 1
)
```

**Arguments**

return\_periods Numeric vector. Vector containing the return periods of the PML curve.  
 amounts Numeric vector. Vector containing the loss amounts corresponding to the return periods.  
 tail\_alpha Numerical. Pareto alpha that is used above the highest amount of the PML curve.  
 truncation Numeric. If truncation is not NULL and truncation > max(t), then the distribution is truncated at truncation.  
 truncation\_type Character. If truncation\_type = "wd" then the whole distribution is truncated. If truncation\_type = "lp" then a truncated Pareto is used for the last piece.  
 dispersion Numerical. Dispersion of the claim count distribution in the resulting PPP\_Model.

**Value**

A PPP\_Model object that contains the information about a collective model with a Panjer distributed claim count and a Piecewise Pareto distributed severity. The object contains the following elements:

- FQ Numerical. Frequency in excess of the lowest threshold of the piecewise Pareto distribution
- t Numeric vector. Vector containing the thresholds for the piecewise Pareto distribution
- alpha Numeric vector. Vector containing the Pareto alphas of the piecewise Pareto distribution
- truncation Numerical. If truncation is not NULL and truncation > max(t), then the distribution is truncated at truncation.
- truncation\_type Character. If truncation\_type = "wd" then the whole distribution is truncated. If truncation\_type = "lp" then a truncated Pareto is used for the last piece.
- dispersion Numerical. Dispersion of the Panjer distribution (i.e. variance to mean ratio).
- Status Numerical indicator: 0 = success, 1 = some information has been ignored, 2 = no solution found
- Comment Character. Information on whether the fit was successful

**Examples**

```
return_periods <- c(1, 5, 10, 20, 50, 100)
amounts <- c(1000, 4000, 7000, 10000, 13000, 14000)

fit <- Fit_PML_Curve(return_periods, amounts)
1 / Excess_Frequency(fit, amounts)

fit <- Fit_PML_Curve(return_periods, amounts, tail_alpha = 1.5,
                    truncation = 20000, truncation_type = "wd")
1 / Excess_Frequency(fit, amounts)
```

**Description**

The function fits a collective model to a wishlist of references (expected layer losses and excess frequencies). The function allows to specify the family of the severity distribution that is used. Depending on this distribution family the function works slightly differently:

- For the severity distribution PiecewisePareto the function returns a PPP\_Model that satisfies all the references
- For the severity distribution Pareto the function returns a PPP\_Model that minimizes the squared relative deviations from the references
- For the severity distribution GenPareto the function returns a PGP\_Model that minimizes the squared relative deviations from the references

**Usage**

```
Fit_References(
  Covers = NULL,
  Attachment_Points = NULL,
  Expected_Layer_Losses = NULL,
  Thresholds = NULL,
  Frequencies = NULL,
  model_threshold = min(c(Attachment_Points, Thresholds)),
  default_alpha = 2,
  dispersion = 1,
  alpha_max = 100,
  severity_distribution = "PiecewisePareto",
  ignore_inconsistent_references = FALSE
)
```

**Arguments**

Covers	Numeric vector. Vector containing the covers of the layers from the wishlist.
Attachment_Points	Numeric vector. Vector containing the attachment points of the layers from the wishlist.
Expected_Layer_Losses	Numeric vector. Vector containing the expected losses of the layers from the wishlist.
Thresholds	Numeric vector. Contains the thresholds from the wishlist for which excess frequencies are given.
Frequencies	Numeric vector. Expected frequencies excess the Thresholds from the wishlist.
model_threshold	Numerical. Lowest threshold of the fitted piecewise Pareto distribution.
default_alpha	Numerical. Default alpha for situations where an alpha has to be selected.
dispersion	Numerical. Dispersion of the claim count distribution in the resulting PPP_Model.
alpha_max	Numerical. Maximum alpha to be used for the matching.
severity_distribution	Character. Implemented distributions: "PiecewisePareto" (default), "Pareto" and "GenPareto".
ignore_inconsistent_references	Logical. If TRUE then inconsistent references are ignored in case of the piecewise Pareto distribution and the other references are used to fit the model

**Value**

For severity\_distribution = "PiecewisePareto" or "Pareto": A PPP\_Model object that contains the information about a collective model with a Panjer distributed claim count and a Piecewise Pareto distributed severity. The object contains the following elements:

- FQ Numerical. Frequency in excess of the lowest threshold of the piecewise Pareto distribution

- `t` Numeric vector. Vector containing the thresholds for the piecewise Pareto distribution
- `alpha` Numeric vector. Vector containing the Pareto alphas of the piecewise Pareto distribution
- `truncation` Numerical. If `truncation` is not NULL and `truncation > max(t)`, then the distribution is truncated at `truncation`.
- `truncation_type` Character. If `truncation_type = "wd"` then the whole distribution is truncated. If `truncation_type = "lp"` then a truncated Pareto is used for the last piece.
- `dispersion` Numerical. Dispersion of the Panjer distribution (i.e. variance to mean ratio).
- `Status` Numerical indicator: 0 = success, 1 = some information has been ignored, 2 = no solution found
- `Comment` Character. Information on whether the fit was successful

For `severity_distribution = "GenPareto"`: A `PGP_Model` object that contains the information about a collective model with a Panjer distributed claim count and a Piecewise Pareto distributed severity. The object contains the following elements:

- `FQ` Expected claim count of the collective model.
- `t` Numeric. Threshold of the Pareto distribution.
- `alpha_ini` Numeric. Initial Pareto alpha (at `t`).
- `alpha_tail` Numeric. Tail Pareto alpha.
- `truncation` If `truncation` is not NULL and `truncation > t`, then the Pareto distribution is truncated at `truncation`.
- `dispersion` Numerical. Dispersion of the Panjer distribution (i.e. variance to mean ratio).
- `Status` Numerical indicator: 0 = success, 1 = some information has been ignored, 2 = no solution found
- `Comment` Character. Information on whether the fit was successful

## Examples

```
covers <- c(1000, 1000, 1000)
att_points <- c(1000, 2000, 5000)
exp_losses <- c(100, 50, 10)
thresholds <- c(4000, 10000)
fqs <- c(0.04, 0.005)
fit <- Fit_References(covers, att_points, exp_losses, thresholds, fqs)
Layer_Mean(fit, covers, att_points)
Excess_Frequency(fit, thresholds)
fit <- Fit_References(covers, att_points, exp_losses, thresholds, fqs,
                     severity_distribution = "Pareto")
Layer_Mean(fit, covers, att_points)
Excess_Frequency(fit, thresholds)
fit <- Fit_References(covers, att_points, exp_losses,
                     severity_distribution = "GenPareto")
Layer_Mean(fit, covers, att_points)
```

---

GenPareto\_Layer\_Mean    *Layer Mean of the generalized Pareto Distribution*

---

### Description

Calculates the expected loss of a generalized Pareto distribution in a reinsurance layer

### Usage

```
GenPareto_Layer_Mean(
  Cover,
  AttachmentPoint,
  t,
  alpha_ini,
  alpha_tail,
  truncation = NULL
)
```

### Arguments

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used.
alpha_ini	Numeric. Initial Pareto alpha (at t).
alpha_tail	Numeric. Tail Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

### Value

The expected loss of the (truncated) Pareto distribution with parameters t and alpha in the layer Cover xs AttachmentPoint

### Examples

```
GenPareto_Layer_Mean(4000, 1000, 1000, 1, 3)
GenPareto_Layer_Mean(4000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3)
GenPareto_Layer_Mean(4000, 1000, t = 5000, alpha_ini = 1, alpha_tail = 3)
GenPareto_Layer_Mean(4000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3, truncation = 5000)
GenPareto_Layer_Mean(9000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3, truncation = 5000)
```

---

GenPareto\_Layer\_SM      *Second Layer Moment of the Generalized Pareto Distribution*

---

### Description

Calculates the second moment of a generalized Pareto distribution in a reinsurance layer

### Usage

```
GenPareto_Layer_SM(  
  Cover,  
  AttachmentPoint,  
  t,  
  alpha_ini,  
  alpha_tail,  
  truncation = NULL  
)
```

### Arguments

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used
alpha_ini	Numeric. Initial Pareto alpha (at t).
alpha_tail	Numeric. Tail Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

### Value

The second moment of the (truncated) generalized Pareto distribution with parameters t, alpha\_ini and alpha\_tail in the layer Cover xs AttachmentPoint

### Examples

```
GenPareto_Layer_SM(4000, 1000, 1000, 1, 2)  
GenPareto_Layer_SM(4000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3)  
GenPareto_Layer_SM(4000, 1000, t = 5000, alpha_ini = 1, alpha_tail = 3)  
GenPareto_Layer_SM(4000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3, truncation = 5000)  
GenPareto_Layer_SM(9000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3, truncation = 5000)
```

---

GenPareto\_Layer\_Var     *Layer Variance of the Generalized Pareto Distribution*

---

### Description

Calculates the variance of a generalized Pareto distribution in a reinsurance layer

### Usage

```
GenPareto_Layer_Var(
  Cover,
  AttachmentPoint,
  t,
  alpha_ini,
  alpha_tail,
  truncation = NULL
)
```

### Arguments

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used
alpha_ini	Numeric. Initial Pareto alpha (at t).
alpha_tail	Numeric. Tail Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

### Value

Variance of the (truncated) generalized Pareto distribution with parameters t, alpha\_ini and alpha\_tail in the layer Cover xs AttachmentPoint

### Examples

```
GenPareto_Layer_Var(4000, 1000, 1000, 1, 2)
GenPareto_Layer_Var(4000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3)
GenPareto_Layer_Var(4000, 1000, t = 5000, alpha_ini = 1, alpha_tail = 3)
GenPareto_Layer_Var(4000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3, truncation = 5000)
GenPareto_Layer_Var(9000, 1000, t = 1000, alpha_ini = 1, alpha_tail = 3, truncation = 5000)
```

---

 GenPareto\_ML\_Estimator\_Alpha

*Maximum Likelihood Estimation of the Pareto Alphas of a Generalized Pareto Distribution*

---

### Description

Calculates the maximum likelihood estimators of the parameters `alpha_ini` and `alpha_tail` of a generalized Pareto distribution with known threshold and (if applicable) known truncation

### Usage

```
GenPareto_ML_Estimator_Alpha(
  losses,
  t,
  truncation = NULL,
  reporting_thresholds = NULL,
  is.censored = NULL,
  weights = NULL,
  alpha_min = 0.001,
  alpha_max = 10
)
```

### Arguments

<code>losses</code>	Numeric vector. Losses that are used for the ML estimation.
<code>t</code>	Numeric or numeric vector. Threshold of the generalized Pareto distribution. Alternatively, <code>t</code> can be a vector of same length as <code>losses</code> . In this case <code>t[i]</code> is the reporting threshold of <code>losses[i]</code> .
<code>truncation</code>	Numeric. If <code>truncation</code> is not <code>NULL</code> and <code>truncation &gt; t</code> , then the generalized Pareto distribution is truncated at <code>truncation</code> .
<code>reporting_thresholds</code>	Numeric vector. Allows to enter loss specific reporting thresholds. If <code>NULL</code> then all reporting thresholds are assumed to be less than or equal to <code>t</code> .
<code>is.censored</code>	Logical vector. <code>TRUE</code> indicates that a loss has been censored by the policy limit. The assumption is that the uncensored losses are Generalized Pareto distributed with the alphas we are looking for. <code>is.censored = NULL</code> means that no losses are censored.
<code>weights</code>	Numeric vector. Weights for the losses. For instance <code>weights[i] = 2</code> and <code>weights[j] = 1</code> for <code>j != i</code> has the same effect as adding another loss of size <code>loss[i]</code> .
<code>alpha_min</code>	Numeric. Lower bound for the estimated alphas.
<code>alpha_max</code>	Numeric. Upper bound for the estimated alphas.

**Value**

Maximum likelihood estimator for the parameters `alpha_ini` and `alpha_tail` of a generalized Pareto distribution with threshold `t` given the observations `losses`

**Examples**

```

losses <- rGenPareto(1000, 1000, 2,3)
GenPareto_ML_Estimator_Alpha(losses, 1000)
losses <- rGenPareto(1000, 1000, 2, 1, truncation = 10000)
GenPareto_ML_Estimator_Alpha(losses, 1000)
GenPareto_ML_Estimator_Alpha(losses, 1000, truncation = 10000)

t <- 1000
alpha_ini <- 1
alpha_tail <- 3
losses <- rGenPareto(5000, t, alpha_ini, alpha_tail)
reporting_thresholds <- rPareto(5000, 1000, 3)
reported <- losses > reporting_thresholds
losses <- losses[reported]
reporting_thresholds <- reporting_thresholds[reported]
GenPareto_ML_Estimator_Alpha(losses, t)
GenPareto_ML_Estimator_Alpha(losses, t, reporting_thresholds = reporting_thresholds)
limit <- 3000
censored <- losses > limit
losses[censored] <- limit
reported <- losses > reporting_thresholds
losses <- losses[reported]
censored <- censored[reported]
reporting_thresholds <- reporting_thresholds[reported]
GenPareto_ML_Estimator_Alpha(losses, t, reporting_thresholds = reporting_thresholds)
GenPareto_ML_Estimator_Alpha(losses, t, reporting_thresholds = reporting_thresholds,
                             is.censored = censored)

losses <- c(190, 600, 120, 270, 180, 120)
w <- rep(1, length(losses))
w[1] <- 3
losses2 <- c(losses, losses[1], losses[1])
GenPareto_ML_Estimator_Alpha(losses, 100, weights = w)
GenPareto_ML_Estimator_Alpha(losses2, 100)

```

---

is.PGP\_Model

*Check if an object is a PGP\_Model*


---

**Description**

Checks if the class of an object is 'PGP\_Model'

**Usage**

```
is.PGP_Model(x)
```

**Arguments**

x                    Object to be checked.

**Examples**

```
PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)
PGPM
is.valid.PGP_Model(PGPM)
is.valid.PGP_Model(PGPM, comment = TRUE)

PGPM$alpha_tail <- -2
is.PGP_Model(PGPM)
is.valid.PGP_Model(PGPM)
is.valid.PGP_Model(PGPM, comment = TRUE)
```

---

is.PPP_Model	<i>Check if an object is a PPP_Model</i>
--------------	--

---

**Description**

Checks if the class of an object is 'PPP\_Model'

**Usage**

```
is.PPP_Model(x)
```

**Arguments**

x                    Object to be checked.

**Examples**

```
PPPM <- PPP_Model(2, c(1000,2000), c(1,2), dispersion = 2)
PPPM
is.valid.PPP_Model(PPPM)

PPPM$alpha <- 2
is.valid.PPP_Model(PPPM)
is.PPP_Model(PPPM)
```

---

`is.valid.PGP_Model`      *Check if an object is a valid PGP\_Model*

---

### **Description**

Checks if an object is a PGP\_Model object and whether it is valid for the use in functions like Layer\_Mean

### **Usage**

```
is.valid.PGP_Model(x, comment = FALSE)
```

### **Arguments**

x	Object to be checked.
comment	If FALSE then the function returns a boolean indicating whether x is a valid PGP_Model. If TRUE then the function returns a comment instead.

### **Examples**

```
PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)
PGPM
is.valid.PGP_Model(PGPM)
is.valid.PGP_Model(PGPM, comment = TRUE)

PGPM$alpha_tail <- -2
is.valid.PGP_Model(PGPM)
is.valid.PGP_Model(PGPM, comment = TRUE)
```

---

`is.valid.PPP_Model`      *Check if an object is a valid PPP\_Model*

---

### **Description**

Checks if an object is a PPP\_Model object and whether it is valid for the use in functions like Layer\_Mean

### **Usage**

```
is.valid.PPP_Model(x, comment = FALSE)
```

### **Arguments**

x	Object to be checked.
comment	If FALSE then the function returns a boolean indicating whether x is a valid PPP_Model. If TRUE then the function returns a comment instead.

**Examples**

```

PPPM <- PPP_Model(2, c(1000,2000), c(1,2), dispersion = 2)
PPPM
is.valid.PPP_Model(PPPM)
is.valid.PPP_Model(PPPM, comment = TRUE)

PPPM$alpha <- 2
is.valid.PPP_Model(PPPM)
is.valid.PPP_Model(PPPM, comment = TRUE)

```

---

Layer_Mean	<i>Expected Loss of a Reinsurance Layer</i>
------------	---

---

**Description**

Calculates the expected loss of a reinsurance layer for a collective model

**Usage**

```
Layer_Mean(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel	A collective model object. Currently only PPP_Models are handled.
Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.

**Value**

The expected loss of the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```

PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Example1_AP
Example1_Cov
Example1_EL
Layer_Mean(PPPM, Example1_Cov, Example1_AP)

```

---

Layer\_Mean.PGP\_Model *Expected Loss of a Reinsurance Layer*

---

### Description

Calculates the expected loss of a reinsurance layer for a PGP\_Model

### Usage

```
## S3 method for class 'PGP_Model'
Layer_Mean(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

### Arguments

CollectiveModel  
PGP\_Model object.

Cover  
Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint  
Numeric. Attachment point of the reinsurance layer.

### Value

The expected loss of the layer Cover xs AttachmentPoint for the given CollectiveModel

### Examples

```
PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)
PGPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Example1_AP
Example1_Cov
Example1_EL
Layer_Mean(PGPM, Example1_Cov, Example1_AP)
```

---

Layer\_Mean.PPP\_Model *Expected Loss of a Reinsurance Layer*

---

### Description

Calculates the expected loss of a reinsurance layer for a PPP\_Model

### Usage

```
## S3 method for class 'PPP_Model'
Layer_Mean(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel  
 PPP\_Model object.

Cover  
 Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint  
 Numeric. Attachment point of the reinsurance layer.

**Value**

The expected loss of the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Example1_AP
Example1_Cov
Example1_EL
Layer_Mean(PPPM, Example1_Cov, Example1_AP)
```

---

 Layer\_Sd

*Standard Deviation of a Reinsurance Layer*


---

**Description**

Calculates the standard deviation of the loss in a reinsurance layer for a collective model

**Usage**

```
Layer_Sd(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel  
 A collective model object. Currently only PPP\_Models are handled.

Cover  
 Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint  
 Numeric. Attachment point of the reinsurance layer.

**Value**

The standard deviation of the loss in the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```

PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Sd(PPPM, Example1_Cov, Example1_AP)

```

---

Layer\_Sd.PGP\_Model      *Standard Deviation of a Reinsurance Layer*

---

**Description**

Calculates the standard deviation of the loss in a reinsurance layer for a PGP\_model

**Usage**

```

## S3 method for class 'PGP_Model'
Layer_Sd(CollectiveModel, Cover = Inf, AttachmentPoint = 0)

```

**Arguments**

CollectiveModel      PGP\_Model object.

Cover      Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint      Numeric. Attachment point of the reinsurance layer.

**Value**

The standard deviation of the loss in the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```

PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)
PGPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Sd(PGPM, Example1_Cov, Example1_AP)

```

---

Layer\_Sd.PPP\_Model      *Standard Deviation of a Reinsurance Layer*

---

**Description**

Calculates the standard deviation of the loss in a reinsurance layer for a PPP\_model

**Usage**

```
## S3 method for class 'PPP_Model'
Layer_Sd(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel      PPP\_Model object.

Cover      Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint      Numeric. Attachment point of the reinsurance layer.

**Value**

The standard deviation of the loss in the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Sd(PPPM, Example1_Cov, Example1_AP)
```

---

Layer\_Var      *Variance of a Reinsurance Layer*

---

**Description**

Calculates the variance of the loss in a reinsurance layer for a collective model

**Usage**

```
Layer_Var(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel  
 A collective model object. Currently only PPP\_Models are handled.

Cover  
 Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint  
 Numeric. Attachment point of the reinsurance layer.

**Value**

The variance of the loss in the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Var(PPPM, Example1_Cov, Example1_AP)
```

---

Layer\_Var.PGP\_Model1     *Variance of a Reinsurance Layer*

---

**Description**

Calculates the variance of the loss in a reinsurance layer for a PGP\_model

**Usage**

```
## S3 method for class 'PGP_Model'
Layer_Var(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel  
 PGP\_Model object.

Cover  
 Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint  
 Numeric. Attachment point of the reinsurance layer.

**Value**

The variance of the loss in the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```
PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)
PGPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Var(PGPM, Example1_Cov, Example1_AP)
```

---

Layer\_Var.PPP\_Model     *Variance of a Reinsurance Layer*

---

**Description**

Calculates the variance of the loss in a reinsurance layer for a PPP\_model

**Usage**

```
## S3 method for class 'PPP_Model'
Layer_Var(CollectiveModel, Cover = Inf, AttachmentPoint = 0)
```

**Arguments**

CollectiveModel     PPP\_Model object.

Cover     Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.

AttachmentPoint     Numeric. Attachment point of the reinsurance layer.

**Value**

The variance of the loss in the layer Cover xs AttachmentPoint for the given CollectiveModel

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Var(PPPM, Example1_Cov, Example1_AP)
```

---

Local\_Pareto\_Alpha     *Local Pareto Alpha*

---

### Description

Calculates the local Pareto alpha of the normal, lognormal and gamma distribution

### Usage

```
Local_Pareto_Alpha(x, distribution, ...)
```

### Arguments

x	Numeric. Vector of thresholds at which the local Pareto alpha is calculated.
distribution	Character. <ul style="list-style-type: none"> <li>• 'lnorm' for lognormal distribution (arguments: meanlog, sdlog)</li> <li>• 'norm' for normal distribution (arguments: mean, sd)</li> <li>• 'gamma' for gamma distribution (arguments: shape, rate, scale)</li> <li>• 'weibull' for Weibull distribution (arguments: shape, scale)</li> <li>• 'exp' for exponential distribution (arguments: rate)</li> <li>• 'Pareto' for Pareto distribution (arguments: t, alpha, truncation = NULL)</li> <li>• 'GenPareto' for exp distribution (arguments: t, alpha_ini, alpha_tail, truncation = NULL)</li> <li>• 'PiecewisePareto' for exp distribution (arguments: t, alpha, truncation = NULL, truncation_type = 'lp')</li> </ul>
...	Arguments for the selected distribution

### Value

Local Pareto alpha of the selected distribution at x

### References

Riegel, U. (2008) Generalizations of common ILF models. *Blaetter der DGVMF* 29: 45–71

### Examples

```
x <- 1:10 * 1e6
Local_Pareto_Alpha(x, "norm", mean = 5e6, sd = 2e6)
Local_Pareto_Alpha(x, "lnorm", meanlog = 0, sdlog = 4)
Local_Pareto_Alpha(x, "gamma", shape = 5, rate = 1e-6)
Local_Pareto_Alpha(x, "weibull", shape = 0.5, scale = 1e6)
Local_Pareto_Alpha(x, "exp", rate = 1e-6)
Local_Pareto_Alpha(x, "Pareto", t = 1e6, alpha = 1, truncation = 20e6)
Local_Pareto_Alpha(x, "GenPareto", t = 1e6, alpha_ini = 1, alpha_tail = 2)
Local_Pareto_Alpha(x, "PiecewisePareto", t = c(1e6, 3e6, 5e6), alpha = c(1, 2, 3),
truncation = 20e6, truncation_type = "wd")
```

---

Pareto\_CDF                      *Distribution Function of the Pareto Distribution*

---

**Description**

Calculates the cumulative distribution function of a Pareto distribution. This function is deprecated. Use `pPareto` instead.

**Usage**

```
Pareto_CDF(x, t, alpha, truncation = NULL)
```

**Arguments**

<code>x</code>	Numeric. The function evaluates the CDF at <code>x</code> .
<code>t</code>	Numeric. Threshold of the Pareto distribution.
<code>alpha</code>	Numeric. Pareto alpha.
<code>truncation</code>	Numeric. If <code>truncation</code> is not <code>NULL</code> and <code>truncation &gt; t</code> , then the Pareto distribution is truncated at <code>truncation</code> .

**Value**

Distribution function of the Pareto distribution with parameters `t` and `alpha` evaluated at `x`

**Examples**

```
x <- 0:10 * 1000
pPareto(x, 1000, 2)
pPareto(x, 1000, 2, truncation = 5000)
```

---

Pareto\_Extrapolation    *Pareto Extrapolation*

---

**Description**

Uses a Pareto distribution to derive the expected loss of a layer from the expected loss of another layer

**Usage**

```
Pareto_Extrapolation(
  Cover_1,
  AttachmentPoint_1,
  Cover_2,
  AttachmentPoint_2,
  alpha,
  ExpLoss_1 = NULL,
  truncation = NULL
)
```

**Arguments**

Cover_1	Numeric. Cover of the layer from which we extrapolate. Use Inf for unlimited layers.
AttachmentPoint_1	Numeric. Attachment point of the layer from which we extrapolate.
Cover_2	Numeric. Cover of the layer to which we extrapolate. Use Inf for unlimited layers.
AttachmentPoint_2	Numeric. Attachment point of the layer to which we extrapolate.
alpha	Numeric. Pareto alpha used for the extrapolation.
ExpLoss_1	Numeric. Expected loss of the layer from which we extrapolate. If NULL (default) then the function provides only the ratio between the expected losses of the layers.
truncation	Numeric. If truncation is not NULL and $\text{truncation} > \text{AttachmentPoint}_1$ , then the Pareto distribution is truncated at truncation.

**Value**

The expected loss of the layer  $\text{Cover}_2$  xs  $\text{AttachmentPoint}_2$  given that  $\text{Cover}_1$  xs  $\text{AttachmentPoint}_1$  has expected loss  $\text{ExpLoss}_1$  and assuming a (truncated) Pareto distribution with parameters  $t$  and  $\alpha$ . If missing then  $\text{ExpLoss}_1 == 1$  is assumed.

**References**

Riegel, U. (2018) Matching tower information with piecewise Pareto. *European Actuarial Journal* 8(2): 437–460

**Examples**

```
Pareto_Extrapolation(1000, 1000, 2000, 2000, 2, ExpLoss_1 = 100)
Pareto_Extrapolation(1000, 1000, 2000, 2000, 2) * 100
Pareto_Extrapolation(1000, 1000, 2000, 2000, 2, truncation = 5000, ExpLoss_1 = 100)
Pareto_Extrapolation(1000, 1000, 2000, 2000, 2, truncation = 5000) * 100
```

---

Pareto\_Find\_Alpha\_bt看\_FQs

*Pareto Alpha Between Two Frequencies*

---

### Description

Finds the Pareto alpha between two excess frequencies

### Usage

```
Pareto_Find_Alpha_bt看_FQs(  
  Threshold_1,  
  Frequency_1,  
  Threshold_2,  
  Frequency_2,  
  max_alpha = 100,  
  tolerance = 1e-10,  
  truncation = NULL  
)
```

### Arguments

Threshold_1	Numeric. Threshold 1
Frequency_1	Numeric. Expected frequency in excess of Threshold_1
Threshold_2	Numeric. Threshold 2
Frequency_2	Numeric. Expected frequency in excess of Threshold_2
max_alpha	Numeric. Upper limit for the alpha that is returned.
tolerance	Numeric. Accuracy of the result.
truncation	Numeric. If truncation is not NULL then the Pareto distribution is truncated at truncation.

### Value

The Pareto alpha between the expected number of claims Frequency\_1 excess Threshold\_1 and the expected number of claims Frequency\_2 excess Threshold\_2

### References

Riegel, U. (2018) Matching tower information with piecewise Pareto. European Actuarial Journal 8(2): 437–460

### Examples

```
Pareto_Find_Alpha_bt看_FQs(1000, 1, 2000, 0.5)  
Pareto_Find_Alpha_bt看_FQs(1000, 1, 2000, 0.5, truncation = 5000)
```

---

 Pareto\_Find\_Alpha\_bt看\_FQ\_Layer

*Pareto Alpha Between a Frequency and a Layer*


---

### Description

Finds the Pareto alpha between an excess frequency and the expected loss of a layer

### Usage

```
Pareto_Find_Alpha_bt看_FQ_Layer(
  Threshold,
  Frequency,
  Cover,
  AttachmentPoint,
  ExpLoss,
  max_alpha = 100,
  tolerance = 1e-10,
  truncation = NULL
)
```

### Arguments

Threshold	Numeric. Threshold
Frequency	Numeric. Expected frequency in excess of Thershold
Cover	Numeric. Cover of the second layer.
AttachmentPoint	Numeric. Attachment point of the layer.
ExpLoss	Numeric. Expected loss of the layer.
max_alpha	Numeric. Upper limit for the alpha that is returned.
tolerance	Numeric. Accuracy of the result.
truncation	Numeric. If truncation is not NULL then the Pareto distribution is truncated at truncation.

### Value

The Pareto alpha between the expected number of claims Frequency excess Threshold and the layer Cover xs AttachmentPoint with expected loss ExpLoss

### References

Riegel, U. (2018) Matching tower information with piecewise Pareto. European Actuarial Journal 8(2): 437–460

**Examples**

```
Pareto_Find_Alpha_bt看_FQ_Layer(1000, 1, 1000, 1000, 500)
Pareto_Find_Alpha_bt看_FQ_Layer(1000, 1, 1000, 1000, 500, truncation = 5000)
```

---

Pareto\_Find\_Alpha\_bt看\_Layers

*Pareto Alpha Between Two Layers*


---

**Description**

Finds the Pareto alpha between two layers

**Usage**

```
Pareto_Find_Alpha_bt看_Layers(
  Cover_1,
  AttachmentPoint_1,
  ExpLoss_1,
  Cover_2,
  AttachmentPoint_2,
  ExpLoss_2,
  max_alpha = 100,
  tolerance = 1e-10,
  truncation = NULL
)
```

**Arguments**

Cover_1	Numeric. Cover of the first layer.
AttachmentPoint_1	Numeric. Attachment point of the first layer.
ExpLoss_1	Numeric. Expected loss of the first layer.
Cover_2	Numeric. Cover of the second layer.
AttachmentPoint_2	Numeric. Attachment point of the second layer.
ExpLoss_2	Numeric. Expected loss of the second layer.
max_alpha	Numeric. Upper limit for the alpha that is returned.
tolerance	Numeric. Accuracy of the result.
truncation	Numeric. If truncation is not NULL then the Pareto distribution is truncated at truncation.

**Value**

The Pareto alpha between the layer Cover\_1 xs AttachmentPoint\_1 with expected loss ExpLoss\_1 and the layer Cover\_2 xs AttachmentPoint\_2 with expected loss ExpLoss\_2

**References**

Riegel, U. (2018) Matching tower information with piecewise Pareto. *European Actuarial Journal* 8(2): 437–460

**Examples**

```
Pareto_Find_Alpha_bt看_Layers(100, 100, 100, 200, 200, 50)
Pareto_Find_Alpha_bt看_Layers(100, 100, 100, 200, 200, 50, truncation = 500)
```

---

Pareto_Layer_Mean	<i>Layer Mean of the Pareto Distribution</i>
-------------------	--

---

**Description**

Calculates the expected loss of a Pareto distribution in a reinsurance layer

**Usage**

```
Pareto_Layer_Mean(Cover, AttachmentPoint, alpha, t = NULL, truncation = NULL)
```

**Arguments**

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
alpha	Numeric. Pareto alpha.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

The expected loss of the (truncated) Pareto distribution with parameters t and alpha in the layer Cover xs AttachmentPoint

**Examples**

```
Pareto_Layer_Mean(4000, 1000, 2)
Pareto_Layer_Mean(4000, 1000, alpha = 2, t = 1000)
Pareto_Layer_Mean(4000, 1000, alpha = 2, t = 5000)
Pareto_Layer_Mean(4000, 1000, alpha = 2, t = 1000, truncation = 5000)
Pareto_Layer_Mean(9000, 1000, alpha = 2, t = 1000, truncation = 5000)
```

---

Pareto_Layer_SM	<i>Second Layer Moment of the Pareto Distribution</i>
-----------------	---

---

**Description**

Calculates the second moment of a Pareto distribution in a reinsurance layer

**Usage**

```
Pareto_Layer_SM(Cover, AttachmentPoint, alpha, t = NULL, truncation = NULL)
```

**Arguments**

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
alpha	Numeric. Pareto alpha.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

The second moment of the (truncated) Pareto distribution with parameters t and alpha in the layer Cover xs AttachmentPoint

**Examples**

```
Pareto_Layer_SM(4000, 1000, 2)
Pareto_Layer_SM(4000, 1000, alpha = 2, t = 1000)
Pareto_Layer_SM(4000, 1000, alpha = 2, t = 5000)
Pareto_Layer_SM(4000, 1000, alpha = 2, t = 1000, truncation = 5000)
Pareto_Layer_SM(9000, 1000, alpha = 2, t = 1000, truncation = 5000)
```

---

Pareto_Layer_Var	<i>Layer Variance of the Pareto Distribution</i>
------------------	--

---

**Description**

Calculates the variance of a Pareto distribution in a reinsurance layer

**Usage**

```
Pareto_Layer_Var(Cover, AttachmentPoint, alpha, t = NULL, truncation = NULL)
```

**Arguments**

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
alpha	Numeric. Pareto alpha.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

The variance of the(truncated) Pareto distribution with parameters t and alpha in the layer Cover  
xs AttachmentPoint

**Examples**

```
Pareto_Layer_Var(4000, 1000, 2)
Pareto_Layer_Var(4000, 1000, alpha = 2, t = 1000)
Pareto_Layer_Var(4000, 1000, alpha = 2, t = 5000)
Pareto_Layer_Var(4000, 1000, alpha = 2, t = 1000, truncation = 5000)
Pareto_Layer_Var(9000, 1000, alpha = 2, t = 1000, truncation = 5000)
```

---

Pareto\_ML\_Estimator\_Alpha

*Maximum Likelihood Estimation of the Alpha of a Pareto distribution*

---

**Description**

Calculates the maximum likelihood estimator for the parameter alpha of a Pareto distribution with a known threshold and (if applicable) a known truncation

**Usage**

```
Pareto_ML_Estimator_Alpha(
  losses,
  t,
  truncation = NULL,
  reporting_thresholds = NULL,
  is.censored = NULL,
  weights = NULL,
  alpha_min = 0.001,
  alpha_max = 10
)
```

**Arguments**

losses	Numeric vector. Losses that are used for the ML estimation.
t	Numeric. Threshold of the Pareto distribution.
truncation	Numeric. If truncation is not NULL, then the Pareto distribution is truncated at truncation.
reporting_thresholds	Numeric vector. Allows to enter loss specific reporting thresholds. If NULL then all reporting thresholds are assumed to be less than or equal to t.
is.censored	Logical vector. TRUE indicates that a loss has been censored by the policy limit. The assumption is that the uncensored losses are Pareto distributed with the alpha we are looking for. is.censored = NULL means that no losses are censored.
weights	Numeric vector. Weights for the losses. For instance weights[i] = 2 and weights[j] = 1 for j != i has the same effect as adding another loss of size loss[i].
alpha_min	Numeric. Lower bound for alpha (only used in truncated case).
alpha_max	Numeric. Upper bound for alpha (only used in truncated case).

**Value**

Maximum likelihood estimator for the parameter alpha of a Pareto distribution with threshold t given the observations losses

**Examples**

```

losses <- rPareto(100, 1000, 2)
Pareto_ML_Estimator_Alpha(losses, 1000)
losses <- rPareto(100, 1000, 2, truncation = 2000)
Pareto_ML_Estimator_Alpha(losses, 1000)
Pareto_ML_Estimator_Alpha(losses, 1000, truncation = 2000)

t <- 100
alpha <- 2
losses <- rPareto(10000, t, alpha)
reporting_thresholds <- rPareto(10000, t, 5)
index <- losses > reporting_thresholds
losses <- losses[index]
reporting_thresholds <- reporting_thresholds[index]
Pareto_ML_Estimator_Alpha(losses, t)
Pareto_ML_Estimator_Alpha(losses, t, reporting_thresholds = reporting_thresholds)

losses <- rPareto(10, 1000, 2)
w <- rep(1, 10)
w[1] <- 3
losses2 <- c(losses, losses[1], losses[1])
Pareto_ML_Estimator_Alpha(losses, 1000, weights = w)
Pareto_ML_Estimator_Alpha(losses2, 1000)

```

---

Pareto\_PDF *Density of the Pareto Distribution*

---

**Description**

Calculates the density function of the Pareto distribution. This function is deprecated. Use dPareto instead.

**Usage**

```
Pareto_PDF(x, t, alpha, truncation = NULL)
```

**Arguments**

x	Numeric. The function evaluates the density at x
t	Numeric. Threshold of the Pareto distribution.
alpha	Numeric. Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

Density function of the Pareto distribution with parameters t and alpha evaluated at x

**Examples**

```
x <- 0:10 * 1000
dPareto(x, 1000, 2)
dPareto(x, 1000, 2, truncation = 5000)
```

---

pGenPareto *Distribution Function of the generalized Pareto Distribution*

---

**Description**

Calculates the cumulative distribution function of a generalized Pareto distribution

**Usage**

```
pGenPareto(x, t, alpha_ini, alpha_tail, truncation = NULL)
```

**Arguments**

x	Numeric. The function evaluates the CDF at x.
t	Numeric. Threshold of the generalized Pareto distribution.
alpha_ini	Numeric. Initial Pareto alpha.
alpha_tail	Numeric. Tail Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the generalized Pareto distribution is truncated at truncation.

**Value**

Distribution function of the generalized Pareto distribution with parameters t, alpha\_ini and alpha\_tail evaluated at x

**Examples**

```
x <- 0:10 * 1000
pGenPareto(x, 1000, 1, 3)
pGenPareto(x, 1000, 1, 3, truncation = 5000)
```

---

PGP\_Model

*PGP\_Model (Collective Panjer & Generalized Pareto Model) Object*

---

**Description**

Constructor function for the PGP\_Model object

**Usage**

```
PGP_Model(  
  FQ = NULL,  
  t = NULL,  
  alpha_ini = NULL,  
  alpha_tail = NULL,  
  truncation = NULL,  
  dispersion = 1,  
  Status = 0,  
  Comment = "OK"  
)
```

**Arguments**

FQ	Numerical. Expected claim count of the collective model.
t	Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used
alpha_ini	Numeric. Initial Pareto alpha (at t).
alpha_tail	Numeric. Tail Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
dispersion	Numerical. Dispersion of the Panjer distribution (i.e. variance to mean ratio).
Status	Numerical indicator if a function returns a PGP_Model object: 0 = success, 1 = some information has been ignored, 2 = no solution found
Comment	Charakter. An optional comment.

**Examples**

```
PGPM <- PGP_Model(2, t = 1000, alpha_ini = 1, alpha_tail = 2 , dispersion = 2)
PGPM
```

---

PiecewisePareto\_CDF     *Distribution Function of the Piecewise Pareto Distribution*

---

**Description**

Calculates the cumulative distribution function of a Piecewise Pareto Distribution. This function is deprecated. Use pPiecewisePareto instead.

**Usage**

```
PiecewisePareto_CDF(x, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

x	Numeric. The function evaluates the CDF at x.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation	Numeric. If truncation is not NULL and truncation > t, then the distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

**Value**

Distribution function of the piecewise Pareto distribution with parameter vectors  $t$  and  $\alpha$  evaluated at  $x$

**References**

Riegel, U. (2018) Matching tower information with piecewise Pareto. *European Actuarial Journal* 8(2): 437–460

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
x <- 0:10 * 1000
pPiecewisePareto(x, t, alpha)
pPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "lp")
pPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

PiecewisePareto\_Layer\_Mean

*Layer Mean of the Piecewise Pareto Distribution*

---

**Description**

Calculates the expected loss of a piecewise Pareto distribution in a reinsurance layer

**Usage**

```
PiecewisePareto_Layer_Mean(
  Cover,
  AttachmentPoint,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp"
)
```

**Arguments**

Cover	Numeric. Cover of the reinsurance layer.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. $\alpha[i]$ is the Pareto alpha in excess of $t[i]$ .
truncation	Numeric. If truncation is not NULL and $\text{truncation} > t$ , then the Pareto distribution is truncated at truncation.

truncation\_type

Character. If truncation\_type = "wd" then the whole distribution is truncated. If truncation\_type = "lp" then a truncated Pareto is used for the last piece.

### Value

The expected loss of the (truncated) piecewise Pareto distribution with parameter vectors `t` and `alpha` in the layer Cover `xs` AttachmentPoint

### References

Riegel, U. (2018) Matching tower information with piecewise Pareto. *European Actuarial Journal* 8(2): 437–460

### Examples

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha)
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha, truncation = 5000)
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha, truncation = 5000, truncation_type = "lp")
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

PiecewisePareto\_Layer\_SM

*Second Layer Moment of the Piecewise Pareto Distribution*

---

### Description

Calculates the second moment of a piecewise Pareto distribution in a reinsurance layer

### Usage

```
PiecewisePareto_Layer_SM(
  Cover,
  AttachmentPoint,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp"
)
```

### Arguments

Cover                Numeric. Cover of the reinsurance layer.  
AttachmentPoint    Numeric. Attachment point of the reinsurance layer.

t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. $\alpha[i]$ is the Pareto alpha in excess of $t[i]$ .
truncation	Numeric. If truncation is not NULL and $\text{truncation} > t$ , then the Pareto distribution is truncated at truncation.
truncation_type	Character. If $\text{truncation\_type} = \text{"wd"}$ then the whole distribution is truncated. If $\text{truncation\_type} = \text{"lp"}$ then a truncated Pareto is used for the last piece.

**Value**

The second moment of the (truncated) piecewise Pareto distribution with parameter vectors  $t$  and  $\alpha$  in the layer  $\text{Cover}$  vs  $\text{AttachmentPoint}$

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha, truncation = 5000)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha, truncation = 5000, truncation_type = "lp")
PiecewisePareto_Layer_SM(4000, 1000, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

PiecewisePareto\_Layer\_Var

*Layer Variance of the Piecewise Pareto Distribution*

---

**Description**

Calculate the variance of a piecewise Pareto distribution in a reinsurance layer

**Usage**

```
PiecewisePareto_Layer_Var(
  Cover,
  AttachmentPoint,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp"
)
```

**Arguments**

Cover	Numeric. Cover of the reinsurance layer.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.

t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

**Value**

The variance of the (truncated) piecewise Pareto distribution with parameter vectors t and alpha in the layer Cover xs AttachmentPoint

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
PiecewisePareto_Layer_Var(4000, 1000, t, alpha)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha) - PiecewisePareto_Layer_Mean(4000, 1000, t, alpha)^2
PiecewisePareto_Layer_Var(4000, 1000, t, alpha, truncation = 5000)
PiecewisePareto_Layer_Var(4000, 1000, t, alpha, truncation = 5000, truncation_type = "lp")
PiecewisePareto_Layer_Var(4000, 1000, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

PiecewisePareto\_Match\_Layer\_Losses

*Match a Tower of Expected Layers Losses*

---

**Description**

Matches the expected losses of a tower of reinsurance layers using a piecewise Pareto severity

**Usage**

```
PiecewisePareto_Match_Layer_Losses(
  Attachment_Points,
  Expected_Layer_Losses,
  Unlimited_Layers = FALSE,
  Frequencies = NULL,
  FQ_at_lowest_AttPt = NULL,
  FQ_at_highest_AttPt = NULL,
  TotalLoss_Frequencies = NULL,
  minimize_ratios = TRUE,
  Use_unlimited_Layer_for_FQ = TRUE,
  truncation = NULL,
  truncation_type = "lp",
  dispersion = 1,
```

```

    tolerance = 1e-10,
    alpha_max = 100,
    merge_tolerance = 1e-06,
    RoL_tolerance = 1e-06
)

```

### Arguments

Attachment_Points	Numeric vector. Vector containing the attachment points of consecutive layers in increasing order
Expected_Layer_Losses	Numeric vector. Vector containing the expected losses of layers xs the attachment points.
Unlimited_Layers	Logical. If TRUE, then Expected_Layer_Losses[i] contains the expected loss of Inf xs Attachment_Points[i]. If FALSE then Expected_Layer_Losses[i] contains the expected loss of the layer Attachment_Points[i+1] xs Attachment_Points[i]
Frequencies	Numeric vector. Expected frequencies excess the attachment points. The vector may contain NAs. If NULL then the function calculates frequencies.
FQ_at_lowest_AttPt	Numerical. Expected frequency excess Attachment_Points[1]. Overrides first entry in Frequencies.
FQ_at_highest_AttPt	Numerical. Expected frequency excess Attachment_Points[k]. Overrides last entry in Frequencies.
TotalLoss_Frequencies	Numeric vector. TotalLoss_Frequencies[i] is the frequency of total losses to layer i (i.e. Attachment_Points[i+1] - Attachment_Points[i] xs Attachment_Points[i]). TotalLoss_Frequencies[i] is the frequency for losses larger than or equal to Attachment_Points[i+1], whereas Frequencies[i] is the frequency of losses larger than Attachment_Points[i]. TotalLoss_Frequencies[i] > Frequencies[i+1] means that there is a point mass of the severity at Attachment_Points[i+1].
minimize_ratios	Logical. If TRUE then ratios between alphas are minimized.
Use_unlimited_Layer_for_FQ	Logical. Only relevant if no frequency is provided for the highest attachment point by the user. If TRUE then the frequency is calculated using the Pareto alpha between the last two layers.
truncation	Numeric. If truncation is not NULL, then the distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.
dispersion	Numerical. Dispersion of the claim count distribution in the resulting PPP_Model.
tolerance	Numeric. Numerical tolerance.
alpha_max	Numerical. Maximum alpha to be used for the matching.

merge_tolerance	Numerical. Consecutive Pareto pieces are merged if the alphas deviate by less than merge_tolerance.
RoL_tolerance	Numerical. Consecutive layers are merged if RoL decreases less than factor $1 - \text{RoL\_tolerance}$ .

## Value

A PPP\_Model object that contains the information about a collective model with a Panjer distributed claim count and a Piecewise Pareto distributed severity. The object contains the following elements:

- FQ Numerical. Frequency in excess of the lowest threshold of the piecewise Pareto distribution
- t Numeric vector. Vector containing the thresholds for the piecewise Pareto distribution
- alpha Numeric vector. Vector containing the Pareto alphas of the piecewise Pareto distribution
- truncation Numerical. If truncation is not NULL and  $\text{truncation} > \max(t)$ , then the distribution is truncated at truncation.
- truncation\_type Character. If truncation\_type = "wd" then the whole distribution is truncated. If truncation\_type = "lp" then a truncated Pareto is used for the last piece.
- dispersion Numerical. Dispersion of the Panjer distribution (i.e. variance to mean ratio).
- Status Numerical indicator: 0 = success, 1 = some information has been ignored, 2 = no solution found
- Comment Character. Information on whether the fit was successful

## References

Riegel, U. (2018) Matching tower information with piecewise Pareto. European Actuarial Journal 8(2): 437–460

## Examples

```
AP <- Example1_AP
EL <- Example1_EL
PiecewisePareto_Match_Layer_Losses(AP, EL)
EL_unlimited <- rev(cumsum(rev(Example1_EL)))
PiecewisePareto_Match_Layer_Losses(AP, EL_unlimited, Unlimited_Layers = TRUE)
PiecewisePareto_Match_Layer_Losses(AP, EL, FQ_at_lowest_AttPt = 0.5)
Example1_FQ <- c(0.3, 0.15, 0.08, 0.02, 0.005)
PiecewisePareto_Match_Layer_Losses(AP, EL, Frequencies = Example1_FQ)
```

---

 PiecewisePareto\_ML\_Estimator\_Alpha

*Maximum Likelihood Estimation of the Alphas of the Piecewise Pareto Distribution*

---

### Description

Calculates the maximum likelihood estimator of the parameter vector alpha for a piecewise Pareto distribution with given vector t and (if applicable) a known truncation

### Usage

```
PiecewisePareto_ML_Estimator_Alpha(
  losses,
  t,
  truncation = NULL,
  truncation_type = "lp",
  reporting_thresholds = NULL,
  is.censored = NULL,
  weights = NULL,
  alpha_min = 0.001,
  alpha_max = 10
)
```

### Arguments

losses	Numeric vector. Losses that are used for the ML estimation.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
truncation	Numeric. If truncation is not NULL and $\text{truncation} > \max(t)$ , then the distribution is truncated at truncation.
truncation_type	Character. If <code>truncation_type = "wd"</code> then the whole distribution is truncated. If <code>truncation_type = "lp"</code> then a truncated Pareto is used for the last piece.
reporting_thresholds	Numeric vector. Allows to enter loss specific reporting thresholds. If NULL then all reporting thresholds are assumed to be less than or equal to $t[1]$ .
is.censored	Logical vector. TRUE indicates that a loss has been censored by the policy limit. The assumption is that the uncensored losses are piecewise Pareto distributed with the alphas we are looking for. <code>is.censored = NULL</code> means that no losses are censored.
weights	Numeric vector. Weights for the losses. For instance <code>weights[i] = 2</code> and <code>weights[j] = 1</code> for $j \neq i$ has the same effect as adding another loss of size <code>loss[i]</code> .
alpha_min	Numeric. Lower bound for the estimated alphas (only used in truncated case).
alpha_max	Numeric. Upper bound for the estimated alphas (only used in truncated case).

**Value**

Maximum likelihood estimator for the parameter alpha of a Pareto distribution with threshold t given the observations losses

**Examples**

```
losses <- rPiecewisePareto(10000, t = c(100,200,300), alpha = c(1,2,3))
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300))

losses <- rPiecewisePareto(10000, t = c(100,200,300), alpha = c(1,2,3),
                           truncation = 500, truncation_type = "wd")
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300))
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300),
                                   truncation = 500, truncation_type = "wd")
reporting_thresholds <- rPareto(10000, 100, 3)
index <- losses > reporting_thresholds
losses <- losses[index]
reporting_thresholds <- reporting_thresholds[index]
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300),
                                   truncation = 500, truncation_type = "wd")
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300),
                                   truncation = 500, truncation_type = "wd",
                                   reporting_thresholds = reporting_thresholds)

losses <- c(140, 240, 490, 200, 110, 710, 120, 190, 210, 310)
w <- rep(1, length(losses))
w[1] <- 2
losses2 <- c(losses, losses[1])
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300), weights = w)
PiecewisePareto_ML_Estimator_Alpha(losses2, c(100,200,300))
```

---

PiecewisePareto\_PDF *Density of the Piecewise Pareto Distribution*

---

**Description**

Calculates the density function of the piecewise Pareto distribution. This function is deprecated. Use dPiecewisePareto instead.

**Usage**

```
PiecewisePareto_PDF(x, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

x	Numeric. The function evaluates the density at x.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].

truncation      Numeric. If truncation is not NULL and truncation > t, then the distribution is truncated at truncation.

truncation\_type      Character. If truncation\_type = "wd" then the whole distribution is truncated. If truncation\_type = "lp" then a truncated Pareto is used for the last piece.

**Value**

Density function of the piecewise Pareto distribution with parameter vectors t and alpha evaluated at x

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
x <- 0:10 * 1000
dPiecewisePareto(x, t, alpha)
dPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "lp")
dPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

pPareto

*Distribution Function of the Pareto Distribution*


---

**Description**

Calculates the cumulative distribution function of a Pareto distribution

**Usage**

```
pPareto(x, t, alpha, truncation = NULL)
```

**Arguments**

x                  Numeric. The function evaluates the CDF at x.

t                  Numeric. Threshold of the Pareto distribution.

alpha              Numeric. Pareto alpha.

truncation        Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

Distribution function of the Pareto distribution with parameters t and alpha evaluated at x

**Examples**

```
x <- 0:10 * 1000
pPareto(x, 1000, 2)
pPareto(x, 1000, 2, truncation = 5000)
```

---

pPiecewisePareto

*Distribution Function of the Piecewise Pareto Distribution*


---

**Description**

Calculates the cumulative distribution function of a Piecewise Pareto Distribution

**Usage**

```
pPiecewisePareto(x, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

x	Numeric. The function evaluates the CDF at x.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation	Numeric. If truncation is not NULL and truncation > t, then the distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

**Value**

Distribution function of the piecewise Pareto distribution with parameter vectors t and alpha evaluated at x

**References**

Riegel, U. (2018) Matching tower information with piecewise Pareto. *European Actuarial Journal* 8(2): 437–460

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
x <- 0:10 * 1000
pPiecewisePareto(x, t, alpha)
pPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "lp")
pPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

 PPP\_Model

*PPP\_Model (Collective Panjer & Piecewise Pareto Model) Object*


---

**Description**

Constructor function for the PPP\_Model object

**Usage**

```

PPP_Model(
  FQ = NULL,
  t = NULL,
  alpha = NULL,
  truncation = NULL,
  truncation_type = "lp",
  dispersion = 1,
  Status = 0,
  Comment = "OK"
)

```

**Arguments**

FQ	Numerical. Expected claim count of the collective model.
t	Numeric vector. Vector containing the thresholds of the Piecewise Pareto distribution.
alpha	Numeric vector. Vector containing the alphas of the Piecewise Pareto distribution.
truncation	Numerical. If truncation is not NULL and $\text{truncation} > \max(t)$ , then the distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.
dispersion	Numerical. Dispersion of the Panjer distribution (i.e. variance to mean ratio).
Status	Numerical indicator if a function returns a PPP_Model object: 0 = success, 1 = some information has been ignored, 2 = no solution found
Comment	Charakter. An optional comment.

**Examples**

```

PPPM <- PPP_Model(2, c(1000,2000), c(1,2), dispersion = 2)
PPPM

```

---

 PPP\_Model\_Excess\_Frequency

*Expected Frequency in Excess of a Threshold*


---

### Description

Calculates the expected frequency in excess of a threshold for a PPP\_Model

### Usage

```
PPP_Model_Excess_Frequency(x, PPP_Model)
```

### Arguments

x	Numeric. Threshold.
PPP_Model	PPP_Model object.

### Value

The expected frequency in excess of x for the given PPP\_Model

### Examples

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Excess_Frequency(PPPM, c(-Inf, 0, 1000, 2000, 3000, Inf))
```

---

 PPP\_Model\_Exp\_Layer\_Loss

*Expected Loss of a Reinsurance Layer*


---

### Description

Calculates the expected loss of a reinsurance layer for a PPP\_Model. This function is deprecated. Use Layer\_Mean instead.

### Usage

```
PPP_Model_Exp_Layer_Loss(Cover, AttachmentPoint, PPP_Model)
```

### Arguments

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
PPP_Model	PPP_Model object.

**Value**

The expected loss of the layer Cover xs AttachmentPoint for the given PPP\_Model

**Examples**

```

PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Example1_AP
Example1_Cov
Example1_EL
Layer_Mean(PPPM, Example1_Cov, Example1_AP)

```

---

PPP\_Model\_Layer\_Sd      *Standard Deviation of a Reinsurance Layer*

---

**Description**

Calculates the standard deviation of the loss in a reinsurance layer for a PPP\_Model

**Usage**

```
PPP_Model_Layer_Sd(Cover, AttachmentPoint, PPP_Model)
```

**Arguments**

Cover	Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint	Numeric. Attachment point of the reinsurance layer.
PPP_Model	PPP_Model object.

**Value**

The standard deviation of the loss in the layer Cover xs AttachmentPoint for the given PPP\_Model

**Examples**

```

PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Sd(PPPM, Example1_Cov, Example1_AP)

```

---

PPP\_Model\_Layer\_Var     *Variance of a Reinsurance Layer*

---

### Description

Calculates the variance of the loss in a reinsurance layer for a PPP\_Model. This function is deprecated. Use Layer\_Var instead.

### Usage

```
PPP_Model_Layer_Var(Cover, AttachmentPoint, PPP_Model)
```

### Arguments

Cover                    Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.  
AttachmentPoint        Numeric. Attachment point of the reinsurance layer.  
PPP\_Model                PPP\_Model object.

### Value

The variance of the loss in the layer Cover xs AttachmentPoint for the given PPP\_Model

### Examples

```
PPPM <- PiecewisePareto_Match_Layer_Losses(Example1_AP, Example1_EL)
PPPM
Example1_Cov <- c(diff(Example1_AP), Inf)
Layer_Var(PPPM, Example1_Cov, Example1_AP)
```

---

PPP\_Model\_Simulate     *Simulate Losses with a PPP\_Model*

---

### Description

Simulates losses of a PPP\_Model

### Usage

```
PPP_Model_Simulate(n, PPP_Model)
```

### Arguments

n                        Integer. Number of Simulations.  
PPP\_Model                PPP\_Model object.

**Value**

A matrix where row k contains the simulated losses of the kth simulation.

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(c(1000, 2000, 3000), c(2000, 1000, 500),
                                          truncation = 10000, truncation_type = "wd")
PPPM
Simulate_Losses(PPPM, 100)
```

---

print.PGP_Model	<i>Print a PGP_Model Object (Collective Panjer &amp; Generalized Pareto Model) Object</i>
-----------------	---

---

**Description**

Print method for PGP\_Model objects

**Usage**

```
## S3 method for class 'PGP_Model'
print(x, ...)
```

**Arguments**

x	PGP_Model object.
...	Other arguments, all currently ignored.

---

print.PPP_Model	<i>Print a PPP_Model Object (Collective Panjer &amp; Piecewise Pareto Model) Object</i>
-----------------	---

---

**Description**

Print method for PPP\_Model objects

**Usage**

```
## S3 method for class 'PPP_Model'
print(x, ...)
```

**Arguments**

x	PPP_Model object.
...	Other arguments, all currently ignored.

---

`qGenPareto`*Quantile Function of the generalized Pareto Distribution*

---

**Description**

Calculates the quantile function of a generalized Pareto distribution

**Usage**

```
qGenPareto(p, t, alpha_ini, alpha_tail, truncation = NULL)
```

**Arguments**

<code>p</code>	Numeric. The function evaluates the inverse CDF at <code>p</code> .
<code>t</code>	Numeric. Threshold of the piecewise Pareto distribution.
<code>alpha_ini</code>	Numeric. Initial Pareto alpha.
<code>alpha_tail</code>	Numeric. Tail Pareto alpha.
<code>truncation</code>	Numeric. If <code>truncation</code> is not <code>NULL</code> and <code>truncation &gt; t</code> , then the generalized Pareto distribution is truncated at <code>truncation</code> .

**Value**

Quantile function of the Pareto distribution with parameters `t`, `alpha_ini` and `alpha_tail`, evaluated at `p`

**Examples**

```
p <- 0:10 * 0.1
qGenPareto(p, 1000, 2, 3)
qGenPareto(p, 1000, 2, 3, truncation = 5000)
```

---

`qPareto`*Quantile Function of the Pareto Distribution*

---

**Description**

Calculates the quantile function of a Pareto distribution

**Usage**

```
qPareto(p, t, alpha, truncation = NULL)
```

**Arguments**

p	Numeric. The function evaluates the inverse CDF at p.
t	Numeric. Threshold of the piecewise Pareto distribution.
alpha	Numeric. Pareto alpha.
truncation	Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**

Quantile function of the Pareto distribution with parameters t and alpha, evaluated at p

**Examples**

```
p <- 0:10 * 0.1
qPareto(p, 1000, 2)
qPareto(p, 1000, 2, truncation = 5000)
```

---

qPiecewisePareto      *Quantile Function of the Piecewise Pareto Distribution*

---

**Description**

Calculates the quantile function of a piecewise Pareto distribution

**Usage**

```
qPiecewisePareto(p, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

p	Numeric. The function evaluates the quantile function at p.
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation	Numeric. If truncation is not NULL and truncation > t, then the distribution is truncated at truncation.
truncation_type	Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

**Value**

Quantile function of the piecewise Pareto distribution with parameter vectors t and alpha evaluated at p

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
p <- 0:10 * 0.1
qPiecewisePareto(p, t, alpha)
qPiecewisePareto(p, t, alpha, truncation = 5000, truncation_type = "lp")
qPiecewisePareto(p, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

rGenPareto

*Simulation of the generalized Pareto Distribution*


---

**Description**

Generates random deviates of a generalized Pareto distribution

**Usage**

```
rGenPareto(n, t, alpha_ini, alpha_tail, truncation = NULL)
```

**Arguments**

n	Numeric. Number of observations.
t	Numeric vector. Thresholds of the generalized Pareto distributions
alpha_ini	Numeric vector. Initial Pareto alphas of the generalized Pareto distributions.
alpha_tail	Numeric vector. Tail Pareto alphas of the generalized Pareto distributions.
truncation	NULL or Numeric vector. If truncation is not NULL and truncation > t, then the generalized Pareto distributions are truncated at truncation (resampled generalized Pareto)

**Value**

A vector of n samples from the (truncated) generalized Pareto distribution with parameters t, alpha\_ini and alpha\_tail

**Examples**

```
rGenPareto(100, 1000, 2, 3)
rGenPareto(100, 1000, 2, 3, truncation = 2000)
rGenPareto(100, t = c(1, 10, 100, 1000), alpha_ini = 1, alpha_tail = c(2, 5))
```

---

rPareto                      *Simulation of the Pareto Distribution*

---

**Description**

Generates random deviates of a Pareto distribution

**Usage**

```
rPareto(n, t, alpha, truncation = NULL)
```

**Arguments**

n	Numeric. Number of observations.
t	Numeric vector. Thresholds of the Pareto distributions
alpha	Numeric vector. Pareto alphas of the Pareto distributions.
truncation	NULL or Numeric vector. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation (resampled Pareto)

**Value**

A vector of n samples from the (truncated) Pareto distribution with parameters t and alpha

**Examples**

```
rPareto(100, 1000, 2)
rPareto(100, 1000, 2, truncation = 2000)
rPareto(100, t = c(1, 10, 100, 1000, 10000), alpha = c(1, 2, 4, 8, 16))
```

---

rPiecewisePareto                      *Simulation of the Piecewise Pareto Distribution*

---

**Description**

Generates random deviates of a piecewise Pareto distribution

**Usage**

```
rPiecewisePareto(
  n,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp",
  scale_pieces = NULL
)
```

**Arguments**

n	Numeric. Number of simulations
t	Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha	Numeric vector. $\alpha[i]$ is the Pareto alpha in excess of $t[i]$ .
truncation	Numeric. If truncation is not NULL and $\text{truncation} > t$ , then the distribution is truncated at truncation.
truncation_type	Character. If $\text{truncation\_type} = \text{"wd"}$ then the whole distribution is truncated. If $\text{truncation\_type} = \text{"lp"}$ then a truncated Pareto is used for the last piece.
scale_pieces	Numeric vector. If not NULL then the density of the $i$ -th Pareto piece (on the interval $(t[i], t[i+1])$ ) is scaled with the factor $\text{const} * \text{scale\_pieces}[i]$ (where $\text{const}$ is a normalization constant)

**Value**

A vector of  $n$  samples from the (truncated) piecewise Pareto distribution with parameter vectors  $t$  and  $\alpha$

**Examples**

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
rPiecewisePareto(100, t, alpha)
rPiecewisePareto(100, t, alpha, truncation = 5000)
rPiecewisePareto(100, t, alpha, truncation = 5000, truncation_type = "lp")
rPiecewisePareto(100, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

 Simulate\_Losses

*Simulate Losses with a Collective Model*


---

**Description**

Simulates losses with a collective model

**Usage**

```
Simulate_Losses(CollectiveModel, nyears = 1)
```

**Arguments**

CollectiveModel	A collective model object. Currently only PPP_Models are handled.
nyears	Integer. Number of simulated years.

**Value**

A matrix where row k contains the simulated losses of the kth simulated year.

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(c(1000, 2000, 3000), c(2000, 1000, 500),
                                           truncation = 10000, truncation_type = "wd")
PPPM
Simulate_Losses(PPPM, 100)
```

---

Simulate\_Losses.PGP\_Model

*Simulate Losses with a PGP\_Model*

---

**Description**

Simulates losses with a PGP\_Model

**Usage**

```
## S3 method for class 'PGP_Model'
Simulate_Losses(CollectiveModel, nyears = 1)
```

**Arguments**

CollectiveModel  
PGP\_Model object.

nyears            Integer. Number of simulated years.

**Value**

A matrix where row k contains the simulated losses of the kth simulated year.

**Examples**

```
PGPM <- PGP_Model(2, 1000, 1, 2, dispersion = 2)
PGPM
Simulate_Losses(PGPM, 100)
```

---

`Simulate_Losses.PPP_Model`*Simulate Losses with a PPP\_Model*

---

**Description**

Simulates losses with a PPP\_Model

**Usage**

```
## S3 method for class 'PPP_Model'  
Simulate_Losses(CollectiveModel, nyears = 1)
```

**Arguments**

`CollectiveModel`  
PPP\_Model object.  
`nyears` Integer. Number of simulated years.

**Value**

A matrix where row *k* contains the simulated losses of the *k*th simulated year.

**Examples**

```
PPPM <- PiecewisePareto_Match_Layer_Losses(c(1000, 2000, 3000), c(2000, 1000, 500),  
                                           truncation = 10000, truncation_type = "wd")  
PPPM  
Simulate_Losses(PPPM, 100)
```

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