

# Package ‘UniversalCVI’

May 7, 2026

**Type** Package

**Title** Hard and Soft Cluster Validity Indices

**Version** 1.4.0

**Imports** e1071, mclust

**Description** Algorithms for checking the accuracy of a clustering result with known classes, computing cluster validity indices, and generating plots for comparing them.

The package is compatible with K-means, fuzzy C means, EM clustering, and hierarchical clustering (single, average, and complete linkage).

The details of the indices in this package can be found in:

J. C. Bezdek, M. Moshtaghi, T. Runkler, C. Leckie (2016) <[doi:10.1109/TFUZZ.2016.2540063](https://doi.org/10.1109/TFUZZ.2016.2540063)>,

T. Calinski, J. Harabasz (1974) <[doi:10.1080/03610927408827101](https://doi.org/10.1080/03610927408827101)>,

C. H. Chou, M. C. Su, E. Lai (2004) <[doi:10.1007/s10044-004-0218-1](https://doi.org/10.1007/s10044-004-0218-1)>,

D. L. Davies, D. W. Bouldin (1979) <[doi:10.1109/TPAMI.1979.4766909](https://doi.org/10.1109/TPAMI.1979.4766909)>,

J. C. Dunn (1973) <[doi:10.1080/01969727308546046](https://doi.org/10.1080/01969727308546046)>,

F. Haouas, Z. Ben Dhiab, A. Hammouda, B. Solaiman (2017) <[doi:10.1109/FUZZ-IEEE.2017.8015651](https://doi.org/10.1109/FUZZ-IEEE.2017.8015651)>,

M. Kim, R. S. Ramakrishna (2005) <[doi:10.1016/j.patrec.2005.04.007](https://doi.org/10.1016/j.patrec.2005.04.007)>,

S. H. Kwon (1998) <[doi:10.1049/EL:19981523](https://doi.org/10.1049/EL:19981523)>,

S. H. Kwon, J. Kim, S. H. Son (2021) <[doi:10.1049/ell2.12249](https://doi.org/10.1049/ell2.12249)>,

G. W. Miligan (1980) <[doi:10.1007/BF02293907](https://doi.org/10.1007/BF02293907)>,

M. K. Pakhira, S. Bandyopadhyay, U. Maulik (2004) <[doi:10.1016/j.patcog.2003.06.005](https://doi.org/10.1016/j.patcog.2003.06.005)>,

M. Popescu, J. C. Bezdek, T. C. Havens, J. M. Keller (2013) <[doi:10.1109/TSMCB.2012.2205679](https://doi.org/10.1109/TSMCB.2012.2205679)>,

S. Saitta, B. Raphael, I. Smith (2007) <[doi:10.1007/978-3-540-73499-4\\_14](https://doi.org/10.1007/978-3-540-73499-4_14)>,

A. Starczewski (2017) <[doi:10.1007/s10044-015-0525-8](https://doi.org/10.1007/s10044-015-0525-8)>,

Y. Tang, F. Sun, Z. Sun (2005) <[doi:10.1109/ACC.2005.1470111](https://doi.org/10.1109/ACC.2005.1470111)>,

N. Wiroonsri (2024) <[doi:10.1016/j.patcog.2023.109910](https://doi.org/10.1016/j.patcog.2023.109910)>,

N. Wiroonsri, O. Preedasawakul (2023) <[doi:10.48550/arXiv.2308.14785](https://doi.org/10.48550/arXiv.2308.14785)>,

C. H. Wu, C. S. Ouyang, L. W. Chen, L. W. Lu (2015) <[doi:10.1109/TFUZZ.2014.2322495](https://doi.org/10.1109/TFUZZ.2014.2322495)>,

X. Xie, G. Beni (1991) <[doi:10.1109/34.85677](https://doi.org/10.1109/34.85677)> and

P.J. Rousseeuw (1987) and L. Kaufman and P.J. Rousseeuw(2009) <[doi:10.1016/0377-0427\(87\)90125-7](https://doi.org/10.1016/0377-0427(87)90125-7)> and

<[doi:10.1002/9780470316801](https://doi.org/10.1002/9780470316801)>

C. Alok. (2010).

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AccClust	<i>Accuracy detection for a clustering result with known classes</i>
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**Description**

Computes the accuracy of a clustering result of a dataset with known classes from the k-means, fuzzy c-means, or EM algorithm.

**Usage**

```
AccClust(x, label.names = "label", algorithm = "FCM", fzm = 2,
         scale = TRUE, nstart = 100, iter = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
label.names	a character string indicating the true label column name. The default is "label"
algorithm	a character string indicating which clustering methods to be used ("FCM", "EM", "Kmeans"). More than one methods may be selected. The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
scale	logical, if TRUE (default), the dataset is normalized before clustering.
nstart	a maximum number of initial random sets for FCM for method = "FCM" or "Kmeans" or c("Kmeans", "FCM"). The default is 20.
iter	a maximum number of iterations for method = "FCM". The default is 100.

**Value**

kmeans	Accuracy score from 0 to 1 of the k-means result
FCM	Accuracy score from 0 to 1 of the FCM result
EM	Accuracy score from 0 to 1 of the EM result

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[R1\\_data](#), [D1\\_data](#), [FzzyCVIs](#), [WP.IDX](#), [XB.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data

# Check accuracy of clustering results obtained by kmeans, FCM, and EM clustering
AccClust(x, label.names = "label", algorithm = c("Kmeans", "FCM", "EM"), fzm = 2,
  scale = TRUE, nstart = 20, iter = 100)

# Check accuracy of a clustering result obtained by the FCM algorithm
AccClust(x, label.names = "label", algorithm = "FCM", fzm = 2,
  scale = TRUE, nstart = 20, iter = 100)
```

---

CCV.IDX

*Correlation Cluster Validity (CCV) index*

---

**Description**

Computes the CCVP and CCVS (M. Popescu et al., 2013) indexes for a result of either FCM or EM clustering from user specified cmin to cmax.

**Usage**

```
CCV.IDX(x, cmax, cmin = 2, indexlist = "all", method = 'FCM', fzm = 2,
  iter = 100, nstart = 20)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
indexlist	a character string indicating which The generalized C index be computed ("all", "CCVP", "CCVS"). More than one indexes can be selected.

method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
iter	a maximum number of iterations for method = "FCM". The default is 100.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.

### Details

A new cluster validity framework that compares the structure in the data to the structure of dissimilarity matrices induced by a matrix transformation of the partition being tested. The largest value of  $CCV(c)$  indicates a valid optimal partition.

### Value

Each of the followings shows the values of each index for  $c$  from  $c_{min}$  to  $c_{max}$  in a data frame.

CCVP	the Pearson Correlation Cluster Validity index.
CCVS	the Spearman's (rho) Correlation Cluster Validity index.

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

### References

M. Popescu, J. C. Bezdek, T. C. Havens and J. M. Keller (2013). "A Cluster Validity Framework Based on Induced Partition Dissimilarity." <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&number=6246717&isnumber=6340245>

### See Also

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

### Examples

```
library(UniversalCVI)

# Iris data
x = iris[,1:4]

# ---- FCM algorithm ----

# Compute all the indices by CCV.IDX
FCM.ALL.CCV = CCV.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "all",
  method = 'FCM', fzm = 2, iter = 100, nstart = 20)
print(FCM.ALL.CCV)
```

```

# Compute CCVP index
FCM.CCVP = CCV.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "CCVP",
  method = 'FCM', fzm = 2, iter = 100, nstart = 20)
print(FCM.CCVP)

# ---- EM algorithm ----

# Compute all the indices by CCV.IDX
EM.ALL.CCV = CCV.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "all",
  method = 'EM', iter = 100, nstart = 20)
print(EM.ALL.CCV)

# Compute CCVP index
EM.CCVP = CCV.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "CCVP",
  method = 'EM', iter = 100, nstart = 20)
print(EM.CCVP)

```

---

CH.IDX

*Calinski–Harabasz (CH) index*


---

### Description

Computes the CH (T. Calinski and J. Harabasz, 1974) index for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

### Usage

```
CH.IDX(x, kmax, kmin = 2, method = "kmeans", nstart = 100)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

### Details

The CH index is defined as

$$CH(k) = \frac{n - k}{k - 1} \frac{\sum_{i=1}^k |C_i| d(v_i, \bar{x})}{\sum_{i=1}^k \sum_{x_j \in C_i} d(x_j, v_i)}$$

The largest value of  $CH(k)$  indicates a valid optimal partition.

**Value**

CH                    the CH index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the CH index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

T. Calinski, J. Harabasz, "A dendrite method for cluster analysis," *Communications in Statistics*, 3, 1-27 (1974).

**See Also**

[Hvalid](#), [Wvalid](#), [DI.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute the CH index
K.CH = CH.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans", nstart = 100)
print(K.CH)

# The optimal number of cluster
K.CH[which.max(K.CH$CH),]

# ---- Hierarchical ----

# Average linkage

# Compute the CH index
H.CH = CH.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average")
print(H.CH)

# The optimal number of cluster
H.CH[which.max(H.CH$CH),]
```

---

 CSL.IDX

*Chou-Su-Lai (CSL) index*


---

### Description

Computes the CSL (C. H. Chou et al., 2004) index for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

### Usage

```
CSL.IDX(x, kmax, kmin = 2, method = "kmeans", nstart = 100)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

### Details

The CSL index is defined as

$$CSL(k) = \frac{\sum_{i=1}^k \left\{ \frac{1}{|C_i|} \sum_{x_j \in C_i} \max_{x_l \in C_i} d(x_j, x_l) \right\}}{\sum_{i=1}^k \{ \min_{j:j \neq i} d(v_i, v_j) \}}.$$

The smallest value of  $CSL(k)$  indicates a valid optimal partition.

### Value

CSL the CSL index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the CSL index, respectively.

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

### References

C. H. Chou, M. C. Su, E. Lai, "A new cluster validity measure and its application to image compression," *Pattern Anal Applic*, 7, 205-220 (2004).

**See Also**

[Hvalid](#), [Wvalid](#), [DI.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute the CSL index
K.CSL = CSL.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans", nstart = 100)
print(K.CSL)

# The optimal number of cluster
K.CSL[which.min(K.CSL$CSL),]

# ---- Hierarchical ----

# Average linkage

# Compute the CSL index
H.CSL = CSL.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average")
print(H.CSL)

# The optimal number of cluster
H.CSL[which.min(H.CSL$CSL),]
```

---

D10\_data

*D10 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 3 different Gaussian and 2 Uniform distributions labeled as 1-5.

**Usage**

```
D10_data
```

**Format**

A data frame with 1250 data points and 3 variables

x Numeric values generated from Gaussian and Uniform distributions

y Numeric values generated from Gaussian and Uniform distributions

label Categorical labels 1,2,3,4,5

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D1\_data

*D1 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian distributions labeled as 1-6.

**Usage**

D1\_data

**Format**

A data frame with 1500 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label1 Categorical labels 1,2,3,4,5,6

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D2\_data

*D2 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian distributions labeled as 1–6.

**Usage**

D2\_data

**Format**

A data frame with 1200 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5,6

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D3\_data

*D3 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 4 different Gaussian distributions labeled as 1–4.

**Usage**

D3\_data

**Format**

A data frame with 1400 data points and 3 variables  
x Numeric values generated from Gaussian distributions  
y Numeric values generated from Gaussian distributions  
label Categorical labels 1,2,3,4

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D4\_data

*D4 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 4 different Gaussian distributions labeled as 1-4.

**Usage**

D4\_data

**Format**

A data frame with 2400 data points and 3 variables  
x Numeric values generated from Gaussian distributions  
y Numeric values generated from Gaussian distributions  
label Categorical labels 1,2,3,4

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D5\_data

*D5 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 5 different Gaussian distributions labeled as 1-5.

**Usage**

D5\_data

**Format**

A data frame with 350 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D6\_data

*D6 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 5 different Gaussian distributions labeled as 1–5.

**Usage**

D6\_data

**Format**

A data frame with 1100 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D7\_data

*D7 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian distributions labeled as 1–6.

**Usage**

D7\_data

**Format**

A data frame with 1500 data points and 3 variables  
x Numeric values generated from Gaussian distributions  
y Numeric values generated from Gaussian distributions  
label1 Categorical labels 1,2,3,4,5,6

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D8\_data

*D8 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian distributions labeled as 1-6.

**Usage**

D8\_data

**Format**

A data frame with 2000 data points and 3 variables  
x Numeric values generated from Gaussian distributions  
y Numeric values generated from Gaussian distributions  
label1 Categorical labels 1,2,3,4,5,6

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

D9\_data

*D9 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 3 different Uniform distributions labeled as 1-3.

**Usage**

D9\_data

**Format**

A data frame with 1000 data points and 3 variables

x Numeric values generated from Uniform distributions

y Numeric values generated from Uniform distributions

label Categorical labels 1,2,3

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

DB.IDX

*Davies–Bouldin (DB) and DB\* (DBs) indexes***Description**

Computes the DB (D. L. Davies and D. W. Bouldin, 1979) and DBs (M. Kim and R. S. Ramakrishna, 2005) indexes for a result either kmeans or hierarchical clustering from user specified `kmin` to `kmax`.

**Usage**

```
DB.IDX(x, kmax, kmin = 2, method = "kmeans",
       indexlist = "all", p = 2, q = 2, nstart = 100)
```

**Arguments**

<code>x</code>	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
<code>kmax</code>	a maximum number of clusters to be considered.
<code>kmin</code>	a minimum number of clusters to be considered. The default is 2.
<code>method</code>	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
<code>indexlist</code>	a character string indicating which cluster validity indexes to be computed ("all", "DB", "DBs"). More than one indexes can be selected.
<code>p</code>	the power of the Minkowski distance between centroids of clusters. The default is 2.
<code>q</code>	the power of dispersion measure of a cluster. The default is 2.
<code>nstart</code>	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

**Details**

The lowest value of  $DB(k)$ ,  $DBs(k)$  indicates a valid optimal partition.

**Value**

DB	the DB index for <code>k</code> from <code>kmin</code> to <code>kmax</code> shown in a data frame where the first and the second columns are <code>k</code> and the DB index, respectively.
DBs	the DBs index for <code>k</code> from <code>kmin</code> to <code>kmax</code> shown in a data frame where the first and the second columns are <code>k</code> and the DBs index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

## References

- D. L. Davies, D. W. Bouldin, "A cluster separation measure," *IEEE Trans Pattern Anal Machine Intell*, 1, 224-227 (1979).
- M. Kim, R. S. Ramakrishna, "New indices for cluster validity assessment," *Pattern Recognition Letters*, 26, 2353-2363 (2005).

## See Also

[Hvalid](#), [Wvalid](#), [DI.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

## Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute all the indices by DB.IDX
K.ALL = DB.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans",
  indexlist = "all", p = 2, q = 2, nstart = 100)
print(K.ALL)

# Compute DB index
K.DB = DB.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans",
  indexlist = "DB", p = 2, q = 2, nstart = 100)
print(K.DB)

# ---- Hierarchical ----

# Average linkage

# Compute all the indices by DB.IDX
H.ALL = DB.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average",
  indexlist = "all", p = 2, q = 2)
print(H.ALL)

# Compute DB index
H.DB = DB.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average",
  indexlist = "DB", p = 2, q = 2)
print(H.DB)
```

---

DI.IDX

*Dunn index*

---

## Description

Computes the DI (J. C. Dunn, 1973) index for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

**Usage**

```
DI.IDX(x, kmax, kmin = 2, method = "kmeans", nstart = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

**Details**

The DI index is defined as

$$DI(k) = \min_{i \neq j \in [k]} \left\{ \frac{\min \{d(x_u, x_v) | x_u \in C_i, x_v \in C_j\}}{\max_{l \in [k]} \max \{d(x_u, x_v) | x_u, x_v \in C_l\}} \right\}.$$

The largest value of  $DI(k)$  indicates a valid optimal partition.

**Value**

DI	the DI index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the DI index, respectively.
----	---

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

J. C. Dunn, "A fuzzy relative of the ISODATA process and its use in detecting compact well-separated clusters," *J Cybern*, 3(3), 32-57 (1973).

**See Also**

[Hvalid](#), [Wvalid](#), [DB.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]
```

```

# ---- Kmeans ----

# Compute the DI index
K.DI = DI.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans", nstart = 100)
print(K.DI)

# The optimal number of cluster
K.DI[which.max(K.DI$DI),]

# ---- Hierarchical ----

# Average linkage

# Compute the DI index
H.DI = DI.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average")
print(H.DI)

# The optimal number of cluster
H.DI[which.max(H.DI$DI),]

```

---

FuzzyCVIs

*Fuzzy cluster validity indexes used in Wiroonsri and Preedasawakul (2023)*


---

## Description

Computes the cluster validity indexes for a result of either FCM or EM clustering from user specified `cmin` to `cmax` used in Wiroonsri and Preedasawakul (2023). It includes the XB (X. L. Xie and G. Beni, 1991) index, KWON (S. H. Kwon, 1998) index, KWON2 (S. H. Kwon et al., 2021) index, TANG (Y. Tang et al., 2005) index, HF (F. Haouas et al., 2017) index, WL (C. H. Wu et al., 2015) index, PBM (M. K. Pakhira et al., 2004) index, KPBM (C. Alok, 2010) index, CCVP and CCVS (M. Popescu et al., 2013) index, GC1, GC2, GC3, and GC4 (J. C. Bezdek et al., 2016) indexes, WPC, WP, WPC11, and, WPC12 (N. Wiroonsri and O. Preedasawakul, 2023) indexes.

## Usage

```

FuzzyCVIs(x, cmax, cmin = 2, indexlist = 'all', corr = 'pearson',
  method = 'FCM', fzm = 2, gamma = (fzm^2*7)/4, sampling = 1,
  iter = 100, nstart = 20, NCstart = TRUE)

```

## Arguments

<code>x</code>	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
<code>cmax</code>	a maximum number of clusters to be considered.
<code>cmin</code>	a minimum number of clusters to be considered. The default is 2.

indexlist	a character string indicating which cluster validity indexes to be computed ("all", "WPC", "WP", "WPCI1", "WPCI2", "XB", "KWON", "KWON2", "TANG", "HF", "WL", "PBM", "KPBM", "CCVP", "CCVS", "GC1", "GC2", "GC3", "GC4"). More than one indexes can be selected.
corr	a character string indicating which correlation coefficient is to be computed ("pearson", "kendall" or "spearman") for indexlist=("WP", "WPC", "WPCI1", "WPCI2", "CCVP", "CCVS", "GC1", "GC2", "GC3" or "GC4"). The default is "pearson".
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
gamma	adjusted fuzziness parameter for indexlist=("WP", "WPC", "WPCI1", "WPCI2"). The default is $7fzm^2/4$ .
sampling	a number greater than 0 and less than or equal to 1 indicating the undersampling proportion of data to be used. This argument is intended for handling a large dataset. The default is 1.
iter	a maximum number of iterations for method = "FCM". The default is 100.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
NCstart	logical for indexlist includes either of the "WP", "WPC", "WPCI1", and "WPCI2"), if TRUE, the WP correlation at c=1 is defined as the ratio introduced in the reference. Otherwise, it is assigned as 0.

## Details

The well-known cluster validity indexes for either FCM or EM clustering. It includes the XB (X. L. Xie and G. Beni., 1991) index, KWON (S. H. Kwon, 1998) index, KWON2 (S. H. Kwon et al., 2021) index, TANG (Y. Tang et al., 2005) index, HF (F. Haouas et al., 2017) index, WL (C. H. Wu et al., 2015) index, PBM (M. K. Pakhira et al., 2004) index, KPBM (C. Alok, 2010) index, CCVP and CCVS (M. Popescu et al., 2013) index, GC1, GC2, GC3, and GC4 (J. C. Bezdek et al., 2016) indexes, WPC, WP, WPCI1, and, WPCI2 (N. Wiroonsri and O. Preedasawakul, 2023) indexes.

The WPC computes the correlation between the actual distance between a pair of data points and the distance between adjusted centroids with respect to the pair. WPCI1 and WPCI2 are the proportion and the subtraction, respectively, of the same two ratios. The first ratio is the WPC improvement from c-1 clusters to c clusters over the entire room for improvement. The second ratio is the WPC improvement from c clusters to c+1 clusters over the entire room for improvement. WP is defined as a combination of WPCI1 and WPCI2.

## Value

WPC	the WP correlation from c from cmin-1 to cmax+1 shown in a data frame. Each of the followings shows the values of each index for c from cmin to cmax in a data frame.
WP	the WP index.
WPCI1	the WPCI1 index.
WPCI2	the WPCI2 index.

XB	the XB index.
KWON	the KWON index.
KWON2	the KWON2 index.
TANG	the TANG index.
HF	the HF index.
WL	the WL index.
PBM	the PBM index
KPBM	the KPBM index
CCVP	the Pearson Correlation Cluster Validity index.
CCVS	the Spearman's (rho) Correlation Cluster Validity index.
GC1	the generalized C index ( $\sum \cdot \sim$ Sum-Product).
GC2	the generalized C index ( $\sum \wedge \sim$ Sum-Min).
GC3	the generalized C index ( $\vee \cdot \sim$ Max-Product).
GC4	the generalized C index ( $\vee \wedge \sim$ Max-Min).

#### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

#### References

- C. Alok. (2010). "An investigation of clustering algorithms and soft computing approaches for pattern recognition," Department of Computer Science, Assam University.
- J. C. Bezdek, M. Moshtaghi, T. Runkler, C. Leckie, "The generalized c index for internal fuzzy cluster validity," IEEE Transactions on Fuzzy Systems, vol. 24, no. 6, pp. 1500–1512, 2016.
- F. Haouas, Z. Ben Dhiyf, A. Hammouda, B. Solaiman, "A new efficient fuzzy cluster validity index: Application to images clustering," 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Naples, Italy, 2017, pp. 1-6.
- S. H. Kwon, "Cluster validity index for fuzzy clustering," Electronics letters, vol. 34, no. 22, pp. 2176–2177, 1998.
- S. H. Kwon, J. Kim, S. H. Son, "Improved cluster validity index for fuzzy clustering," Electronics Letters, vol. 57, no. 21, pp. 792–794, 2021.
- M. K. Pakhira, S. Bandyopadhyay, U. Maulik, "Validity index for crisp and fuzzy clusters," Pattern recognition, vol. 37, no. 3, pp. 487–501, 2004.

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Y. Tang, F. Sun, Z. Sun, "Improved validation index for fuzzy clustering," in Proceedings of the 2005, American Control Conference, 2005., pp. 1120–1125 vol. 2, 2005.

N. Wiroonsri, O. Preedasawakul, "A correlation-based fuzzy cluster validity index with secondary options detector," arXiv:2308.14785, 2023

C. H. Wu, C. S. Ouyang, L. W. Chen, L. W. Lu, "A new fuzzy clustering validity index with a median factor for centroid-based clustering," IEEE Transactions on Fuzzy Systems, vol. 23, no. 3, pp. 701–718, 2015.

X. Xie, G. Beni, "A validity measure for fuzzy clustering," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 13, no. 8, pp. 841–847, 1991.

### See Also

[WP.IDX](#), [GC.IDX](#), [CCV.IDX](#), [R1\\_data](#)

### Examples

```
library(UniversalCVI)

# Iris data
x = iris[,1:4]

# ---- FCM algorithm ----

# Compute selected a set of indices ("WPC","WP","XB") using default gamma
F.s = FuzzyCVIs(scale(x), cmax = 10, cmin = 2, indexlist = c("WPC","WP","XB"),
  corr = 'pearson', method = 'FCM', fzm = 2, iter = 100, nstart = 20, NCstart = TRUE)

# Plot the computed indexes
plot_idx(F.s)

# ---- EM algorithm ----

# Compute all the indices by FuzzyCVIs using default gamma
```

```
E.all = FzzyCVIs(scale(x), cmax = 10, cmin = 2, indexlist = 'all', corr = 'pearson',
  method = 'EM', iter = 100, nstart = 20, NCstart = TRUE)

# Plot the computed indexes
plot_idx(E.all)
```

---

GC.IDX

*The generalized C index*


---

### Description

Computes the GC1 GC2 GC3 and GC4 (J. C. Bezdek et al., 2016) indexes for a result of either FCM or EM clustering from user specified cmin to cmax.

### Usage

```
GC.IDX(x, cmax, cmin = 2, indexlist = "all", method = 'FCM', fzm = 2,
  iter = 100, nstart = 20)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
indexlist	a character string indicating which The generalized C index be computed ("all", "GC1", "GC2", "GC3", "GC4"). More than one indexes can be selected.
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
iter	a maximum number of iterations for method = "FCM". The default is 100.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.

### Details

The GC index is a soft version of the C-index, formulated based on relational transformations of the membership degree matrix  $\mu$ . It comprises four distinct variants, each with its own definition. The smallest value of  $GC(c)$  indicates a valid optimal partition.

**Value**

Each of the followings shows the values of each index for  $c$  from  $c_{min}$  to  $c_{max}$  in a data frame.

GC1	the generalized C index ( $\sum \cdot \sim$ Sum-Product).
GC2	the generalized C index ( $\sum \wedge \sim$ Sum-Min).
GC3	the generalized C index ( $\vee \cdot \sim$ Max-Product).
GC4	the generalized C index ( $\vee \wedge \sim$ Max-Min).

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

J. C. Bezdek, M. Moshtaghi, T. Runkler, and C. Leckie, "The generalized  $c$  index for internal fuzzy cluster validity," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 6, pp. 1500–1512, 2016.  
<https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7429723&isnumber=7797168>

**See Also**

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# Iris data
x = iris[,1:4]

# ---- FCM algorithm ----

# Compute all the indices by GC.IDX
FCM.all.GC = GC.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "all",
  method = 'FCM', fzm = 2, iter = 100, nstart = 5)
print(FCM.all.GC)

# Compute GC2 index
FCM.GC2 = GC.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "GC2",
  method = 'FCM', fzm = 2, iter = 100, nstart = 5)
print(FCM.GC2)

# ---- EM algorithm ----

# Compute all the indices by GC.IDX
EM.all.GC = GC.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "all",
  method = 'EM', iter = 100, nstart = 5)
print(EM.all.GC)

# Compute GC2 index
EM.GC2 = GC.IDX(scale(x), cmax = 10, cmin = 2, indexlist = "GC2",
```

```
method = 'EM', iter = 100, nstart = 5)
print(EM.GC2)
```

HF.IDX

*HF index***Description**

Computes the HF (F. Haouas et al., 2017) index for a result of either FCM or EM clustering from user specified cmin to cmax.

**Usage**

```
HF.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

**Arguments**

**x** a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.

**cmax** a maximum number of clusters to be considered.

**cmin** a minimum number of clusters to be considered. The default is 2.

**method** a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".

**fzm** a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.

**nstart** a maximum number of initial random sets for FCM for method = "FCM". The default is 20.

**iter** a maximum number of iterations for method = "FCM". The default is 100.

**Details**

The HF index is defined as

$$HF(c) = \frac{\sum_{j=1}^c \sum_{i=1}^n \mu_{ij}^m \|x_i - v_j\|^2 + \frac{1}{c(c-1)} \sum_{j \neq k} \|v_j - v_k\|^2}{\frac{n}{2c} (\min_{j \neq k} \{\|v_j - v_k\|^2\} + \text{median}_{j \neq k} \{\|v_j - v_k\|^2\})}$$

The smallest value of  $HF(c)$  indicates a valid optimal partition.

**Value**

**HF** the HF index for c from cmin to cmax shown in a data frame where the first and the second columns are c and the HF index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

## References

F. Haouas, Z. Ben Dhiab, A. Hammouda and B. Solaiman, "A new efficient fuzzy cluster validity index: Application to images clustering," 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Naples, Italy, 2017, pp. 1-6. <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=8015651&isnumber=8015374>

## See Also

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

## Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the HF index
FCM.HF = HF.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.HF)

# The optimal number of cluster
FCM.HF[which.min(FCM.HF$HF),]

# ---- EM algorithm ----

# Compute the HF index
EM.HF = HF.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.HF)

# The optimal number of cluster
EM.HF[which.min(EM.HF$HF),]
```

---

Hvalid	<i>Wiroonsri(2024) correlation-based cluster validity indices and other well-known cluster validity indices</i>
--------	---

---

## Description

Computes the cluster validity indexes for a result of either kmeans or hierarchical clustering from user specified kmin to kmax used in Wiroonsri(2024). It includes the DI (J. C. Dunn, 1973) index, CH (T. Calinski and J. Harabasz, 1974) index, DB (D. L. Davies and D. W. Bouldin, 1979) index, PB (G. W. Miligan, 1985) index, CSL (C. H. Chou et al., 2004) index, PBM (M. K. Pakhira et al., 2004) index, DBs (M. Kim and R. S. Ramakrishna, 2005), Score function (S. Saitta et al., 2007), STR (A. Starczewski, 2017) index, NC, NCI, NCI1, and, NCI2 (N. Wiroonsri, 2024) indexes.

**Usage**

```
Hvalid(x, kmax, kmin = 2, indexlist = "all", method = "kmeans",
      p = 2, q = 2, corr = "pearson", nstart = 100, sampling = 1, NCstart = TRUE)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
indexlist	a character string indicating which cluster validity indexes to be computed ("all", "NC", "NCI", "NCI1", "NCI2", "PB", "CSL", "CH", "DB", "DBs", "SH", "SF", "DI", "STR", "PBM"). More than one indexes can be selected.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_complete", "hclust_ward.D", "hclust_ward.D2", "hclust_average", "hclust_single"). The default is "kmeans".
p	the power of the Minkowski distance between centroids of clusters for indexlist = c("DB", "DBs"). The default is 2.
q	the power of dispersion measure of a cluster for indexlist = c("DB", "DBs"). The default is 2.
corr	a character string indicating which correlation coefficient is to be computed ("pearson", "kendall" or "spearman"). The default is "pearson".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.
sampling	a number greater than 0 and less than or equal to 1 indicating the undersampling proportion of data to be used. This argument is intended for handling a large dataset. The default is 1.
NCstart	logical for indexlist includes the "NC", "NCI", "NCI1", and "NCI2"), if TRUE, the NC correlation at k=1 is defined as the ratio introduced in the reference. Otherwise, it is assigned as 0.

**Details**

The well-known cluster validity indices used in Wiroonsri(2024). It includes the DI (J. C. Dunn, 1973) index, CH (T. Calinski and J. Harabasz, 1974) index, DB (D. L. Davies and D. W. Bouldin, 1979) index, PB (G. W. Miligan, 1980) index, CSL (C. H. Chou et al., 2004) index, PBM (M. K. Pakhira et al., 2004) index, DBs (M. Kim and R. S. Ramakrishna, 2005), Silhouette (Rousseeuw, 1987; Kaufman and Rousseeuw, 2009), Score function (S. Saitta et al., 2007), STR (A. Starczewski, 2017), NC, NCI, NCI1, and, NCI2 (N. Wiroonsri, 2024) indexes.

The NC correlation computes the correlation between an actual distance between a pair of data points and a centroid distance of clusters that the two points locate in. NCI1 and NCI2 are the proportion and the subtraction, respectively, of the same two ratios. The first ratio is the NC improvement from k-1 clusters to k clusters over the entire room for improvement. The second ratio is the NC improvement from k clusters to k+1 clusters over the entire room for improvement. NCI is a combination of NCI1 and NCI2.

**Value**

NC the NC correlations for k from  $k_{min}-1$  to  $k_{max}+1$  shown in a data frame where the first and the second columns are k and the NC, respectively.

Each of the followings shows the values of each index for k from  $k_{min}$  to  $k_{max}$  in a data frame.

NCI	the NCI index.
NCI1	the NCI1 index.
NCI2	the NCI2 index.
PB	the PB index.
DI	the DI index.
DB	the DB index.
DBs	the DBs index.
CSL	the CSL index.
CH	the CH index.
SH	the Score function.
SF	the Score function.
STR	the STR index.
PBM	the PBM index.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

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- D. L. Davies, D. W. Bouldin, "A cluster separation measure," *IEEE Trans Pattern Anal Machine Intell*, 1, 224-227 (1979).
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A. Starczewski, "A new validity index for crisp clusters," *Pattern Anal Applic* 20, 687–700 (2017).

N. Wiroonsri, "Clustering performance analysis using a new correlation based cluster validity index," *Pattern Recognition*, 145, 109910, 2024.

### See Also

[Wvalid](#), [FzzyCVIs](#), [DI.IDX](#), [R1\\_data](#)

### Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute all the indices by Hvalid
## Not run:
Hvalid(scale(x), kmax = 15, kmin = 2, indexlist = "all",
        method = "kmeans", p = 2, q = 2, corr = "pearson", nstart = 100, NCstart = TRUE)

## End(Not run)

# Compute selected a set of indices ("NC", "NCI", "DI", "DB")
Hvalid(scale(x), kmax = 15, kmin = 2, indexlist = c("NC", "NCI", "DI", "DB"),
        method = "kmeans", p = 2, q = 2, corr = "pearson", nstart = 100, NCstart = TRUE)

# ---- Hierarchical ----

# Average linkage

# Compute all the indices by Hvalid
## Not run:
Hvalid(scale(x), kmax = 15, kmin = 2, indexlist = "all",
        method = "hclust_average", p = 2, q = 2, corr = "pearson", nstart = 100, NCstart = TRUE)

## End(Not run)

# Compute selected a set of indices ("NC", "NCI", "DI", "DB")
Hvalid(scale(x), kmax = 15, kmin = 2, indexlist = c("NC", "NCI", "DI", "DB"),
        method = "hclust_average", p = 2, q = 2, corr = "pearson", nstart = 100, NCstart = TRUE)
```

```
#---Plot and compare the indexes---

# Compute six cluster validity indexes of a kmeans clustering result for k from 2 to 15
IDX.list = c("NCI", "DI", "DB", "DBs", "CSL", "CH")

Hvalid.result = Hvalid(scale(x), kmax = 15, kmin = 2, indexlist = IDX.list,
  method = "hclust_average", p = 2, q = 2, corr = "pearson", nstart = 100, NCstart = TRUE)

# Plot the computed indexes
plot_idx(Hvalid.result)
```

KPBM.IDX *Modified Kernel form of Pakhira-Bandyopadhyay-Maulik (KPBM) index*

### Description

Computes the KPBM (C. Alok, 2010) index for a result of either FCM or EM clustering from user specified cmin to cmax.

### Usage

```
KPBM.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
iter	a maximum number of iterations for method = "FCM". The default is 100.

### Details

The KPBM index is defined as

$$KPBM(c) = \left( \frac{\max_{j \neq k} \|v_j - v_k\|}{c \sum_{j=1}^c \sum_{i=1}^n \mu_{ij} \|x_i - v_j\|} \right)^2.$$

The largest value of  $KPBM(c)$  indicates a valid optimal partition.

**Value**

KPBM                    the KPBM index for  $c$  from  $c_{min}$  to  $c_{max}$  shown in a data frame where the first and the second columns are  $c$  and the KPBM index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

C. Alok. (2010). "An investigation of clustering algorithms and soft computing approaches for pattern recognition", Department of Computer Science, Assam University.

**See Also**

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the KPBM index
FCM.KPBM = KPBM.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.KPBM)

# The optimal number of cluster
FCM.KPBM[which.max(FCM.KPBM$KPBM),]

# ---- EM algorithm ----

# Compute the KPBM index
EM.KPBM = KPBM.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.KPBM)

# The optimal number of cluster
EM.KPBM[which.max(EM.KPBM$KPBM),]
```

---

KWON.IDX	<i>KWON index</i>
----------	-------------------

---

### Description

Computes the KWON (S. H. Kwon, 1998) index for a result of either FCM or EM clustering from user specified `cmin` to `cmax`.

### Usage

```
KWON.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

### Arguments

<code>x</code>	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
<code>cmax</code>	a maximum number of clusters to be considered.
<code>cmin</code>	a minimum number of clusters to be considered. The default is 2.
<code>method</code>	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
<code>fzm</code>	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
<code>nstart</code>	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
<code>iter</code>	a maximum number of iterations for method = "FCM". The default is 100.

### Details

The KWON index is defined as

$$KWON(c) = \frac{\sum_{j=1}^c \sum_{i=1}^n \mu_{ij}^2 \|x_i - v_j\|^2 + \frac{1}{c} \sum_{j=1}^c \|v_j - v_0\|^2}{\min_{i \neq j} \|v_i - v_j\|^2}.$$

The smallest value of  $KWON(c)$  indicates a valid optimal partition.

### Value

KWON	the KWON index for <code>c</code> from <code>cmin</code> to <code>cmax</code> shown in a data frame where the first and the second columns are <code>c</code> and the KWON index, respectively.
------	---

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

## References

S. H. Kwon, "Cluster validity index for fuzzy clustering," *Electronics letters*, vol. 34, no. 22, pp. 2176–2177, 1998. doi:10.1049/el:19981523

## See Also

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

## Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the KWON index
FCM.KWON = KWON.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.KWON)
# The optimal number of cluster
FCM.KWON[which.min(FCM.KWON$KWON),]

# ---- EM algorithm ----

# Compute the KWON index
EM.KWON = KWON.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.KWON)
# The optimal number of cluster
EM.KWON[which.min(EM.KWON$KWON),]
```

---

KWON2.IDX

*KWON2 index*

---

## Description

Computes the KWON2 (S. H. Kwon et al., 2021) index for a result of either FCM or EM clustering from user specified cmin to cmax.

## Usage

```
KWON2.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
iter	a maximum number of iterations for method = "FCM". The default is 100.

**Details**

KWON2 is defined as

$$KWON2(c) = \frac{w_1 \left[ w_2 \sum_{j=1}^c \sum_{i=1}^n \mu_{ij}^2 \sqrt{\frac{m}{2}} \|x_i - v_j\|^2 + \frac{\sum_{j=1}^c \|v_j - v_0\|^2}{\max_j \|v_j - v_0\|^2} + w_3 \right]}{\min_{i \neq j} \|v_i - v_j\|^2 + \frac{1}{c} + \frac{1}{c^m - 1}}.$$

where  $w_1 = \frac{n-c+1}{n}$ ,  $w_2 = \left(\frac{c}{c-1}\right)^{\sqrt{2}}$  and  $w_3 = \frac{nc}{(n-c+1)^2}$ .

The smallest value of  $KWON2(c)$  indicates a valid optimal partition.

**Value**

KWON2 the KWON2 index for c from cmin to cmax shown in a data frame where the first and the second columns are c and the KWON2 index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

S. H. Kwon, J. Kim, and S. H. Son, "Improved cluster validity index for fuzzy clustering," *Electronics Letters*, vol. 57, no. 21, pp. 792–794, 2021.

**See Also**

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

**Examples**

```

library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the KWON2 index
FCM.KWON2 = KWON2.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.KWON2)

# The optimal number of cluster
FCM.KWON2[which.min(FCM.KWON2$KWON2),]

# ---- EM algorithm ----

# Compute the KWON2 index
EM.KWON2 = KWON2.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.KWON2)

# The optimal number of cluster
EM.KWON2[which.min(EM.KWON2$KWON2),]

```

---

PB.IDX

*Point biserial correlation (PB)*


---

**Description**

Computes the PB (G. W. Miligan, 1980) index for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

**Usage**

```
PB.IDX(x, kmax, kmin = 2, method = "kmeans", corr = "pearson", nstart = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".

corr	a character string indicating which correlation coefficient is to be computed ("pearson", "kendall" or "spearman"). The default is "pearson".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

### Details

The largest value of  $PB(k)$  indicates a valid optimal partition.

### Value

PB	the PB index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the PB index, respectively.
----	---

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

### References

G. W. Miligan, "An examination of the effect of six types of error perturbation on fifteen clustering algorithms," *Psychometrika*, 45, 325-342 (1980).

### See Also

[Hvalid](#), [Wvalid](#), [DI.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

### Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute PB index
K.PB = PB.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans",
  corr = "pearson", nstart = 100)
print(K.PB)

# The optimal number of cluster
K.PB[which.max(K.PB$PB),]

# ---- Hierarchical ----

# Average linkage

# Compute PB index
H.PB = PB.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average",
  corr = "pearson")
```

```
print(H.PB)

# The optimal number of cluster
H.PB[which.max(H.PB$PB),]
```

---

PBM.IDX

*Pakhira-Bandyopadhyay-Maulik (PBM) index*


---

### Description

Computes the PBM (M. K. Pakhira et al., 2004) index for a result of either FCM or EM clustering from user specified cmin to cmax.

### Usage

```
PBM.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
iter	a maximum number of iterations for method = "FCM". The default is 100.

### Details

The PBM index is defined as

$$PBM(c) = \left( \frac{\sum_{i=1}^n \|x_i - v_0\| \cdot \max_{j \neq k} \|v_j - v_k\|}{c \sum_{j=1}^c \sum_{i=1}^n \mu_{ij} \|x_i - v_j\|} \right)^2.$$

The largest value of  $PBM(c)$  indicates a valid optimal partition.

### Value

PBM	the PBM index for c from cmin to cmax shown in a data frame where the first and the second columns are c and the PBM index, respectively.
-----	---

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

M. K. Pakhira, S. Bandyopadhyay, and U. Maulik, "Validity index for crisp and fuzzy clusters," Pattern recognition, vol. 37, no. 3, pp. 487–501, 2004.

**See Also**

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the PBM index
FCM.PBM = PBM.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.PBM)

# The optimal number of cluster
FCM.PBM[which.max(FCM.PBM$PBM),]

# ---- EM algorithm ----

# Compute the PBM index
EM.PBM = PBM.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.PBM)

# The optimal number of cluster
EM.PBM[which.max(EM.PBM$PBM),]
```

---

plot\_idx

*Plots for visualizing CVIs*

---

**Description**

Plot and compare upto 8 indices computed by the algorithms in this package.

**Usage**

```
plot_idx(idxresult,selected.idx = NULL)
```

**Arguments**

- `idxresult` a result from one of the algorithms FzzyCVIs, WP.IDX, GC.IDX, CCV.IDX, XB.IDX, WL.IDX, TANG.IDX, PBM.IDX, KWON.IDX, KWON2.IDX, KPBM.IDX, HF.IDX, Hvalid, Wvalid, SF.IDX, SH.IDX, PB.IDX, DI.IDX, DB.IDX, CSL.IDX, CH.IDX or STRPBM.IDX.
- `selected.idx` a numeric vector indicates a part of the indexes from the `idxresult` in respective order selected by a user. For instance, `selected.idx = 3` or `selected.idx = c(1, 3, 5)` may be selected. If not specified, the full `idxresult` will be considered.

**Value**

Plots of upto 8 cluster validity indices computed from FzzyCVIs, WP.IDX, GC.IDX, CCV.IDX, XB.IDX, WL.IDX, TANG.IDX, PBM.IDX, KWON.IDX, KWON2.IDX, KPBM.IDX, HF.IDX, Hvalid, Wvalid, SH.IDX, SF.IDX, PB.IDX, DI.IDX, DB.IDX, CSL.IDX, CH.IDX or STRPBM.IDX. When using the isolated index algorithm, all the plots computed by that algorithm will be shown. When using FzzyCVIs or Hvalid with more than 8 selected indices, the first 8 indices will be plotted.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, "A correlation-based fuzzy cluster validity index with secondary options detector," arXiv:2308.14785, 2023

**See Also**

[FzzyCVIs](#), [WP.IDX](#), [XB.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# Iris data
x = iris[,1:4]

# ----Compute all the indices by FzzyCVIs ----
FCVIs = FzzyCVIs(scale(x), cmax = 10, cmin = 2, indexlist = 'all', corr = 'pearson',
                 method = 'FCM', fzm = 2, iter = 100, nstart = 20, NCstart = TRUE)

# plots of the eight indices by default
plot_idx(idxresult = FCVIs)

# plots of a specific selected.idx
plot_idx(idxresult = FCVIs, selected.idx = c(2,5,7))

# ----Compute all the indices by Wvalid ----
FCM.NC = Wvalid(scale(x), kmax = 10, kmin=2, method = 'kmeans',
                corr='pearson', nstart=100, NCstart = TRUE)
```

```
# plots of the four indices by default
plot_idx(idxresult = FCM.NC)

# ----Compute all the indices by XB.IDX ----

FCM.XB = XB.IDX(scale(x), cmax = 10, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
plot_idx(idxresult = FCM.XB)
```

---

R1\_data

*R1 Artificial Dataset*

---

### Description

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 9 different Gaussian distributions labeled as 1-9.

### Usage

R1\_data

### Format

A data frame with 450 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5,6,7,8,9

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

### References

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

### See Also

[FzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

R2\_data

*R2 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 7 different Gaussian distributions labeled as 1–7.

**Usage**

R2\_data

**Format**

A data frame with 1750 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5,6,7

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

R3\_data

*R3 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 16 different Gaussian distributions labeled as 1–16.

**Usage**

R3\_data

**Format**

A data frame with 1600 data points and 3 variables  
x Numeric values generated from Gaussian distributions  
y Numeric values generated from Gaussian distributions  
label1 Categorical labels 1,2,3,...,16

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

R4\_data

*R4 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 5 different Gaussian distributions labeled as 1-5.

**Usage**

R4\_data

**Format**

A data frame with 1250 data points and 3 variables  
x Numeric values generated from Gaussian distributions  
y Numeric values generated from Gaussian distributions  
label1 Categorical labels 1,2,3,4,5

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

R5\_data

*R5 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian distributions labeled as 1-6.

**Usage**

R5\_data

**Format**

A data frame with 1200 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5,6

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FuzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

R6\_data

*R6 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian distributions labeled as 1–6.

**Usage**

R6\_data

**Format**

A data frame with 1500 data points and 3 variables

x Numeric values generated from Gaussian distributions

y Numeric values generated from Gaussian distributions

label Categorical labels 1,2,3,4,5,6

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

R7\_data

*R7 Artificial Dataset*

---

**Description**

A 2-dimensional dataset from Wiroonsri and Preedasawakul (2023) generated from 6 different Gaussian and 3 Uniform distributions labeled as 1–3.

**Usage**

R7\_data

**Format**

A data frame with 1200 data points and 3 variables

x Numeric values generated from Gaussian and Uniform distributions

y Numeric values generated from Gaussian and Uniform distributions

label Categorical labels 1,2,3

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

N. Wiroonsri, O. Preedasawakul, A correlation-based fuzzy cluster validity index with secondary options detector, arXiv:2308.14785, 2023

**See Also**

[FzzyCVIs](#), [WP.IDX](#), [D1\\_data](#), [Hvalid](#), [DI.IDX](#)

---

SF.IDX

*The score function*

---

**Description**

Computes the SF (S. Saitta et al., 2007) index for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

**Usage**

```
SF.IDX(x, kmax, kmin = 2, method = "kmeans", nstart = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

**Details**

The smallest value of  $SF(k)$  indicates a valid optimal partition.

**Value**

SF                    the Score function index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the SF index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

S. Saitta, B. Raphael, I. Smith, "A bounded index for cluster validity," *In Perner, P.: Machine Learning and Data Mining in Pattern Recognition, Lecture Notes in Computer Science*, 4571, Springer (2007).

**See Also**

[Hvalid](#), [Wvalid](#), [DI.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute the SF index
K.SF = SF.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans", nstart = 100)
print(K.SF)

# The optimal number of cluster
K.SF[which.min(K.SF$SF),]

# ---- Hierarchical ----

# Average linkage

# Compute the SF index
H.SF = SF.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average")
print(H.SF)

# The optimal number of cluster
H.SF[which.min(H.SF$SF),]
```

SH.IDX

*Silhouette index***Description**

Computes the SH (Rousseeuw, 1987; Kaufman and Rousseeuw, 2009) index for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

**Usage**

```
SH.IDX(x, kmax, kmin = 2, method = "kmeans", nstart = 100)
```

**Arguments**

**x** a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.

**kmax** a maximum number of clusters to be considered.

**kmin** a minimum number of clusters to be considered. The default is 2.

**method** a character string indicating which clustering method to be used ("kmeans", "hclust\_ward.D", "hclust\_ward.D2", "hclust\_complete", "hclust\_average", "hclust\_single"). The default is "kmeans".

**nstart** a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

**Details**

For  $i \in [n]$ ,  $l \in [k]$ , and  $x_i \in C_l$ , let

$$a(i) = \frac{1}{|C_l| - 1} \sum_{y \in C_l} \|x_i - y\| \text{ and}$$

$$b(i) = \min_{r \neq l} \frac{1}{|C_r|} \sum_{y \in C_r} \|x_i - y\|.$$

The silhouette value of one data point  $x_j$  is defined as:

$$s(j) = \begin{cases} \frac{b(j) - a(j)}{\max\{a(j), b(i)\}} & \text{if } |C_j| > 1 \\ 0 & \text{if } |C_j| = 1 \end{cases}.$$

The silhouette index is defined as

$$SH(k) = \frac{1}{n} \sum_{i=1}^n s(i).$$

The largest value of  $SH(k)$  indicates a valid optimal partition.

**Value**

SH                    the SH index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the SH index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

Rousseeuw, P.J., 1987. Silhouettes: a graphical aid to the interpretation and validation of cluster analysis. *J. Comput. Appl. Math.* 20, 53–65.

P.J. Rousseeuw, "Silhouettes: a graphical aid to the interpretation and validation of cluster analysis". *J. Comput. Appl. Math* 20, 53–65 (1987).

**See Also**

[Hvalid](#), [Wvalid](#), [DI.IDX](#), [FzzyCVIs](#), [R1\\_data](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Hierarchical ----

# Average linkage

# Compute the SH index
H.SH = SH.IDX(scale(x), kmax = 10, kmin = 2, method = "hclust_average", nstart = 1)
print(H.SH)

# The optimal number of cluster
H.SH[which.max(H.SH$SH),]
```

---

STRPBM.IDX

*Starczewski and Pakhira-Bandyopadhyay-Maulik for crisp clustering indexes*

---

**Description**

Computes the STR (A. Starczewski, 2017) and PBM (M. K. Pakhira et al., 2004) indexes for a result either kmeans or hierarchical clustering from user specified kmin to kmax.

**Usage**

```
STRPBM.IDX(x, kmax, kmin = 2, method = "kmeans", indexlist = "all", nstart = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
indexlist	a character string indicating which cluster validity indexes to be computed ("all", "STR", "PBM"). More than one indexes can be selected.
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.

**Details**

PBM index can be used with both crisp and fuzzy clustering algorithms.  
 The largest value of  $STR(k)$  indicates a valid optimal partition.  
 The largest value of  $PBM(k)$  indicates a valid optimal partition.

**Value**

STR	the STR index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the STR index, respectively.
PBM	the PBM index for k from kmin to kmax shown in a data frame where the first and the second columns are k and the PBM index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

- M. K. Pakhira, S. Bandyopadhyay and U. Maulik, "Validity index for crisp and fuzzy clusters," *Pattern Recogn* 37(3):487–501 (2004).  
 A. Starczewski, "A new validity index for crisp clusters," *Pattern Anal Applic* 20, 687–700 (2017).

**See Also**

[Wvalid](#), [FzzyCVIs](#), [DI.IDX](#), [R1\\_data](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----
```

```

# Compute all the indices by STRPBM.IDX
K.ALL = STRPBM.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans",
  indexlist = "all", nstart = 100)
print(K.ALL)

# Compute STR index
K.STR = STRPBM.IDX(scale(x), kmax = 15, kmin = 2, method = "kmeans",
  indexlist = "STR", nstart = 100)
print(K.STR)

# ---- Hierarchical ----

# Average linkage

# Compute all the indices by STRPBM.IDX
H.ALL = STRPBM.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average",
  indexlist = "all")
print(H.ALL)

# Compute STR index
H.STR = STRPBM.IDX(scale(x), kmax = 15, kmin = 2, method = "hclust_average",
  indexlist = "STR")
print(H.STR)

```

---

TANG.IDX

*Tang index*


---

### Description

Computes the TANG (Y. Tang et al., 2005) index for a result of either FCM or EM clustering from user specified *cmin* to *cmax*.

### Usage

```
TANG.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

### Arguments

<i>x</i>	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
<i>cmax</i>	a maximum number of clusters to be considered.
<i>cmin</i>	a minimum number of clusters to be considered. The default is 2.
<i>method</i>	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
<i>fzm</i>	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.

`nstart` a maximum number of initial random sets for FCM for method = "FCM". The default is 20.

`iter` a maximum number of iterations for method = "FCM". The default is 100.

### Details

The Tang index is defined as

$$TANG(c) = \frac{\sum_{j=1}^c \sum_{i=1}^n \mu_{ij}^2 \|x_i - v_j\|^2 + \frac{1}{c(c-1)} \sum_{j \neq k} \|v_j - v_k\|^2}{\min_{j \neq k} \{\|v_j - v_k\|^2\} + \frac{1}{c}}.$$

The smallest value of  $TANG(c)$  indicates a valid optimal partition.

### Value

TANG the TANG index for  $c$  from  $c_{min}$  to  $c_{max}$  shown in a data frame where the first and the second columns are  $c$  and the TANG index, respectively.

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

### References

Y. Tang, F. Sun, and Z. Sun, "Improved validation index for fuzzy clustering," in Proceedings of the 2005, American Control Conference, 2005., pp. 1120–1125 vol. 2, 2005. <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=1470111&isnumber=31519>

### See Also

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

### Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the TANG index
FCM.TANG = TANG.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.TANG)

# The optimal number of cluster
FCM.TANG[which.min(FCM.TANG$TANG),]

# ---- EM algorithm ----
```

```
# Compute the TANG index
EM.TANG = TANG.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.TANG)

# The optimal number of cluster
EM.TANG[which.min(EM.TANG$TANG),]
```

---

WL.IDX

*Wu and Li (WL) index*


---

### Description

Computes the WL (C. H. Wu et al., 2015) index for a result of either FCM or EM clustering from user specified cmin to cmax.

### Usage

```
WL.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
iter	a maximum number of iterations for method = "FCM". The default is 100.

### Details

The WL index is defined as

$$WL(c) = \frac{\sum_{j=1}^c \left( \frac{\sum_{i=1}^n \mu_{ij}^2 \|x_i - v_j\|^2}{\sum_{i=1}^n \mu_{ij}} \right)}{\min_{j \neq k} \{\|v_j - v_k\|^2\} + \text{median}_{j \neq k} \{\|v_j - v_k\|^2\}}.$$

The smallest value of  $WL(c)$  indicates a valid optimal partition.

**Value**

WL                    the WL index for  $c$  from  $c_{min}$  to  $c_{max}$  shown in a data frame where the first and the second columns are  $c$  and the WL index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

C. H. Wu, C. S. Ouyang, L. W. Chen, and L. W. Lu, "A new fuzzy clustering validity index with a median factor for centroid-based clustering," IEEE Transactions on Fuzzy Systems, vol. 23, no. 3, pp. 701–718, 2015. <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6811211&isnumber=7115244>

**See Also**

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the WL index
FCM.WL = WL.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.WL)

# The optimal number of cluster
FCM.WL[which.min(FCM.WL$WL),]

# ---- EM algorithm ----

# Compute the WL index
EM.WL = WL.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.WL)

# The optimal number of cluster
EM.WL[which.min(EM.WL$WL),]
```

WP.IDX

*Wiroonsri and Preedasawakul (WP) index***Description**

Computes the WPC (WP correlation), WP, WPCI1 and WPCI2 (N. Wiroonsri and O. Preeda-sawakul, 2023) indexes for a result of either FCM or EM clustering from user specified `cmin` to `cmax`.

**Usage**

```
WP.IDX(x, cmax, cmin = 2, corr = 'pearson', method = 'FCM', fzm = 2,
       gamma = (fzm^2*7)/4, sampling = 1, iter = 100, nstart = 20, NCstart = TRUE)
```

**Arguments**

<code>x</code>	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
<code>cmax</code>	a maximum number of clusters to be considered.
<code>cmin</code>	a minimum number of clusters to be considered. The default is 2.
<code>corr</code>	a character string indicating which correlation coefficient is to be computed ("pearson", "kendall" or "spearman"). The default is "pearson".
<code>method</code>	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
<code>fzm</code>	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
<code>gamma</code>	adjusted fuzziness parameter for <code>indexlist = ("WP", "WPC", "WPCI1", "WPCI2")</code> . The default is computed from $7fzm^2/4$ .
<code>sampling</code>	a number greater than 0 and less than or equal to 1 indicating the undersampling proportion of data to be used. This argument is intended for handling a large dataset. The default is 1.
<code>iter</code>	a maximum number of iterations for method = "FCM". The default is 100.
<code>nstart</code>	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
<code>NCstart</code>	logical for <code>indexlist = ("WP", "WPC", "WPCI1", "WPCI2")</code> , if TRUE, the WP correlation at <code>c=1</code> is defined as an adjusted sd of the distances between all data points and their mean. Otherwise, the WP correlation at <code>c=1</code> is defined as 0.

**Details**

The newly introduced index was inspired by the recently introduced Wiroonsri index which is only compatible with hard clustering methods.

The WPC computes the correlation between the actual distance between a pair of data points and the distance between adjusted centroids with respect to the pair. WPCI1 and WPCI2 are the proportion and the subtraction, respectively, of the same two ratios. The first ratio is the WPC improvement from  $c-1$  clusters to  $c$  clusters over the entire room for improvement. The second ratio is the WPC improvement from  $c$  clusters to  $c+1$  clusters over the entire room for improvement. WP is defined as a combination of WPCI1 and WPCI2.

The largest value of  $WP(c)$  indicates a valid optimal partition.

### Value

WPC                    the WP correlations for  $c$  from  $c_{min}-1$  to  $c_{max}+1$  shown in a data frame where the first and the second columns are  $c$  and the WPC, respectively.

Each of the followings show the value of each index for  $c$  from  $c_{min}$  to  $c_{max}$  in a data frame.

WP                    the WP index.  
 WPCI1                the WPCI1 index.  
 WPCI2                the WPCI2 index.

### Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

### References

N. Wiroonsri, O. Preedasawakul, "A correlation-based fuzzy cluster validity index with secondary options detector," arXiv:2308.14785, 2023

### See Also

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

### Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute all the indices by WP.IDX using default gamma
FCM.WP = WP.IDX(scale(x), cmax = 10, cmin = 2, corr = 'pearson', method = 'FCM', fzm = 2,
  iter = 100, nstart = 20, NCstart = TRUE)
print(FCM.WP$WP)

# The optimal number of cluster
FCM.WP$WP[which.max(FCM.WP$WP$WPI),]
```

```
# ---- EM algorithm ----

# Compute all the indices by WP.IDX using default gamma
EM.WP = WP.IDX(scale(x), cmax = 10, cmin = 2, corr = 'pearson', method = 'EM',
  iter = 100, nstart = 20, NCstart = TRUE)
print(EM.WP$WP)

# The optimal number of cluster
EM.WP$WP[which.max(EM.WP$WP$WPI),]
```

---

Wvalid

*Wiroonsri(2024) correlation-based cluster validity indices*


---

### Description

Computes the NC correlation, NCI, NCI1 and NCI2 cluster validity indices for the number of clusters from user specified kmin to kmax obtained from either K-means or hierarchical clustering based on the recent paper by Wiroonsri(2024).

### Usage

```
Wvalid(x, kmax, kmin = 2, method = "kmeans",
  corr = "pearson", nstart = 100, sampling = 1, NCstart = TRUE)
```

### Arguments

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
kmax	a maximum number of clusters to be considered.
kmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("kmeans", "hclust_ward.D", "hclust_ward.D2", "hclust_complete", "hclust_average", "hclust_single"). The default is "kmeans".
corr	a character string indicating which correlation coefficient is to be computed ("pearson", "kendall" or "spearman"). The default is "pearson".
nstart	a maximum number of initial random sets for kmeans for method = "kmeans". The default is 100.
sampling	a number greater than 0 and less than or equal to 1 indicating the undersampling proportion of data to be used. This argument is intended for handling a large dataset. The default is 1.
NCstart	logical for indexlist includes the "NC", "NCI", "NCI1", and "NCI2"), if TRUE, the NC correlation at k=1 is defined as the ratio introduced in the reference. Otherwise, it is assigned as 0.

## Details

The NC correlation computes the correlation between an actual distance between a pair of data points and a centroid distance of clusters that the two points locate in. NCI1 and NCI2 are the proportion and the subtraction, respectively, of the same two ratios. The first ratio is the NC improvement from  $k-1$  clusters to  $k$  clusters over the entire room for improvement. The second ratio is the NC improvement from  $k$  clusters to  $k+1$  clusters over the entire room for improvement. NCI is a combination of NCI1 and NCI2.

## Value

NC                    the NC correlations for  $k$  from  $k_{min}-1$  to  $k_{max}+1$  shown in a data frame where the first and the second columns are  $k$  and the NC, respectively.

Each of the followings shows the values of each index for  $k$  from  $k_{min}$  to  $k_{max}$  in a data frame.

NCI                    the NCI index.

NCI1                   the NCI1 index.

NCI2                   the NCI2 index.

## Author(s)

Nathakhun Wiroonsri and Onthada Preedasawakul

## References

N. Wiroonsri, "Clustering performance analysis using a new correlation based cluster validity index," *Pattern Recognition*, 145, 109910, 2024. [doi:10.1016/j.patcog.2023.109910](https://doi.org/10.1016/j.patcog.2023.109910)

## See Also

[Hvalid](#), [FzzyCVIs](#), [DB.IDX](#), [R1\\_data](#)

## Examples

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- Kmeans ----

# Compute all the indices by Wvalid
K.NC = Wvalid(scale(x), kmax = 15, kmin=2, method = 'kmeans',
  corr='pearson', nstart=100, NCstart = TRUE)
print(K.NC)

# The optimal number of cluster
K.NC$NCI[which.max(K.NC$NCI$NCI),]

# ---- Hierarchical ----
```

```
# Average linkage

# Compute all the indices by Wvalid
H.NC = Wvalid(scale(x), kmax = 15, kmin=2, method = 'hclust_average',
  corr='pearson', nstart=100, NCstart = TRUE)
print(H.NC)

# The optimal number of cluster
H.NC$NCI[which.max(H.NC$NCI$NCI),]
```

XB.IDX

*Xie and Beni (XB) index***Description**

Computes the XB (X. L. Xie and G. Beni, 1991) index for a result of either FCM or EM clustering from user specified cmin to cmax.

**Usage**

```
XB.IDX(x, cmax, cmin = 2, method = "FCM", fzm = 2, nstart = 20, iter = 100)
```

**Arguments**

x	a numeric data frame or matrix where each column is a variable to be used for cluster analysis and each row is a data point.
cmax	a maximum number of clusters to be considered.
cmin	a minimum number of clusters to be considered. The default is 2.
method	a character string indicating which clustering method to be used ("FCM" or "EM"). The default is "FCM".
fzm	a number greater than 1 giving the degree of fuzzification for method = "FCM". The default is 2.
nstart	a maximum number of initial random sets for FCM for method = "FCM". The default is 20.
iter	a maximum number of iterations for method = "FCM". The default is 100.

**Details**

The XB index is defined as

$$XB(c) = \frac{\sum_{j=1}^c \sum_{i=1}^n \mu_{ij}^2 \|x_i - v_j\|^2}{n \cdot \min_{j \neq k} \{\|v_j - v_k\|^2\}}.$$

The lowest value of  $XB(c)$  indicates a valid optimal partition.

**Value**

XB                    the XB index for  $c$  from  $c_{min}$  to  $c_{max}$  shown in a data frame where the first and the second columns are  $c$  and the XB index, respectively.

**Author(s)**

Nathakhun Wiroonsri and Onthada Preedasawakul

**References**

X. Xie and G. Beni, "A validity measure for fuzzy clustering," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 13, no. 8, pp. 841–847, 1991.

**See Also**

[R1\\_data](#), [TANG.IDX](#), [FzzyCVIs](#), [WP.IDX](#), [Hvalid](#)

**Examples**

```
library(UniversalCVI)

# The data is from Wiroonsri (2024).
x = R1_data[,1:2]

# ---- FCM algorithm ----

# Compute the XB index
FCM.XB = XB.IDX(scale(x), cmax = 15, cmin = 2, method = "FCM",
  fzm = 2, nstart = 20, iter = 100)
print(FCM.XB)

# The optimal number of cluster
FCM.XB[which.min(FCM.XB$XB),]

# ---- EM algorithm ----

# Compute the XB index
EM.XB = XB.IDX(scale(x), cmax = 15, cmin = 2, method = "EM",
  nstart = 20, iter = 100)
print(EM.XB)

# The optimal number of cluster
EM.XB[which.min(EM.XB$XB),]
```

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