

# Package ‘VBel’

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**Type** Package

**Title** Variational Bayes for Fast and Accurate Empirical Likelihood Inference

**Version** 1.1.7

**Description** Computes the Gaussian variational approximation of the Bayesian empirical likelihood posterior. This is an implementation of the function found in Yu, W., & Bondell, H. D. (2023) <[doi:10.1080/01621459.2023.2169701](https://doi.org/10.1080/01621459.2023.2169701)>.

**License** GPL (>= 3)

**Imports** Rcpp (>= 1.0.12), stats

**LinkingTo** Rcpp, RcppEigen

**Encoding** UTF-8

**RoxygenNote** 7.3.3

**URL** <https://github.com/jlimrasc/VBel>

**BugReports** <https://github.com/jlimrasc/VBel/issues>

**Suggests** mvtnorm, testthat (>= 3.0.0)

**Config/testthat/edition** 3

**NeedsCompilation** yes

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**Repository** CRAN

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 VBel-package

*Variational Bayes for Fast and Accurate Empirical Likelihood Inference*


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### Description

Computes the Gaussian variational approximation of the Bayesian empirical likelihood posterior. This is an implementation of the function found in Yu, W., & Bondell, H. D. (2023) <doi:10.1080/01621459.2023.2169701>.

### Author(s)

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### References

References: Yu, W., & Bondell, H. D. (2023), “Variational Bayes for Fast and Accurate Empirical Likelihood Inference”, *Journal of the American Statistical Association*, 119(546), 1089–1101. Page 3 doi:[10.1080/01621459.2023.2169701](https://doi.org/10.1080/01621459.2023.2169701)

### See Also

[compute\\_AEL\(\)](#) for C++ computation of AEL

[compute\\_GVA\(\)](#) for C++ computation of GVA

[diagnostic\\_plot\(\)](#) for verifying convergence of computed GVA data

### Examples

```
#ansGVARcppPure <- compute_GVA(mu, C_0, h, delthh, delth_logpi, z, lam0, rho,
#elip, a, SGD_iters, AEL_iters)
#diagnostic_plot(ansGVARcppPure)
```

---

 compute\_AEL

*Compute the Adjusted Empirical Likelihood*


---

### Description

Evaluates the Log-Adjusted Empirical Likelihood (AEL) (Chen, Variyath, and Abraham 2008) for a given data set, moment conditions and parameter values. The AEL function is formulated as

$$\log \text{AEL}(\boldsymbol{\theta}) = \max_{\mathbf{w}'} \sum_{i=1}^{n+1} \log(w'_i),$$

where  $\mathbf{z}_{n+1}$  is a pseudo-observation that satisfies

$$h(\mathbf{z}_{n+1}, \boldsymbol{\theta}) = -\frac{a_n}{n} \sum_{i=1}^n h(\mathbf{z}_i, \boldsymbol{\theta})$$

for some constant  $a_n > 0$  that may (but not necessarily) depend on  $n$ , and  $\mathbf{w}' = (w'_1, \dots, w'_n, w'_{n+1})$  is a vector of probability weights that define a discrete distribution on  $\{\mathbf{z}_1, \dots, \mathbf{z}_n, \mathbf{z}_{n+1}\}$ , and are subject to the constraints

$$\sum_{i=1}^{n+1} w'_i h(\mathbf{z}_i, \boldsymbol{\theta}) = 0, \quad \text{and} \quad \sum_{i=1}^{n+1} w'_i = 1.$$

Here, the maximizer  $\tilde{\mathbf{w}}$  is of the form

$$\tilde{w}_i = \frac{1}{n+1} \frac{1}{1 + \lambda_{\text{AEL}}^\top h(\mathbf{z}_i, \boldsymbol{\theta})},$$

where  $\lambda_{\text{AEL}}$  satisfies the constraints

$$\frac{1}{n+1} \sum_{i=1}^{n+1} \frac{h(\mathbf{z}_i, \boldsymbol{\theta})}{1 + \lambda_{\text{AEL}}^\top h(\mathbf{z}_i, \boldsymbol{\theta})} = 0, \quad \text{and} \quad \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{1}{1 + \lambda_{\text{AEL}}^\top h(\mathbf{z}_i, \boldsymbol{\theta})} = 1.$$

### Usage

```
compute_AEL(th, h, lam0, a, z, iters = 500)
```

### Arguments

th	p x 1 parameter vector to evaluate the AEL function at
h	User-defined moment-condition function. Note that output should be an n x K matrix where K is necessarily $\geq p$ . Input format for h should be (zi, th) where zi corresponds to the ith observation's data and th is the parameter vector
lam0	Initial vector for Lagrange multiplier lambda
a	Positive scalar adjustment constant
z	n x d data matrix. Note that $\{z_i\}_{i=1}^n$ is a sequence of d-dimensional data vectors
iters	Number of iterations using Newton-Raphson for estimation of lambda. Default: 500

### Details

Note that theta (th) is a p-dimensional vector, h is a K-dimensional vector and  $K \geq p$

### Value

A numeric value for the Adjusted Empirical Likelihood function computed evaluated at a given theta value

### Author(s)

Weichang Yu, Jeremy Lim

## References

Chen, J., Variyath, A. M., and Abraham, B. (2008), “Adjusted Empirical Likelihood and its Properties”, *Journal of Computational and Graphical Statistics*, 17, 426–443. Pages 2,3,4,5,6,7 [doi:10.1198/106186008X321068](https://doi.org/10.1198/106186008X321068)

## Examples

```
# Generating 30 data points from a simple linear-regression model
set.seed(1)
x   <- runif(30, min = -5, max = 5)
vari <- rnorm(30, mean = 0, sd = 1)
y   <- 0.75 - x + vari
z   <- cbind(x, y)

lam0 <- matrix(c(0,0), nrow = 2)
th   <- matrix(c(0.8277, -1.0050), nrow = 2)

# Specify AEL constant and Newton-Rhapson iteration
a   <- 0.00001
iters <- 10

# Specify moment condition functions for linear regression
h <- function(zi, th) {
  xi   <- zi[1]
  yi   <- zi[2]
  h_zith <- c(yi - th[1] - th[2] * xi, xi*(yi - th[1] - th[2] * xi))
  matrix(h_zith, nrow = 2)
}
result <- compute_AEL(th, h, lam0, a, z, iters)
```

---

compute\_GVA

*Compute the Full-Covariance Gaussian VB Empirical Likelihood Posterior*

---

## Description

Requires a given data set, moment conditions and parameter values and returns a list of the final mean and variance-covariance along with an array of the in-between calculations at each iteration for analysis of convergence

## Usage

```
compute_GVA(
  mu0,
  C0,
  h,
  delthh,
  delth_logpi,
  z,
```

```

    lam0,
    rho,
    epsil,
    a,
    SGD_iters = 10000,
    AEL_iters = 500,
    verbosity = 500
)

```

### Arguments

mu0	p x 1 initial vector of Gaussian VB mean
C0	p x p initial lower triangular matrix of Gaussian VB Cholesky
h	User-defined moment-condition function. Note that output should be an n x K matrix where K is necessarily $\geq$ p. Input format for h should be (zi, th) where zi corresponds to the ith observation's data and th is the parameter vector
delthh	User-defined first-order derivative of moment-condition function. Note that output should be a K x p matrix of h(zi,th) with respect to theta. Input format for h should be (zi, th) where zi corresponds to the ith observation's data and th is the parameter vector
delth_logpi	User-defined first-order derivative of log-prior function. Note that output should be a p x 1 vector. Input format for h should be (th) the parameter vector
z	Data matrix, n x d matrix
lam0	Initial vector for Lagrange multiplier lambda
rho	Scalar numeric between 0 to 1. ADADELTA accumulation constant
epsil	Positive numeric scalar stability constant. Should be specified with a small value
a	Positive scalar adjustment constant. For more accurate calculations, small values are recommended
SGD_iters	Number of Stochastic Gradient-Descent iterations for optimising mu and C. Default: 10,000
AEL_iters	Number of iterations using Newton-Raphson for optimising AEL lambda. Default: 500
verbosity	Integer for how often to print updates on current iteration number. To turn off, set to 0. Default:500

### Value

A list containing:

1. mu\_FC: VB Posterior Mean at final iteration. A vector of size p x 1
2. C\_FC: VB Posterior Variance-Covariance (Cholesky) at final iteration. A lower-triangular matrix of size p x p
3. mu\_FC\_arr: VB Posterior Mean for each iteration. A matrix of size p x (SGD\_iters + 1)
4. C\_FC\_arr: VB Posterior Variance-Covariance (Cholesky) for each iteration. An array of size p x p x (SGD\_iters + 1)

**Author(s)**

Weichang Yu, Jeremy Lim

**References**

Yu, W., & Bondell, H. D. (2023). "Variational Bayes for Fast and Accurate Empirical Likelihood Inference", *Journal of the American Statistical Association*, 1–13. doi:[10.1080/01621459.2023.2169701](https://doi.org/10.1080/01621459.2023.2169701)

**Examples**

```
# -----
# Initialise Inputs
# -----
# Generating 30 data points from a simple linear-regression model
set.seed(1)
x <- runif(30, min = -5, max = 5)
vari <- rnorm(30, mean = 0, sd = 1)
y <- 0.75 - x + vari
lam0 <- matrix(c(0,0), nrow = 2)
z <- cbind(x, y)

# Specify moment condition functions for linear regression and its corresponding derivative
h <- function(zi, th) {
  xi <- zi[1]
  yi <- zi[2]
  h_zith <- c(yi - th[1] - th[2] * xi, xi*(yi - th[1] - th[2] * xi))
  matrix(h_zith, nrow = 2)
}

delthh <- function(zi, th) {
  xi <- zi[1]
  matrix(c(-1, -xi, -xi, -xi^2), 2, 2)
}

# Specify derivative of log prior
delth_logpi <- function(theta) { -0.0001 * mu0 }

# Specify AEL constant and Newton-Rhapson iteration
a <- 0.00001
AEL_iters <- 10

# Specify initial values for GVA mean vector and Cholesky
reslm <- lm(y ~ x)
mu0 <- matrix(unname(reslm$coefficients),2,1)
C0 <- unname(t(chol(vcov(reslm))))

# Specify details for ADADELTA (Stochastic Gradient-Descent)
SGD_iters <- 50
epsil <- 10^-5
rho <- 0.9

# -----
```

```
# Main
# -----
result <-compute_GVA(mu0, C0, h, delthh, delth_logpi, z, lam0,
rho, epsil, a, SGD_iters, AEL_iters)
```

---

diagnostic\_plot      *Check the convergence of a data set computed by compute\_GVA*

---

### Description

Plots mu and variance in a time series plot to check for convergence of the computed data (i.e. Full-Covariance Gaussian VB Empirical Likelihood Posterior)

### Usage

```
diagnostic_plot(dataList, muList, cList)
```

### Arguments

dataList	Named list of data generated from <a href="#">compute_GVA</a>
muList	Vector of indices of mu_arr to plot. (default: if data resolution $(p) \leq 3$ , use all, else use 1, $\lfloor p/2 \rfloor$ and p)
cList	Matrix of indices of variance to plot, 2xn matrix, each row is (col,row) of variance matrix. (default: if data resolution $(p) \leq 3$ , use (1, 1) and (1, p), else use (1, p), $(\lfloor p/2 \rfloor, 1)$ and $(\lfloor p/2 \rfloor, \lfloor p/2 \rfloor + 1)$ )

### Value

Matrix of variance of C\_FC

### Examples

```
# -----
# Initialise Inputs
# -----
# Generating 30 data points from a simple linear-regression model
seedNum <- 100
set.seed(seedNum)
n      <- 100
p      <- 10
lam0   <- matrix(0, nrow = p)
mean   <- rep(1, p)
sigStar <- matrix(0.4, p, p) + diag(0.6, p)
z      <- rmvnorm::rmvnorm(n = n-1, mean = mean, sigma = sigStar)

# Specify moment condition functions for linear regression and its corresponding derivative
h      <- function(zi, th) { matrix(zi - th, nrow = 10) }
delthh <- function(z, th) { -diag(p) }
```

```

# Specify derivative of log prior
delth_logpi <- function(theta) {-0.0001 * theta}

# Specify AEL constant and Newton-Rhapson iteration
AEL_iters <- 5 # Number of iterations for AEL
a          <- 0.00001

# Specify initial values for GVA mean vector and Cholesky
zbar      <- 1/(n-1) * matrix(colSums(z), nrow = p)
mu_0      <- matrix(zbar, p, 1)

sumVal    <- matrix(0, nrow = p, ncol = p)
for (i in 1:p) {
  zi       <- matrix(z[i,], nrow = p)
  sumVal   <- sumVal + (zi - zbar) %*% matrix(zi - zbar, ncol = p)
}
sigHat    <- 1/(n-2) * sumVal
C_0       <- 1/sqrt(n) * t(chol(sigHat))

# Specify details for ADADELTA (Stochastic Gradient-Descent)
SGD_iters <- 5 # Number of iterations for GVA
epsil     <- 10^-5
rho       <- 0.9

# -----
# Main
# -----
# Compute GVA
ansGVA <- compute_GVA(mu_0, C_0, h, delthh, delth_logpi, z, lam0, rho, epsil,
a, SGD_iters, AEL_iters)

# Plot graphs
diagnostic_plot(ansGVA) # Plot all graphs
diagnostic_plot(ansGVA, muList = c(1,4)) # Limit number of graphs
diagnostic_plot(ansGVA, cList = matrix(c(1,1, 5,6, 3,3), ncol = 2)) # Limit number of graphs

```

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