

# Package ‘RCBR’

May 7, 2026

**Title** Random Coefficient Binary Response Estimation

**Description** Nonparametric maximum likelihood estimation methods for random coefficient binary response models and some related functionality for sequential processing of hyperplane arrangements. See J. Gu and R. Koenker (2020) <[DOI:10.1080/01621459.2020.1802284](https://doi.org/10.1080/01621459.2020.1802284)>.

**Version** 0.6.2

**Maintainer** Roger Koenker <[rkoenker@uiuc.edu](mailto:rkoenker@uiuc.edu)>

**Depends** R (>= 2.10), Matrix

**Suggests** knitr, digest

**Imports** methods, Rmosek, REBayes, orthopolynom, Formula, mvtnorm

**LazyData** TRUE

**SystemRequirements** MOSEK (<http://www.mosek.com>) and MOSEK License for use of Rmosek,

**License** GPL (>= 2)

**URL** <https://www.r-project.org>

**NeedsCompilation** no

**Encoding** UTF-8

**Author** Roger Koenker [aut, cre],  
Jiaying Gu [aut]

**RoxygenNote** 7.2.3

**Repository** CRAN

**Date/Publication** 2023-11-08 09:40:02 UTC

## Contents

bounds.KW2 . . . . .	2
GH . . . . .	3
GH.se . . . . .	4
GK.control . . . . .	4
Horowitz93 . . . . .	5

KW.control . . . . .	6
KWDual . . . . .	7
logLik.GK . . . . .	8
logLik.KW1 . . . . .	9
neighbours . . . . .	9
NICER . . . . .	10
NICERd . . . . .	11
plot.GK . . . . .	13
plot.KW2 . . . . .	13
polycount . . . . .	14
polyzone . . . . .	14
rcbr . . . . .	15
predict.GK . . . . .	16
predict.KW2 . . . . .	17
rcbr . . . . .	17
rcbr.fit . . . . .	19
rcbr.fit.GK . . . . .	21
rcbr.fit.KW1 . . . . .	22
rcbr.fit.KW2 . . . . .	23
witness . . . . .	24

## Index 25

---

bounds.KW2 *Prediction of Bounds on Marginal Effects*

---

### Description

Given a fitted model by the exact NPML procedure prediction is made at a new design point with lower and upper bounds for the prediction due to ambiguity of the assignment of mass within the cell enumerated polygons.

### Usage

```
bounds.KW2(object, ...)
```

### Arguments

object	is the fitted NPML object
...	is expected to contain an argument newdata

### Value

a list consisting of the following components:

- phat** Point prediction
- lower** lower bound prediction
- upper** upper bound prediction
- xpoly** indices of crossed polygons

**Author(s)**

Jiaying Gu

**See Also**

predict.KW2 for a simpler prediction function without bounds

---

GH*Current Status Linear Regression*

---

**Description**

Groeneboom and Hendrickx semiparametric binary response estimator (scalar case) score estimator based on NPMLE avoids any smoothing proposed by Groeneboom and Hendrickx (2018).

**Usage**

GH(b, X, y, eps = 0.001)

**Arguments**

b	parameter vector (fix last entry as a known number, usually 1 or -1, for normalization)
X	design matrix
y	binary response vector
eps	trimming tolerance parameter

**Value**

A list with components:

- evaluation of a score function at parameter value
- estimated standard error
- single index linear predictor

**References**

Groeneboom, P. and K. Hendrickx (2018) Current Status Linear Regression, *Annals of Statistics*, 46, 1415-1444,

---

 GH.se

*Current Status Linear Regression Standard Errors*


---

### Description

Groeneboom and Hendrickx semiparametric binary response estimator (scalar case) score estimator based on NPMLE avoids any smoothing proposed by Groneboom and Hendrickx (2018).

### Usage

```
GH.se(bstar, X, y, eps = 0.001, hc = 2)
```

### Arguments

bstar	parameter vector (fix last entry as a known number, usually 1 or -1, for normalization)
X	design matrix
y	binary response vector
eps	trimming tolerance parameter
hc	kernel bandwidth (used for the standard error estimation)

### Value

A list with components:

- evaluation of a score function at parameter value
- estimated standard error
- sindex single index linear predictor

### References

Groeneboom, P. and K. Hendrickx (2018) Current Status Linear Regression, *Annals of Statistics*, 46, 1415-1444,

---

 GK.control

*Control parameters for Gautier-Kitamura bivariate random coefficient binary response*


---

### Description

These parameters can be passed via the ... argument of the rcbf function. defaults as suggested in Gautier and Kitamura matlab code

**Usage**

```
GK.control(n, u = -20:20/10, v = -20:20/10, T = 3, TX = 10, Mn = 1/log(n)^2)
```

**Arguments**

n	the sample size
u	grid values for intercept coordinate
v	grid values for slope coordinate
T	Truncation parameter for numerator must grow "sufficiently slowly with n"
TX	Truncation parameter for denominator must grow "sufficiently slowly with n"
Mn	Trimming parameter "chosen to go to 0 slowly with n"

**Value**

updated list

---

Horowitz93

*Horowitz (1993) Modal Choice Data*

---

**Description**

Modal choice data for journey to work in the Washington DC area from the late 1960's. The variables are: \* 'DCOST': difference in cost of car versus transit (transit - car) \* 'CARS': number of cars at home \* 'DOVTT': difference in out of vehicle time (transit - car) \* 'DIVTT': difference in in vehicle time (transit - car) \* 'DEPEND': coded 1 if by car, 0 if by mass transit

**Usage**

```
Horowitz93
```

**Format**

A data frame with 842 observations on 5 variables:

**Source**

[https://www.gams.com/latest/gamslib\\_ml/libhtml/gamslib\\_mws.html](https://www.gams.com/latest/gamslib_ml/libhtml/gamslib_mws.html)

**References**

Horowitz, J L, (1993) Semiparametric estimation of a work-trip mode choice model. *Journal of Econometrics*, 58, 49-70.

---

KW.control	<i>Control parameters for NPMLE of bivariate random coefficient binary response</i>
------------	---

---

### Description

These parameters can be passed via the . . . argument of the rcbf function. The first three arguments are only relevant if full cell enumeration is employed for bivariate version of the NPMLE.

### Usage

```
KW.control(
  uv = NULL,
  u = NULL,
  v = NULL,
  initial = c(0, 0),
  epsbound = 1,
  epstol = 1e-07,
  presolve = 1,
  verb = 0
)
```

### Arguments

uv	matrix of evaluation points for potential mass points
u	grid of evaluation points for potential mass points
v	grid of evaluation points for potential mass points
initial	initial point for cell enumeration algorithm
epsbound	controls how close witness points can be to vertices of a cell
epstol	zero tolerance for witness solutions
presolve	controls whether Mosek does a presolve of the LP
verb	controls verbosity of Mosek solver 0 implies it is quiet

### Value

updated list

## Description

Interface function for calls to optimizer from various REBayes functions. There is currently only one option for the optimization that based on Mosek. It relies on the **Rmosek** interface to R see installation instructions in the Readme file in the inst directory of this package. This version of the function is intended to work with versions of Mosek after 7.0. A more experimental option employing the **pogs** package available from <https://github.com/foges/pogs> and employing an ADMM (Alternating Direction Method of Multipliers) approach has been deprecated, those interested could try installing version 1.4 of REBayes, and following the instructions provided there.

## Usage

```
KWDual(A, d, w, ...)
```

## Arguments

A	Linear constraint matrix
d	constraint vector
w	weights for x should sum to one.
...	other parameters passed to control optimization: These may include <code>rtol</code> the relative tolerance for dual gap convergence criterion, <code>verb</code> to control verbosity desired from mosek, <code>verb = 0</code> is quiet, <code>verb = 5</code> produces a fairly detailed iteration log, <code>control</code> is a control list consisting of sublists <code>iparam</code> , <code>dparam</code> , and <code>sparam</code> , containing elements of various mosek control parameters. See the <code>Rmosek</code> and <code>Mosek</code> manuals for further details. A prime example is <code>rtol</code> which should eventually be deprecated and folded into <code>control</code> , but will persist for a while for compatibility reasons. The default for <code>rtol</code> is <code>1e-6</code> , but in some cases it is desirable to tighten this, say to <code>1e-10</code> . Another example that motivated the introduction of <code>control</code> would be <code>control = list(iparam = list(num_threads = 1))</code> , which forces Mosek to use a single threaded process. The default allows Mosek to use multiple threads (cores) if available, which is generally desirable, but may have unintended (undesirable) consequences when running simulations on clusters.

## Value

Returns a list with components:

f	dual solution vector, the mixing density
g	primal solution vector, the mixture density evaluated at the data points
logLik	log likelihood
status	return status from Mosek

. Mosek termination messages are treated as warnings from an R perspective since solutions producing, for example, MSK\_RES\_TRM\_STALL: The optimizer is terminated due to slow progress, may still provide a satisfactory solution, especially when the return status variable is "optimal".

### Author(s)

R. Koenker

### References

Koenker, R and I. Mizera, (2013) "Convex Optimization, Shape Constraints, Compound Decisions, and Empirical Bayes Rules," *JASA*, 109, 674–685.

Mosek Aps (2015) Users Guide to the R-to-Mosek Optimization Interface, <https://docs.mosek.com/8.1/rmosek/index.html>.

Koenker, R. and J. Gu, (2017) REBayes: An R Package for Empirical Bayes Mixture Methods, *Journal of Statistical Software*, 82, 1–26.

---

logLik.GK

*log likelihood for Gautier Kitamura procedure*

---

### Description

log likelihood for Gautier Kitamura procedure

### Usage

```
## S3 method for class 'GK'
logLik(object, ...)
```

### Arguments

object	a fitted object of class "GK"
...	other parameters for logLik

### Value

a scalar log likelihood

---

logLik.KW1	<i>log likelihood for KW1 procedure</i>
------------	---

---

**Description**

log likelihood for KW1 procedure

**Usage**

```
## S3 method for class 'KW1'
logLik(object, ...)
```

**Arguments**

object	a fitted object of class "KW1"
...	other parameters for logLik

**Value**

a scalar log likelihood

---

neighbours	<i>Check Neighbouring Cell Counts</i>
------------	---------------------------------------

---

**Description**

Compare cell counts for each cell with its neighbours and return indices of the locally maximal cells.

**Usage**

```
neighbours(SignVector)
```

**Arguments**

SignVector	n by m matrix of signs produced by NICER
------------	--

**Value**

Column indices of the cells that are locally maximal, i.e. those whose neighbours have strictly fewer cell counts. The corresponding interior points of these cells can be used as potential mass points for the NPMLE function `rcbr.fit.KW`.

---

 NICER

*New Incremental Cell Enumeration (in) R*


---

### Description

Find interior points and cell counts of the polygons (cells) formed by a line arrangement.

### Usage

```
NICER(A, b, initial = c(0, 0), verb = TRUE, epsbound = 1, epstol = 1e-07)
```

### Arguments

A	is a n by 2 matrix of slope coefficients
b	is an n vector of intercept coefficients
initial	origin for the interior point vectors w
verb	controls verbosity of Mosek solution
epsbound	is a scalar tolerance controlling how close the witness point can be to an edge of the polytope
epstol	is a scalar tolerance for the LP convergence

### Details

Modified version of the algorithm of Rada and Cerny (2018). The main modifications include preprocessing as hyperplanes are added to determine which new cells are created, thereby reducing the number of calls to the witness function to solve LPs, and treatment of degenerate configurations as well as those in "general position." When the hyperplanes are in general position the number of polytopes (cells) is determined by the elegant formula of Zaslavsky (1975)

$$m = \binom{n}{d} + n + 1$$

. In degenerate cases, i.e. when hyperplanes are not in general position, the number of cells is more complicated as considered by Alexanderson and Wetzel (1981). The function `polycount` is provided to check agreement with their results in an effort to aid in the selection of tolerances for the witness function. Current version is intended for use with  $d = 2$ , but the algorithm is adaptable to  $d > 2$ , and there is an experimental version called `NICERd` in the package.

### Value

A list with components:

- `SignVector` a n by m matrix of signs determining position of cell relative to each hyperplane.
- `w` a d by m matrix of interior points for the m cells

## References

Alexanderson, G.L and J.E. Wetzel, (1981) Arrangements of planes in space, *Discrete Math*, 34, 219–240. Gu, J. and R. Koenker (2020) Nonparametric Maximum Likelihood Methods for Binary Response Models with Random Coefficients, *J. Am. Stat Assoc* Rada, M. and M. Cerny (2018) A new algorithm for the enumeration of cells of hyperplane arrangements and a comparison with Avis and Fukada’s reverse search, *SIAM J. of Discrete Math*, 32, 455-473. Zaslavsky, T. (1975) Facing up to arrangements: Face-Count Formulas for Partitions of Space by Hyperplanes, *Memoirs of the AMS*, Number 154.

## Examples

```
{
  if(packageVersion("Rmosek") > "8.0.0"){
    A = cbind(c(1,-1,1,-2,2,1,3), c(1,1,1,1,1,-1,-2))
    B = matrix(c(3,1,7,-2,7,-1,1), ncol = 1)
    plot(NULL,xlim = c(-10,10),ylim = c(-10,10))
    for (i in 1:nrow(A))
      abline(a = B[i,1]/A[i,2], b = -A[i,1]/A[i,2],col = i)
    f = NICER(A, B)
    for (j in 1:ncol(f$SignVector))
      points(f$w[1,j], f$w[2,j], cex = 0.5)
  }
}
```

---

 NICERd

*New (Accelerated) Incremental Cell Enumeration (in) R*


---

## Description

Find interior points and cell counts of the polygons (polytopes) formed by a hyperplane arrangement.

## Usage

```
NICERd(
  A,
  b,
  initial = rep(0, ncol(A)),
  verb = TRUE,
  accelerate = FALSE,
  epsbound = 1,
  epstol = 1e-07
)
```

**Arguments**

A	is a n by d matrix of hyperplane slope coefficients
b	is an n vector of hyperplane intercept coefficients
initial	origin for the interior point vectors w
verb	controls verbosity of Mosek solution
accelerate	allows the option to turn off acceleration step (turned off by default)
epsbound	is a scalar tolerance controlling how close the witness point can be to an edge of the polytope
epstol	is a scalar tolerance for the LP convergence

**Details**

Modified version of the algorithm of Rada and Cerny (2018). The main modifications include preprocessing as hyperplanes are added to determine which new cells are created, thereby reducing the number of calls to the witness function to solve LPs, and treatment of degenerate configurations as well as those in "general position." (for  $d = 2$  for now). When the hyperplanes are in general position the number of cells (polytopes) is determined by the elegant formula of Zaslavsky (1975)

$$m = \binom{n}{d} + n + 1$$

. In degenerate cases, i.e. when hyperplanes are not in general position, the number of cells is more complicated as considered by Alexanderson and Wetzel (1981). The function `polycount` is provided to check agreement with their results in an effort to aid in the selection of tolerances for the witness function for arrangement in  $d = 2$ . The current version is intended mainly for use with  $d = 2$ , but the algorithm is adapted to the general position setting with  $d > 2$ , although it requires hyperplanes in general position and may require some patience when both the sample size is large. if hyperplanes not general position (i.e. all cross at origin), turn off `accelerate`

**Value**

A list with components:

- `SignVector` a n by m matrix of signs determining position of cell relative to each hyperplane.
- `w` a d by m matrix of interior points for the m cells

**References**

Alexanderson, G.L and J.E. Wetzel, (1981) Arrangements of planes in space, *Discrete Math*, 34, 219–240. Rada, M. and M. Cerny (2018) A new algorithm for the enumeration of cells of hyperplane arrangements and a comparison with Avis and Fukada’s reverse search, *SIAM J. of Discrete Math*, 32, 455-473. Zaslavsky, T. (1975) Facing up to arrangements: Face-Count Formulas for Partitions of Space by Hyperplanes, *Memoirs of the AMS*, Number 154.

---

plot.GK	<i>Plot a GK object</i>
---------	-------------------------

---

**Description**

Given a fitted model by the Guatier-Kitamura procedure plot the estimated density contours

**Usage**

```
## S3 method for class 'GK'
plot(x, ...)
```

**Arguments**

x	is the fitted GK object
...	other arguments to pass to contour, notably e.g. add = TRUE

**Value**

nothing (invisibly)

---

plot.KW2	<i>Plot a KW2 object</i>
----------	--------------------------

---

**Description**

Given a fitted model by the rchr NPMLE procedure plot the estimated mass points

**Usage**

```
## S3 method for class 'KW2'
plot(x, smooth = 0, pal = NULL, inches = 1/6, N = 25, tol = 0.001, ...)
```

**Arguments**

x	is the fitted NPMLE object
smooth	is a parameter to control bandwidth of the smoothing if a contour plot of the estimated density is desired, default is no smoothing and only the mass points of the discrete estimate are plotted.
pal	a color palette
inches	as used in symbols to control size of mass points
N	scaling of the color palette
tol	tolerance for size of mass points
...	other arguments to pass to symbols, notably e.g. add = TRUE

**Value**

nothing (invisibly)

---

polycount

*Check Cell Count for degenerate hyperplane arrangements*

---

**Description**

When the hyperplane arrangement is degenerate, i.e. not in general position, the number of distinct cells can be checked against the formula of Alexanderson and Wetzel (1981).

**Usage**

polycount(A, b, maxints = 10)

**Arguments**

A is a n by m matrix of hyperplane slope coefficients  
 b is an n vector of hyperplane intercept coefficients  
 maxints is maximum number of lines allowed to cross at the same vertex

**Value**

number of distinct cells

**References**

Alexanderson, G.L and J.E. Wetzel, (1981) Arrangements of planes in space, Discrete Math, 34, 219–240.

---

polyzone

*Identify crossed polygons from existing cells when adding a new line  
 (works only for dim = 2)*

---

**Description**

Given an existing cell configuration represented by the Signvector and associated interior points w, identify the polygons crossed by the next new line.

**Usage**

polyzone(SignVector, w, A, b)

**Arguments**

SignVector	current SignVctor matrix
w	associated interior points
A	design matrix for full problem aka [1,z]
b	associated final column of design matrix aka [v]

**Value**

vector of indices of crossed polygons

**Author(s)**

Jiaying Gu

---

prcbr

*Profiling estimation methods for RCBR models*


---

**Description**

Profile likelihood and (GEE) score methods for estimation of random coefficient binary response models. This function is a wrapper for rcbbr that uses the offset argument to implement estimation of additional fixed parameters. It may be useful to restrict the domain of the optimization over the profiled parameters, this can be accomplished, at least for box constraints by setting omethod = "L-BFGS-B" and specifying the lo and up accordingly.

**Usage**

```
prcbr(
  formula,
  b0,
  data,
  logL = TRUE,
  omethod = "BFGS",
  lo = -Inf,
  up = Inf,
  ...
)
```

**Arguments**

formula is of the extended form enabled by the **Formula** package. In the Cosslett, or current status, model the formula takes the form  $y \sim v \mid z$  where  $v$  is the covariate designated to have coefficient one, and  $z$  is another covariate or group of covariates that are assumed fixed coefficients that are to be estimated.

b0	is either an initial value of the parameter for the Z covariates or a matrix of such values, in which case optimization occurs over this discrete set, when there is only one covariate then b0 is either scalar, or a vector.
data	data frame for formula variables
logL	if logL is TRUE the log likelihood is optimized, otherwise a GEE score criterion is minimized.
omethod	optimization method for optim, default "BFGS".
lo	lower bound(s) for the parameter domain
up	upper bound(s) for the parameter domain
...	other arguments to be passed to rcbf.fit to control fitting.

**Value**

a list comprising the components:

**bopt** output of the optimizer for the profiled parameters beta

**fopt** output of the optimizer for the random coefficients eta

---

predict.GK

*Prediction of Marginal Effects*

---

**Description**

Given a fitted model by the Gautier Kitamura procedure predictions are made at new design points given by the newdata argument.

**Usage**

```
## S3 method for class 'GK'
predict(object, ...)
```

**Arguments**

object	is the fitted object of class "GK"
...	is expected to contain an argument newdata

**Value**

a vector pf predicted probabilities

---

predict.KW2	<i>Prediction of Marginal Effects</i>
-------------	---------------------------------------

---

**Description**

Given a fitted model by the rcbr NPMLE procedure predictions are made at new design points given by the newdata argument.

**Usage**

```
## S3 method for class 'KW2'  
predict(object, ...)
```

**Arguments**

object	is the fitted NPMLE object
...	is expected to contain an argument newdata

**Value**

a vector of predicted probabilities

**See Also**

bound.KW2 for a prediction function with bounds

---

rcbr	<i>Estimation of Random Coefficient Binary Response Models</i>
------	--

---

**Description**

Two methods are implemented for estimating binary response models with random coefficients: A nonparametric maximum likelihood method proposed by Cosslett (1986) and extended by Ichimura and Thompson (1998), and a (hemispherical) deconvolution method proposed by Gautier and Kitamura (2013). The former is closely related to the NPMLE for mixture models of Kiefer and Wolfowitz (1956). The latter is an R translation of the matlab implementation of Gautier and Kitamura.

**Usage**

```
rcbr(formula, data, subset, offset, mode = "GK", ...)
```

## Arguments

formula	an expression of the generic form $y \sim z + v$ where $y$ is the observed binary response, $z$ is an observed covariate with a random coefficient, and $v$ is an observed covariate with coefficient normalize to be one. If $z$ is not present then the model has only a random "intercept" coefficient and thus corresponds to the basic model of Cosslett (1983); this model is also referred to as the current status model in the biostatistics literature, see Groeneboom and Hendrikx (2016). When $z$ is present there are random coefficients associated with both the intercept and $z$ .
data	is a <code>data.frame</code> containing the data referenced in the formula.
subset	specifies a subsample of the data used for fitting the model
offset	specifies a fixed shift in $v$ representing the potential effect of other covariates having fixed coefficients that may be useful for profile likelihood computations. (Should be vector of the same length as $v$ .)
mode	controls whether the Gautier and Kitamura, "GK", or Kiefer and Wolfowitz, "KW" methods are used.
...	miscellaneous other arguments to control fitting. See <code>GK.control</code> and <code>KW.control</code> for further details.

## Details

The `predict` method produces estimates of the probability of a "success" ( $y = 1$ ) for a particular vector,  $(z, v)$ , when aggregated over the estimated distribution of random coefficients.

The `logLik` produces an evaluation of the log likelihood value associated with a fitted model.

## Value

of object of class `GK`, `KW1`, with components described in further detail in the respective fitting functions.

## Author(s)

Jiaying Gu and Roger Koenker

## References

- Kiefer, J. and J. Wolfowitz (1956) Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters, *Ann. Math. Statist.*, 27, 887-906.
- Cosslett, S. (1983) Distribution Free Maximum Likelihood Estimator of the Binary Choice Model, *Econometrica*, 51, 765-782.
- Gautier, E. and Y. Kitamura (2013) Nonparametric estimation in random coefficients binary choice models, *Econometrica*, 81, 581-607.
- Gu, J. and R. Koenker (2020) Nonparametric Maximum Likelihood Methods for Binary Response Models with Random Coefficients, *J. Am. Stat Assoc.*
- Groeneboom, P. and K. Hendrickx (2016) Current Status Linear Regression, preprint available from <https://arxiv.org/abs/1601.00202>.

Ichimura, H. and T. S. Thompson, (1998) Maximum likelihood estimation of a binary choice model with random coefficients of unknown distribution," *Journal of Econometrics*, 86, 269-295.

### Examples

```
{
if(packageVersion("Rmosek") > "8.0.0"){
  # Simple Test Problem for rcbr
  n <- 60
  B0 = rbind(c(0.7,-0.7,1),c(-0.7,0.7,1))
  z <- rnorm(n)
  v <- rnorm(n)
  s <- sample(0:1, n, replace = TRUE)
  XB0 <- cbind(1,z,v) %*% t(B0)
  u <- s * XB0[,1] + (1-s) * XB0[,2]
  y <- (u > 0) - 0
  D <- data.frame(z = z, v = v, y = y)
  f <- rcbr(y ~ z + v, mode = "KW", data = D)
  plot(f)
  # Simple Test Problem for rcbr
  set.seed(15)
  n <- 100
  B0 = rbind(c(0.7,-0.7,1),c(-0.7,0.7,1))
  z <- rnorm(n)
  v <- rnorm(n)
  s <- sample(0:1, n, replace = TRUE)
  XB0 <- cbind(1,z,v) %*% t(B0)
  u <- s * XB0[,1] + (1-s) * XB0[,2]
  y <- (u > 0) - 0
  D <- data.frame(z = z, v = v, y = y)
  f <- rcbr(y ~ z + v, mode = "GK", data = D)
  contour(f$u, f$v, matrix(f$w, length(f$u)))
  points(x = 0.7, y = -0.7, col = 2)
  points(x = -0.7, y = 0.7, col = 2)
  f <- rcbr(y ~ z + v, mode = "GK", data = D, T = 7)
  contour(f$u, f$v, matrix(f$w, length(f$u)))
  points(x = 0.7, y = -0.7, col = 2)
  points(x = -0.7, y = 0.7, col = 2)
}
}
```

### Description

Two methods are implemented for estimating binary response models with random coefficients: A nonparametric maximum likelihood method proposed by Cosslett (1986) and extended by Ichimura and Thompson (1998), and a (hemispherical) deconvolution method proposed by Gautier and Kitamura (2013). The former is closely related to the NPMLE for mixture models of Kiefer and

Wolfowitz (1956). The latter is an R translation of the matlab implementation of Gautier and Kitamura.

### Usage

```
rcbr.fit(x, y, offset = NULL, mode = "KW", control)
```

### Arguments

x	design matrix
y	binary response vector
offset	specifies a fixed shift in $v$ representing the potential effect of other covariates having fixed coefficients that may be useful for profile likelihood computations. (Should be vector of the same length as $v$ .)
mode	controls whether the Gautier and Kitamura, "GK", or Kiefer and Wolfowitz, "KW" methods are used.
control	control parameters for fitting methods See GK.control and KW.control for further details.

### Details

The predict method produces estimates of the probability of a "success" ( $y = 1$ ) for a particular vector,  $(z, v)$ , when aggregated over the estimated distribution of random coefficients.

### Value

of object of class GK, KW1, with components described in further detail in the respective fitting functions.

### Author(s)

Jiaying Gu and Roger Koenker

### References

- Kiefer, J. and J. Wolfowitz (1956) Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters, *Ann. Math. Statist*, 27, 887-906.
- Cosslett, S. (1983) Distribution Free Maximum Likelihood Estimator of the Binary Choice Model, *Econometrica*, 51, 765-782. Gautier, E. and Y. Kitamura (2013) Nonparametric estimation in random coefficients binary choice models, *Econometrica*, 81, 581-607.
- Groeneboom, P. and K. Hendrickx (2016) Current Status Linear Regression, preprint available from <https://arxiv.org/abs/1601.00202>.
- Ichimura, H. and T. S. Thompson, (1998) Maximum likelihood estimation of a binary choice model with random coefficients of unknown distribution," *Journal of Econometrics*, 86, 269-295.

---

rcbr.fit.GK	<i>Gautier and Kitamura (2013) bivariate random coefficient binary response</i>
-------------	---

---

### Description

This is an implementation based on the matlab version of Gautier and Kitamura's deconvolution method for the bivariate random coefficient binary response model. Methods based on the fitted object are provided for `predict`, `logLik` and `plot`.requires orthopolynom package for Gegenbauer polynomials

### Usage

```
rcbr.fit.GK(X, y, control)
```

### Arguments

<code>X</code>	the design matrix expected to have an intercept column of ones as the first column.
<code>y</code>	the binary response.
<code>control</code>	is a list of tuning parameters for the fitting,see <code>GK.control</code> for further details.

### Value

a list with components:

- `u` grid values
- `v` grid values
- `w` estimated function values on 2d  $u \times v$  grid
- `X` design matrix
- `y` response vector

### Author(s)

Gautier and Kitamura for original matlab version, Jiaying Gu and Roger Koenker for the R translation.

### References

Gautier, E. and Y. Kitamura (2013) Nonparametric estimation in random coefficients binary choice models, *Econometrica*, 81, 581-607.

---

rcbr.fit.KW1	<i>NPML</i> E fitting for the Cosslett random coefficient binary response model
--------------	---

---

### Description

This is the original one dimensional version of the Cosslett model, also known as the current status model:

$$P(y = 1|v) = \int I(\eta > v) dF(\eta).$$

invoked with the formula  $y \sim v$ . By default the algorithm computes a vector of potential locations for the mass points of  $\hat{F}$  by finding interior points of the intervals between the ordered  $v$ , and then solving a convex optimization problem to determine these masses. Alternatively, a vector of predetermined locations can be passed via the control argument. Additional covariate effects can be accommodated by either specifying a fixed offset in the call to rcbr or by using the profile likelihood function prcbr.

### Usage

```
rcbr.fit.KW1(X, y, control)
```

### Arguments

X	the design matrix expected to have an intercept column of ones as the first column, the last column is presumed to contain values of the covariate that is designated to have coefficient one.
y	the binary response.
control	is a list of parameters for the fitting, see KW.control for further details.

### Value

a list with components:

- x evaluation points for the fitted distribution
- y estimated mass associated with the  $v$  points
- logLik the loglikelihood value of the fit
- status mosek solution status

### Author(s)

Jiaying Gu and Roger Koenker

### References

Gu, J. and R. Koenker (2018) Nonparametric maximum likelihood estimation of the random coefficients binary choice model, preprint.

rcbr.fit.KW2

*NPMLE fitting for random coefficient binary response model***Description**

Exact NPMLE fitting requires that the `uv` argument contain a matrix whose rows represent points in the interior of the locally maximal polytopes determined by the hyperplane arrangement of the observations. If it is not provided it will be computed afresh here; since this can be somewhat time consuming, `uv` is included in the returned object so that it can be reused if desired. Approximate NPMLE fitting can be achieved by specifying an equally spaced grid of points at which the NPMLE can assign mass using the arguments `u` and `v`. If the design matrix  $X$  contains only 2 columns, so we have the Cosslett, aka current status, model then the polygons in the prior description collapse to intervals and the default method computes the locally maximal count intervals and passes their interior points to the optimizer of the log likelihood. Alternatively, as in the bivariate case one can specify a grid to obtain an approximate solution.

**Usage**

```
rcbr.fit.KW2(x, y, control)
```

**Arguments**

<code>x</code>	the design matrix expected to have an intercept column of ones as the first column, the last column is presumed to contain values of the covariate that is designated to have coefficient one.
<code>y</code>	the binary response.
<code>control</code>	is a list of parameters for the fitting, see <code>KW.control</code> for further details.

**Value**

a list with components:

- `uv` evaluation points for the fitted distribution
- `W` estimated mass associated with the `uv` points
- `logLik` the loglikelihood value of the fit
- `status` mosek solution status

**Author(s)**

Jiaying Gu and Roger Koenker

**References**

Gu, J. and R. Koenker (2018) Nonparametric maximum likelihood estimation of the random coefficients binary choice model, preprint.

---

witness	<i>Find witness point</i>
---------	---------------------------

---

### Description

Find (if possible) an interior point of a polytope solving a linear program

### Usage

```
witness(A, b, s, epsbound = 1, epstol = 1e-07, presolve = 1, verb = 0)
```

### Arguments

A	Is a n by d matrix of hyperplane slope coefficients.
b	Is an n vector of hyperplane intercept coefficients.
s	Is an n vector of signs.
epsbound	Is a scalar tolerance controlling how close the witness point can be to an edge of the polytope.
epstol	Is a scalar tolerance for the LP convergence.
presolve	Controls whether Mosek should presolve the LP.
verb	Controls verbosity of Mosek solution.

### Details

Solves LP:  $\max_{w, \text{eps}} |SAw - \text{eps}| \geq Sb, 0 < \text{eps} \leq \text{epsbound}$  S is diag(s), if at the solution  $\text{eps} > 0$ , then w is a valid interior point otherwise the LP fails to find an interior point, another s must be tried. Constructs a problem formulation that can be passed to Rmosek for solution.

### Value

List with components:

- w proposed interior point at solution
- fail indicator of whether w is a valid interior point

# Index

## \* datasets

Horowitz93, 5

## \* nonparametrics

KWDual, 7

rcbr.fit.GK, 21

rcbr.fit.KW1, 22

rcbr.fit.KW2, 23

## \* nonparametric

rcbr, 17

rcbr.fit, 19

bounds.KW2, 2

GH, 3

GH.se, 4

GK.control, 4

Horowitz93, 5

KW.control, 6

KWDual, 7

logLik.GK, 8

logLik.KW1, 9

neighbours, 9

NICER, 10

NICERd, 11

plot.GK, 13

plot.KW2, 13

polycount, 14

polyzone, 14

prcbr, 15

predict.GK, 16

predict.KW2, 17

rcbr, 17

rcbr.fit, 19

rcbr.fit.GK, 21

rcbr.fit.KW1, 22

rcbr.fit.KW2, 23

witness, 24