

# Package ‘ald’

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**Type** Package

**Title** The Asymmetric Laplace Distribution

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**Description** It provides the density, distribution function, quantile function, random number generator, likelihood function, moments and Maximum Likelihood estimators for a given sample, all this for the three parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999). This is a special case of the skewed family of distributions available in Galarza et.al. (2017) <doi:10.1002/sta4.140> useful for quantile regression.

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## Contents

ald-package . . . . .	2
ALD . . . . .	3
likALD . . . . .	5
mleALD . . . . .	7
momentsALD . . . . .	8

<b>Index</b>	<b>11</b>
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ald-package

*The Asymmetric Laplace Distribution*

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## Description

It provides the density, distribution function, quantile function, random number generator, likelihood function, moments and Maximum Likelihood estimators for a given sample, all this for the three parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression.

## Details

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## Author(s)

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## References

- Koenker, R., Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. *J. Amer. Statist. Assoc.* 94(3):1296-1309.
- Yu, K. & Moyeed, R. (2001). Bayesian quantile regression. *Statistics & Probability Letters*, 54(4), 437-447.
- Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics-Theory and Methods*, 34(9-10), 1867-1879.

## See Also

[ALD,momentsALD,likALD,mleALD](#)

## Examples

```
## Let's plot an Asymmetric Laplace Distribution!  
  
##Density  
sseq = seq(-40,80,0.5)  
dens = dALD(y=sseq,mu=50,sigma=3,p=0.75)  
plot(sseq,dens,type="l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")
```

```

## Distribution Function
df = pALD(q=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,df,type="l",lwd=2,col="blue",xlab="x",ylab="F(x)", main="ALD Distribution function")
abline(h=1,lty=2)

##Inverse Distribution Function
prob = seq(0,1,length.out = 1000)
idf = qALD(prob=prob,mu=50,sigma=3,p=0.75)
plot(prob,idf,type="l",lwd=2,col="gray30",xlab="x",ylab=expression(F^{-1}~(x)))
title(main="ALD Inverse Distribution function")
abline(v=c(0,1),lty=2)

#Random Sample Histogram
sample = rALD(n=10000,mu=50,sigma=3,p=0.75)
hist(sample,breaks = 70,freq = FALSE,ylim=c(0,max(dens)),main="")
title(main="Histogram and True density")
lines(sseq,dens,col="red",lwd=2)

## Let's compute the MLE's

param = c(-323,40,0.9)
y = rALD(10000,mu = param[1],sigma = param[2],p = param[3]) #A random sample
res = mleALD(y)

#Comparing
cbind(param,res$par)

#Let's plot

seqq = seq(min(y),max(y),length.out = 1000)
dens = dALD(y=seqq,mu=res$par[1],sigma=res$par[2],p=res$par[3])
hist(y,breaks=50,freq = FALSE,ylim=c(0,max(dens)))
lines(seqq,dens,type="l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")

```

**Description**

Density, distribution function, quantile function and random generation for a Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to  $\mu$ , scale parameter  $\sigma$  and skewness parameter  $p$ . This is a special case of the skewed family of distributions in Galarza (2016) available in `lqr::SKD`.

**Usage**

```

dALD(y, mu = 0, sigma = 1, p = 0.5)
pALD(q, mu = 0, sigma = 1, p = 0.5, lower.tail = TRUE)
qALD(prob, mu = 0, sigma = 1, p = 0.5, lower.tail = TRUE)
rALD(n, mu = 0, sigma = 1, p = 0.5)

```

**Arguments**

<code>y, q</code>	vector of quantiles.
<code>prob</code>	vector of probabilities.
<code>n</code>	number of observations.
<code>mu</code>	location parameter.
<code>sigma</code>	scale parameter.
<code>p</code>	skewness parameter.
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ .

**Details**

If `mu`, `sigma` or `p` are not specified they assume the default values of 0, 1 and 0.5, respectively, belonging to the Symmetric Standard Laplace Distribution denoted by  $ALD(0, 1, 0.5)$ .

As discussed in Koenker and Machado (1999) and Yu and Moyeed (2001) we say that a random variable  $Y$  is distributed as an ALD with location parameter  $\mu$ , scale parameter  $\sigma > 0$  and skewness parameter  $p$  in  $(0,1)$ , if its probability density function (pdf) is given by

$$f(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp -\rho_p\left(\frac{y-\mu}{\sigma}\right)$$

where  $\rho_p(\cdot)$  is the so called check (or loss) function defined by

$$\rho_p(u) = u(p - I_{u < 0})$$

, with  $I$  denoting the usual indicator function. This distribution is denoted by  $ALD(\mu, \sigma, p)$  and its  $p$ -th quantile is equal to  $\mu$ .

The scale parameter `sigma` must be positive and non zero. The skew parameter `p` must be between zero and one ( $0 < p < 1$ ).

**Value**

`dALD` gives the density, `pALD` gives the distribution function, `qALD` gives the quantile function, and `rALD` generates a random sample.

The length of the result is determined by `n` for `rALD`, and is the maximum of the lengths of the numerical arguments for the other functions `dALD`, `pALD` and `qALD`.

**Note**

The numerical arguments other than `n` are recycled to the length of the result.

**Author(s)**

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## References

- Galarza Morales, C., Lachos Davila, V., Barbosa Cabral, C., and Castro Cepero, L. (2017) Robust quantile regression using a generalized class of skewed distributions. *Stat*,6: 113-130 doi: 10.1002/sta4.140.
- Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics-Theory and Methods*, 34(9-10), 1867-1879.

## See Also

[momentsALD](#),[likALD](#),[mleALD](#)

## Examples

```
## Let's plot an Asymmetric Laplace Distribution!

##Density
library(ald)
sseq = seq(-40,80,0.5)
dens = dALD(y=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,dens,type = "l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")

#Look that is a special case of the skewed family in Galarza (2017)
# available in lqr package, dSKD(...,sigma = 2*3,dist = "laplace")

## Distribution Function
df = pALD(q=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,df,type="l",lwd=2,col="blue",xlab="x",ylab="F(x)", main="ALD Distribution function")
abline(h=1,lty=2)

##Inverse Distribution Function
prob = seq(0,1,length.out = 1000)
idf = qALD(prob=prob,mu=50,sigma=3,p=0.75)
plot(prob,idf,type="l",lwd=2,col="gray30",xlab="x",ylab=expression(F^{-1}(x)))
title(main="ALD Inverse Distribution function")
abline(v=c(0,1),lty=2)

#Random Sample Histogram
sample = rALD(n=10000,mu=50,sigma=3,p=0.75)
hist(sample,breaks = 70,freq = FALSE,ylim=c(0,max(dens)),main="")
title(main="Histogram and True density")
lines(sseq,dens,col="red",lwd=2)
```

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likALD

*Log-Likelihood function for the Asymmetric Laplace Distribution*

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## Description

Log-Likelihood function for the Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to  $\mu$ , scale parameter  $\sigma$  and skewness parameter  $p$ .

**Usage**

```
likALD(y, mu = 0, sigma = 1, p = 0.5, loglik = TRUE)
```

**Arguments**

y	observation vector.
mu	location parameter $\mu$ .
sigma	scale parameter $\sigma$ .
p	skewness parameter $p$ .
loglik	logical; if TRUE (default), the Log-likelihood is return, if not just the Likelihood.

**Details**

If mu, sigma or p are not specified they assume the default values of 0, 1 and 0.5, respectively, belonging to the Symmetric Standard Laplace Distribution denoted by  $ALD(0, 1, 0.5)$ .

As discussed in Koenker and Machado (1999) and Yu and Moyeed (2001) we say that a random variable Y is distributed as an ALD with location parameter  $\mu$ , scale parameter  $\sigma > 0$  and skewness parameter  $p$  in  $(0,1)$ , if its probability density function (pdf) is given by

$$f(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp -\rho_p\left(\frac{y-\mu}{\sigma}\right)$$

where  $\rho_p(\cdot)$  is the so called check (or loss) function defined by

$$\rho_p(u) = u(p - I_{u < 0})$$

, with  $I$ , denoting the usual indicator function. Then the Log-likelihood function is given by

$$\sum_{i=1}^n \log\left(\frac{p(1-p)}{\sigma} \exp -\rho_p\left(\frac{y_i - \mu}{\sigma}\right)\right)$$

The scale parameter sigma must be positive and non zero. The skew parameter p must be between zero and one ( $0 < p < 1$ ).

**Value**

likeALD returns the Log-likelihood by default and just the Likelihood if loglik = FALSE.

**Author(s)**

Christian E. Galarza <<cgalarza88@gmail.com>> and Victor H. Lachos <<h1lachos@ime.unicamp.br>>

## References

- Koenker, R., Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. *J. Amer. Statist. Assoc.* 94(3):1296-1309.
- Yu, K. & Moyeed, R. (2001). Bayesian quantile regression. *Statistics & Probability Letters*, 54(4), 437-447.
- Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics-Theory and Methods*, 34(9-10), 1867-1879.

## See Also

[ALD](#), [momentsALD](#), [mleALD](#)

## Examples

```
## Let's compute the log-likelihood for a given sample

y = rALD(n=1000)
loglik = likALD(y)

#Changing the true parameters the loglik must decrease
loglik2 = likALD(y,mu=10,sigma=2,p=0.3)

loglik;loglik2
if(loglik>loglik2){print("First parameters are Better")}
```

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mleALD	<i>Maximum Likelihood Estimators (MLE) for the Asymmetric Laplace Distribution</i>
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## Description

Maximum Likelihood Estimators (MLE) for the Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to  $\mu$ , scale parameter  $\sigma$  and skewness parameter  $p$ .

## Usage

```
mleALD(y, initial = NA)
```

## Arguments

<code>y</code>	observation vector.
<code>initial</code>	optional vector of initial values $c(\mu, \sigma, p)$ .

## Details

The algorithm computes iteratively the MLE's via the combination of the MLE expressions for  $\mu$  and  $\sigma$ , and then maximizing with respect to  $p$  the Log-likelihood function (`likALD`) using the well known `optimize` R function. By default the tolerance is  $10^{-5}$  for all parameters.

**Value**

The function returns a list with two objects

`iter`                iterations to reach convergence.  
`par`                 vector of Maximum Likelihood Estimators.

**Author(s)**

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**References**

Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics-Theory and Methods*, 34(9-10), 1867-1879.

**See Also**

[ALD](#), [momentsALD](#), [likALD](#)

**Examples**

```
## Let's try this function

param = c(-323,40,0.9)
y = rALD(10000,mu = param[1],sigma = param[2],p = param[3]) #A random sample
res = mleALD(y)

#Comparing
cbind(param,res$par)

#Let's plot

seqq = seq(min(y),max(y),length.out = 1000)
dens = dALD(y=seqq,mu=res$par[1],sigma=res$par[2],p=res$par[3])
hist(y,breaks=50,freq = FALSE,ylim=c(0,max(dens)))
lines(seqq,dens,type="l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")
```

**Description**

Mean, variance, skewness, kurtosis, central moments w.r.t mu and first absolute central moment for the Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to mu, scale parameter sigma and skewness parameter p.

**Usage**

```

meanALD(mu=0, sigma=1, p=0.5)
varALD(mu=0, sigma=1, p=0.5)
skewALD(mu=0, sigma=1, p=0.5)
kurtALD(mu=0, sigma=1, p=0.5)
momentALD(k=1, mu=0, sigma=1, p=0.5)
absALD(sigma=1, p=0.5)

```

**Arguments**

k	moment number.
mu	location parameter $\mu$ .
sigma	scale parameter $\sigma$ .
p	skewness parameter $p$ .

**Details**

If mu, sigma or p are not specified they assume the default values of 0, 1 and 0.5, respectively, belonging to the Symmetric Standard Laplace Distribution denoted by  $ALD(0, 1, 0.5)$ .

As discussed in Koenker and Machado (1999) and Yu and Moyeed (2001) we say that a random variable Y is distributed as an ALD with location parameter  $\mu$ , scale parameter  $\sigma > 0$  and skewness parameter  $p$  in  $(0,1)$ , if its probability density function (pdf) is given by

$$f(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp -\rho_p\left(\frac{y-\mu}{\sigma}\right)$$

where  $\rho_p(\cdot)$  is the so called check (or loss) function defined by

$$\rho_p(u) = u(p - I_{u < 0})$$

, with  $I$  denoting the usual indicator function. This distribution is denoted by  $ALD(\mu, \sigma, p)$  and its  $p$ th quantile is equal to  $\mu$ . The scale parameter sigma must be positive and non zero. The skew parameter p must be between zero and one ( $0 < p < 1$ ).

**Value**

meanALD gives the mean, varALD gives the variance, skewALD gives the skewness, kurtALD gives the kurtosis, momentALD gives the  $k$ th central moment, i.e.,  $E(y - \mu)^k$  and absALD gives the first absolute central moment denoted by  $E|y - \mu|$ .

**Author(s)**

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**References**

- Koenker, R., Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. *J. Amer. Statist. Assoc.* 94(3):1296-1309.
- Yu, K. & Moyeed, R. (2001). Bayesian quantile regression. *Statistics & Probability Letters*, 54(4), 437-447.
- Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics-Theory and Methods*, 34(9-10), 1867-1879.

**See Also**

[ALD,likALD,mleALD](#)

**Examples**

```
## Let's compute some moments for a Symmetric Standard Laplace Distribution.

#Third raw moment
momentALD(k=3,mu=0,sigma=1,p=0.5)

#The well known mean, variance, skewness and kurtosis
meanALD(mu=0,sigma=1,p=0.5)
varALD(mu=0,sigma=1,p=0.5)
skewALD(mu=0,sigma=1,p=0.5)
kurtALD(mu=0,sigma=1,p=0.5)

# and this guy
absALD(sigma=1,p=0.5)
```

# Index

## \* **ALD**

ALD, 3  
likALD, 5  
mleALD, 7  
momentsALD, 8

## \* **Laplace**

ALD, 3  
likALD, 5  
mleALD, 7  
momentsALD, 8

## \* **Log-likelihood**

likALD, 5

## \* **MLE**

mleALD, 7

## \* **Maximum likelihood estimators**

mleALD, 7

## \* **asymmetric laplace distribution**

ALD, 3  
likALD, 5  
mleALD, 7  
momentsALD, 8

## \* **likelihood**

likALD, 5

## \* **moments**

momentsALD, 8

## \* **package**

ald-package, 2

## \* **quantile regression**

ALD, 3  
likALD, 5  
mleALD, 7  
momentsALD, 8

absALD (momentsALD), 8

ALD, 2, 3, 7, 8, 10

ald (ald-package), 2

ald-package, 2

dALD (ALD), 3

kurtALD (momentsALD), 8

likALD, 2, 5, 5, 8, 10

meanALD (momentsALD), 8

mleALD, 2, 5, 7, 7, 10

momentALD (momentsALD), 8

momentsALD, 2, 5, 7, 8, 8

pALD (ALD), 3

qALD (ALD), 3

rALD (ALD), 3

skewALD (momentsALD), 8

varALD (momentsALD), 8