

# Package ‘asymmetry.measures’

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**Title** Asymmetry Measures for Probability Density Functions

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**Maintainer** Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>

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**Imports** stats, sn, skewt, gamlss.dist

## Description

Provides functions and examples for the weak and strong density asymmetry measures in the articles: “A measure of asymmetry”, Patil, Patil and Bagkavos (2012) <[doi:10.1007/s00362-011-0401-6](https://doi.org/10.1007/s00362-011-0401-6)> and “A measure of asymmetry based on a new necessary and sufficient condition for symmetry”, Patil, Bagkavos and Wood (2014) <[doi:10.1007/s13171-013-0034-z](https://doi.org/10.1007/s13171-013-0034-z)>. The measures provided here are useful for quantifying the asymmetry of the shape of a density of a random variable. The package facilitates implementation of the measures which are applicable in a variety of fields including e.g. probability theory, statistics and economics.

**License** GPL (>= 2)

**NeedsCompilation** no

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**Author** Dimitrios Bagkavos [aut, cre],  
Lucia Gamez [aut]

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d.sample	<i>Switch between a range of probability density functions.</i>
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### Description

Returns the user-specified probability density function out of a range of available options evaluated at selected grid points.

### Usage

```
d.sample(s,dist, p1,p2)
```

### Arguments

s	A scalar or vector: the x-axis grid points where the probability density function will be evaluated.
dist	Character string, used as a switch to the user selected distribution function (see details below).
p1	A scalar. Parameter 1 (vector or object) of the selected density.
p2	A scalar. Parameter 2 (vector or object) of the selected density.

**Details**

Based on user-specified argument `dist`, the function returns the value of the probability density function at `s`.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd \sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2] \sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4] \sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(\text{mean1}, \text{sd1}, \text{mean2}, \text{sd2})$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \{f_t(s/p_1; p_2)I_{\{s \geq 0\}} + f_t(p_1 s; p_2)I_{\{s < 0\}}\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

### Value

A vector containing the user selected density values at the user specified points  $s$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds), Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

### See Also

[r.sample](#), [q.sample](#), [p.sample](#)

### Examples

```
selected.dens <- "weib" #select Weibull as the density
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point where the density is evaluated
d.sample(xout,selected.dens,shape,scale) # calculate density at xout
```

---

`edf`*Empirical cumulative distribution function*

---

**Description**

Empirical (nonparametric) cumulative distribution function for given a random sample.

**Usage**

```
edf(xin, xout)
```

**Arguments**

`xin` A vector of data points - the available sample.  
`xout` A vector of design points where the distribution function will be estimated.

**Details**

The empirical distribution function estimator at  $x$  is defined as the number of observations up to  $x$ , divided by  $n$ , i.e.

$$F_n(x) = \frac{\#\{X_1, \dots, X_n\} \leq x}{n}$$

**Value**

A vector with the estimated distribution function at `xout`.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation:

Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> , Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Hollander, M. and Wolfe, D.A. (1999), *Nonparametric Statistical Methods*, 2nd edition, Wiley.

**Examples**

```
x.in <- rexp(200)
x.out <- seq(0.1, 5, length=60)
dist.est <- edf(x.in, x.out)
plot(x.out, dist.est, col="blue", main="Empirical c.d.f.", xlab="x", ylab="probability")
```

---

Epanechnikov

*Epanechnikov kernel*

---

### Description

Implementation of the Epanechnikov kernel.

### Usage

`Epanechnikov(x)`

### Arguments

`x` A vector of data points between  $-\sqrt{5}$  and  $\sqrt{5}$  where the kernel will be evaluated.

### Details

Implements:

$$K(u) = \frac{3}{4\sqrt{5}} \left(1 - \frac{x^2}{5}\right)$$

for  $|x| \leq \sqrt{5}$

### Value

The value of the kernel at  $x$

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

[Kernel Statistics](#)

### See Also

[IntEpanechnikov](#)

---

eta.s                                      *Strong asymmetry measure eta(X).*

---

### Description

Returns the strong asymmetry measure  $\eta(X)$  of [Patil, Bagkavos and Wood \(2014\)](#).

### Usage

```
eta.s(xin, dist, GridLength, p1, p2)
```

### Arguments

xin	A vector of data points - the available sample.
dist	Character string, specifies selected distribution function.
GridLength	A non-negative number, which will be rounded up if fractional. Desired length of the sequence.
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

### Details

Implements

$$\eta(X) = -0.5 \operatorname{sign}(\rho_1) \max |\rho_p + \rho_p^*|$$

with  $1/2 \leq p \leq 1$ .

Uses maximum likelihood estimates for the unknown functionals in the definition of the measure.

### Value

Returns a scalar, the value of the strong asymmetry measure  $\eta(X)$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

- [Patil P.N., Bagkavos D. and Wood A.T.A., \(2014\). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, Sankhya A, 76, 123–145.](#)
- [Bagkavos D., Patil P.N., Wood A.T.A. \(2016\), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. \(eds\), Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.](#)

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
selected.dist <- "norm" #select norm as the distribution
m.use <- mean(GDP.Per.head.dist.2005)
sd.use<- sd(GDP.Per.head.dist.2005)
grid <- 50

s.use<- GDP.Per.head.dist.1995
eta.s(GDP.Per.head.dist.2005,selected.dist,grid,m.use,sd.use)
```

---

eta.s.exact

*Strong asymmetry measure  $\eta(X)$ .*

---

**Description**

Returns the strong asymmetry measure  $\eta(X)$  of [Patil, Bagkavos and Wood \(2014\)](#).

**Usage**

```
eta.s.exact(xin, dist, GridLength, p1, p2)
```

**Arguments**

xin	A vector of data points - the available sample.
dist	Character string, specifies selected distribution function.
GridLength	A non-negative number, which will be rounded up if fractional.Desired length of the sequence.
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

**Details**

Implements

$$\eta(X) = -0.5 \text{sign}(\rho_1) \max |\rho_p + \rho_p^*|$$

with  $1/2 \leq p \leq 1$  This version uses exact p.d.f. and c.d.f. evaluation and not estimates of the unknown functionals.

**Value**

Returns a scalar, the value of the strong asymmetry measure  $\eta(X)$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., González Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#), [eta.s](#)

**Examples**

```
selected.dist <- "norm" #select norm as the distribution
m.use <- 2
sd.use <- 2
grid <- 50
s.use <- rnorm(100)
eta.s.exact(s.use, selected.dist, grid, m.use, sd.use) # calculate eta.s at xout
```

---

eta.w.breve

*Asymmetry coefficient  $\check{\eta}$*

---

**Description**

Implements the asymmetry coefficient  $\check{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.breve(xin, kfun)
```

**Arguments**

`xin` A vector of data points - the available sample.  
`kfun` The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\check{\eta}$  is defined by

$$\check{\eta} = -\frac{\sum_{i=1}^n U_i W_i - n\bar{U}\bar{W}}{\sqrt{(\sum_{i=1}^n U_i^2 - n\bar{U}^2)(\sum_{i=1}^n W_i^2 - n\bar{W}^2)}}$$

where

$$U_i = \hat{f}(X_i), W_i = F_n(X_i), \bar{U} = n^{-1} \sum_{i=1}^n U_i, \bar{W} = n^{-1} \sum_{i=1}^n W_i.$$

**Value**

Returns a scalar, the estimate of  $\check{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971–985.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.breve(GDP.Per.head.dist.1995,Epanechnikov)
0.329707 #estimate of etabreve
```

eta.w.breve.bc

Asymmetry coefficient  $\check{\eta}$  using boundary correction**Description**

Implements the asymmetry coefficient  $\check{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.breve.bc(xin, kfun)
```

**Arguments**

`xin` A vector of data points - the available sample.  
`kfun` The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ .  $\check{\eta}$  is defined by

$$\check{\eta} = - \frac{\sum_{i=1}^n U_i W_i - n \bar{U} \bar{W}}{\sqrt{(\sum_{i=1}^n U_i^2 - n \bar{U}^2)(\sum_{i=1}^n W_i^2 - n \bar{W}^2)}}$$

where

$$U_i = \hat{f}(X_i), W_i = F_n(X_i), \bar{U} = n^{-1} \sum_{i=1}^n U_i, \bar{W} = n^{-1} \sum_{i=1}^n W_i.$$

eta.w.breve.bc uses reflection to correct the boundary bias of the kernel density estimate kde

**Value**

Returns a scalar, the estimate of  $\check{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
 Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

[Patil, P.N., Patil, P.P. and Bagkavos, D., \(2012\), A measure of asymmetry. Stat. Papers, 53, 971-985.](#)

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.breve.bc(GDP.Per.head.dist.1995,Epanechnikov)
0.329707 #estimate of etabreve
```

---

eta.w.hat	<i>Asymmetry coefficient <math>\hat{\eta}</math></i>
-----------	--

---

**Description**

Implements the asymmetry coefficient  $\hat{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.hat(xin, kfun)
```

**Arguments**

`xin`                    A vector of data points - the available sample.  
`kfun`                    The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\hat{\eta}$  is defined by

$$\hat{\eta} = -\frac{\sum_{i=1}^n U_i V_i - n\bar{U}\bar{V}}{\sqrt{(\sum_{i=1}^n U_i^2 - n\bar{U}^2)(\sum_{i=1}^n V_i^2 - n\bar{V}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad V_i = \hat{F}(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

**Value**

Returns a scalar, the estimate of  $\hat{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971-985.

**See Also**

[eta.w.hat.bc](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.hat(GDP.Per.head.dist.1995,Epanechnikov)
0.3463025 #estimate of etahat
```

---

eta.w.hat.bc

*Asymmetry coefficient  $\hat{\eta}$  using boundary correction*

---

**Description**

Implements the asymmetry coefficient  $\hat{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#)

**Usage**

```
eta.w.hat.bc(xin, kfun)
```

**Arguments**

`xin`                    A vector of data points - the available sample.  
`kfun`                    The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\hat{\eta}$  is defined by

$$\hat{\eta} = -\frac{\sum_{i=1}^n U_i V_i - n\bar{U}\bar{V}}{\sqrt{(\sum_{i=1}^n U_i^2 - n\bar{U}^2)(\sum_{i=1}^n V_i^2 - n\bar{V}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad V_i = \hat{F}(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

`eta.w.hat.bc` uses reflection to correct the boundary bias issue of the kernel estimate `kde`.

**Value**

Returns a scalar, the estimate of  $\hat{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. *Stat. Papers*, 53, 971-985.

**See Also**

[eta.w.hat](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.hat.bc(GDP.Per.head.dist.1995,Epanechnikov)
0.3463025 #estimate of etahat.bc
```

---

eta.w.tilde

*Asymmetry coefficient  $\tilde{\eta}$*

---

**Description**

Implements the asymmetry coefficient  $\tilde{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.tilde(xin, kfun)
```

**Arguments**

`xin` A vector of data points - the available sample.  
`kfun` The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ .  $\tilde{\eta}$  is defined by

$$\tilde{\eta} = -\frac{\sum_{i=1}^n U_i V_i - (n/2)\bar{U}}{\sqrt{(n/12)(\sum_{i=1}^n U_i^2 - n\bar{U}^2)}}$$

where

$$U_i = \hat{f}(X_i), V_i = F(X_i), \bar{U} = n^{-1} \sum_{i=1}^n U_i, \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

**Value**

Returns a scalar, the estimate of  $\tilde{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971-985.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve.bc](#), [eta.w.breve](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.tilde(GDP.Per.head.dist.1995,Epanechnikov)  
0.3333485 #estimate of etatile
```

---

eta.w.tilde.bc	<i>Asymmetry coefficient <math>\tilde{\eta}</math> using boundary correction</i>
----------------	--

---

**Description**

Implements the asymmetry coefficient  $\tilde{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.tilde.bc(xin, kfun)
```

**Arguments**

xin	A vector of data points - the available sample.
kfun	The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\tilde{\eta}$  is defined by

$$\tilde{\eta} = -\frac{\sum_{i=1}^n U_i V_i - (n/2)\bar{U}}{\sqrt{(n/12)(\sum_{i=1}^n U_i^2 - n\bar{U}^2)}}$$

where

$$U_i = \hat{f}(X_i), V_i = F(X_i), \bar{U} = n^{-1} \sum_{i=1}^n U_i, \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

eta.w.tilde.bc uses reflection to correct the boundary bias of kde.

**Value**

Returns a scalar, the estimate of  $\tilde{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. *Stat. Papers*, 53, 971-985.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve.bc](#), [eta.w.breve](#), [eta.w.tilde](#)

**Examples**

```
eta.w.tilde.bc(GDP.Per.head.dist.1995,Epanechnikov)
0.3333485 #estimate of etatile.bc
```

---

GDP.Per.head.dist.1995

*annual Gross Domestic Product (GDP) per head across 15 European Union (EU) countries*

---

**Description**

Contains values of the GDP/head distribution of 216 EU regions (the so called NUTS-2 level of the Eurostat categorization of territories within the EU for the year 1995).

**Usage**

GDP.Per.head.dist.1995

**Format**

A vector with 184 values of the GDP/head distribution for 1995.

**Source**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators, Directorate-General for Regional Policy 1/2008.

**References**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators

**See Also**

[GDP.Per.head.dist.2005](#)

---

GDP.Per.head.dist.2005

*annual Gross Domestic Product (GDP) per head across 15 European Union (EU) countries*

---

**Description**

Contains values of the GDP/head distribution of 216 EU regions (the so called NUTS-2 level of the Eurostat categorization of territories within the EU for the year 2005.

**Usage**

GDP.Per.head.dist.1995

**Format**

A vector with 184 values of the GDP/head distribution for 2005.

**Source**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators, Directorate-General for Regional Policy 1/2008.

**References**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators

**See Also**[GDP.Per.head.dist.1995](#)

---

IntEpanechnikov	<i>Integrated Epanechnikov function</i>
-----------------	---

---

**Description**

Implements the Integrated Epanechnikov kernel.

**Usage**

```
IntEpanechnikov(x)
```

**Arguments**

`x` A vector of design points with values from  $-\sqrt{5}$  to  $\sqrt{5}$ .

**Details**

Implements:

$$K(u) = \int_{-\infty}^u \frac{3}{4\sqrt{5}} \left(1 - \frac{x^2}{5}\right) dx$$

for  $|x| \leq \sqrt{5}$

**Value**

The value of the integrated kernel function at the user designated points.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

[Kernel Statistics](#)

**See Also**

[Epanechnikov](#)

---

IntKde                      *Integrated Kernel density estimator*

---

### Description

Classical univariate integrated kernel density estimator

### Usage

```
IntKde(xin, xout, h, kfun)
```

### Arguments

xin	A vector of data points - the available sample size.
xout	grid points where the distribution function will be estimated.
h	The bandwidth parameter. Defaults to $3.572 * \sigma * n^{-1/3}$ according to Bowman et al.(1998).
kfun	The kernel to use in the distribution function estimate.

### Details

It implements the classical density integrated kernel estimator.

Let  $X_1, X_2, \dots, X_n$  be a univariate independent and identically distributed sample drawn from some unknown distribution function  $F$ . Its kernel density estimator is

$$\hat{F}(x) = n^{-1} \sum_{i=1}^n K \{ (x - X_i)h^{-1} \}$$

where  $K$  is an integrated kernel, and  $h > 0$  is a smoothing parameter called the bandwidth.

### Value

Returns a vector with the estimate of the distribution function at the user specified grid points.

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Bowman, A., Hall, P., and Prvan, T., (1998), Bandwidth Selection for the Smoothing of Distribution Functions, *Biometrika*, 799-808.

**See Also**

[bw.nrd](#), [bw.nrd0](#), [bw.ucv](#), [bw.bcv](#)

**Examples**

```
x.in <- rnorm(100)
x.out <- seq(-3.4,3.4,length=60)
kernel <- IntEpanechnikov
dist.est <- IntKde(xin=x.in,xout=x.out,kfun=kernel)
plot(x.out,dist.est, type="l", col="red", main="Kernel c.d.f. estimator")
```

---

kde	<i>Kernel density estimator.</i>
-----	----------------------------------

---

**Description**

Classical univariate kernel density estimator.

**Usage**

```
kde(xin, xout, h, kfun)
```

**Arguments**

xin	A vector of data points. Missing values not allowed.
xout	A vector of grid points at which the estimate will be calculated.
h	A scalar, the bandwidth to use in the estimate, e.g. <code>bw.nrd(xin)</code> .
kfun	Kernel function to use.

**Details**

Implements the classical density kernel estimator based on a sample  $X_1, X_2, \dots, X_n$  of i.i.d observations from a distribution  $F$  with density  $h$ . The estimator is defined by

$$\hat{f}(x) = n^{-1} \sum_{i=1}^n K_h(x - X_i)$$

where  $h$  is determined by a bandwidth selector such as Silverman's default plug-in rule and  $K$ , the kernel, is a non-negative probability density function.

**Value**

A vector with the density estimates at the designated points `xout`.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Silverman, B.W. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, London.

**See Also**

[bw.nrd](#), [bw.nrd0](#), [bw.ucv](#), [bw.bcv](#)

**Examples**

```
x.in <- rnorm(100)
x.out <- seq(-3.4,3.4,length=60)
bandwidth <- bw.nrd(x.in)
kernel <- Epanechnikov
dens.est <- kde(x.in,x.out,bandwidth,kernel)
plot(x.out,dens.est,col="red",main="Kernel density estimator")
```

---

p.sample

*Switch between a range of available cumulative distribution functions.*

---

**Description**

Returns the value of the selected cumulative distribution function at user supplied grid points.

**Usage**

```
p.sample(s,dist, p1,p2)
```

**Arguments**

s	A scalar or vector: the x-axis grid points where the cumulative distribution function is be evaluated.
dist	Character string, used as a switch to the user selected distribution function (see details below).
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

## Details

Based on the user-specified argument `dist`, the function returns the value of the cumulative distribution function at `s`.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The Weibull distribution is implemented as

$$F(s) = 1 - \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s > 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$F(s) = \Phi \left( \frac{\ln s - p_1}{p_2} \right)$$

where  $p_1$  is the mean,  $p_2$  is the standard deviation and  $\Phi$  is the cumulative distribution function of the standard normal distribution.

- `norm`: The normal distribution is implemented as

$$\Phi(s) = \frac{1}{\sqrt{2\pi}p_2} \int_{-\infty}^s e^{-\frac{(t-p_1)^2}{2p_2^2}} dt$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation.

- `uni`: The uniform distribution is implemented as

$$F(s) = \frac{s - p_1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$F(s; p_1, p_2) = \frac{1}{\pi} \arctan \left( \frac{s - p_1}{p_2} \right) + \frac{1}{2}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$F_S(s; \sigma) = \int_0^s \frac{\sqrt{2/\pi}}{\sigma} \exp \left\{ -\frac{x^2}{2\sigma^2} \right\} dx$$

where  $mean = 0$  and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$F(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} \int_{-\infty}^s e^{-\frac{(t-p_2[1])^2}{2p_2[2]^2}} dt + (1 - p_1) \frac{1}{p_2[4]\sqrt{2\pi}} \int_{-\infty}^s e^{-\frac{(t-p_2[3])^2}{2p_2[4]^2}} dt$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution is implemented as

$$F(y; p_1) = \Phi\left(\frac{y - \xi}{\omega}\right) - 2T\left(\frac{y - \xi}{\omega}, p_1\right)$$

where *location* =  $\xi = 0$ , *scale* =  $\omega = 1$ , *parameter* =  $p_1$  and  $T(h, a)$  is the Owens T function, defined by

$$T(h, a) = \frac{1}{2\pi} \int_0^a \exp\left\{\frac{-0.5h^2(1+x^2)}{1+x^2}\right\} dx, -\infty \leq h, a \leq \infty$$

- fas: The Fernandez and Steel distribution is implemented as

$$F(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ \int_{-\infty}^s f_t(x/p_1; p_2) I_{\{x \geq 0\}} dx + \int_{-\infty}^s f_t(p_1 x; p_2) I_{\{x < 0\}} dx \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the t distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  is the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$F(s; \mu, p_2, p_1, \tau) = \int_{-\infty}^s \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2} dz$$

where  $r = \sinh(\sinh(z) - p_1)$ ,  $c = \cosh(\sinh(z) - p_1)$  and  $z = (s - \mu)/p_2$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector containing the cumulative distribution function values at the user specified points  $s$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

## See Also

[r.sample](#), [q.sample](#), [d.sample](#)

**Examples**

```

selected.d <- "weib" #select Weibull as the CDF
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point where the CDF is evaluated
p.sample(xout,selected.d,shape,scale) # calculate CDF at xout

```

pdfsq

*Calculate  $f^2(x)$* **Description**

Calculates the square of a density.

**Usage**

```
pdfsq(s,dist, p1,p2)
```

**Arguments**

s	A scalar or vector: the x-axis grid points where the probability density function will be evaluated.
dist	Character string, used as a switch to the user selected distribution function (see details below).
p1	A scalar. Parameter 1 (vector or object) of the selected density.
p2	A scalar. Parameter 2 (vector or object) of the selected density.

**Details**

Based on user-specified argument `dist`, the function returns the value of  $f^2(x)dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- norm: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2\sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- uni: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- cauchy: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- fnorm: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- normmixt: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(\text{mean1}, \text{sd1}, \text{mean2}, \text{sd2})$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ f_t(s/p_1; p_2) I_{\{s \geq 0\}} + f_t(p_1 s; p_2) I_{\{s < 0\}} \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

**Value**

A vector containing the user selected density values at the user specified points  $s$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

**See Also**

[r.sample](#), [q.sample](#), [p.sample](#)

**Examples**

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfsq(xout,selected.dens,shape,scale) # calculate the square density at xout
```

---

pdfsqcdf

*Calculate  $f^2(x)F(x)$*

---

**Description**

Return the product  $f^2(x)F(x)$

**Usage**

```
pdfsqcdf(s,dist, p1,p2)
```

**Arguments**

<code>s</code>	A scalar or vector: the x-axis grid points where the probability density function will be evaluated.
<code>dist</code>	Character string, used as a switch to the user selected distribution function (see details below).
<code>p1</code>	A scalar. Parameter 1 (vector or object) of the selected density.
<code>p2</code>	A scalar. Parameter 2 (vector or object) of the selected density.

### Details

Based on user-specified argument `dist`, the function returns the value of  $\int f^2(x)F(x)dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left(\frac{s}{p_2}\right)^{p_1-1} \exp\left\{-\left(\frac{s}{p_2}\right)^{p_1}\right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{1 + \left(\frac{s-p_1}{p_2}\right)^2\right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \{f_t(s/p_1; p_2)I_{\{s \geq 0\}} + f_t(p_1 s; p_2)I_{\{s < 0\}}\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

### Value

A vector containing the user selected density values at the user specified points  $s$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

### See Also

[r.sample](#), [q.sample](#), [p.sample](#)

### Examples

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfsqcdf(xout,selected.dens,shape,scale) # calculate pdfsqcdf function at xout
```

---

pdfsqcdfstar                      Calculate  $f^2(x)(1 - F(x))$ .

---

### Description

Return the product  $f^2(x)(1 - F(x))$ .

### Usage

pdfsqcdfstar(s,dist, p1,p2)

### Arguments

s	A scalar or vector: the x-axis grid points where the probability density function will be evaluated.
dist	Character string, used as a switch to the user selected distribution function (see details below).
p1	A scalar. Parameter 1 (vector or object) of the selected density.
p2	A scalar. Parameter 2 (vector or object) of the selected density.

### Details

Based on user-specified argument `dist`, the function returns the value of  $f^2(x)(1 - F(x))dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- weib: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- lognorm: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- norm: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- uni: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- cauchy: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- fnorm: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- normmixt: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(\text{mean1}, \text{sd1}, \text{mean2}, \text{sd2})$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ f_t(s/p_1; p_2) I_{\{s \geq 0\}} + f_t(p_1 s; p_2) I_{\{s < 0\}} \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

**Value**

A vector containing the user selected density values at the user specified points  $s$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

**See Also**

[r.sample](#), [q.sample](#), [p.sample](#)

**Examples**

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfsqcdfstar(xout,selected.dens,shape,scale) #return f^2(xout)F(xout)
```

---

pdfthird

*Calculate  $f^3(x)$*

---

**Description**

Return the value of  $f^3(x)$ .

**Usage**

```
pdfthird(s,dist, p1,p2)
```

**Arguments**

$s$	A scalar or vector: the x-axis grid points where the probability density function will be evaluated.
$dist$	Character string, used as a switch to the user selected distribution function (see details below).
$p1$	A scalar. Parameter 1 (vector or object) of the selected density.
$p2$	A scalar. Parameter 2 (vector or object) of the selected density.

### Details

Based on user-specified argument `dist`, the function returns the value of  $f^3(x)dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \{f_t(s/p_1; p_2)I_{\{s \geq 0\}} + f_t(p_1 s; p_2)I_{\{s < 0\}}\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

### Value

A vector containing the user selected density values at the user specified points  $s$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

### See Also

[pdfsq](#), [pdfsqcdf](#), [pdfsqcdfstar](#)

### Examples

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfthird(xout,selected.dens,shape,scale) # calculate density to the cube at xout
```

---

q.sample

Switch between a range of available quantile functions.

---

### Description

Returns the quantiles of selected distributions at user specified locations.

### Usage

```
q.sample(s,dist, p1=0,p2=1)
```

### Arguments

s	A scalar or vector: the probabilities where the quantile function will be evaluated.
dist	Character string, used as a switch to the user selected distribution function (see details below).
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

### Details

Based on user-specified argument `dist`, the function returns the value of the quantile function at `s`. Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The quantile function for the weibull distribution is implemented as

$$Q(s) = p_1(-\log(1-s))^{1/p_2}$$

where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution has quantile function implemented as

$$Q(s) = \exp \left\{ p_1 + \sqrt{2p_2^2} \operatorname{erf}^{-1}(2s-1) \right\}$$

where  $p_1$  is the mean,  $p_2$  is the standard deviation and `erf` is the Gauss error function.

- `norm`: The normal distribution has quantile function implemented as

$$Q(p) = \Phi^{-1}(s; p_1, p_2)$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation.

- `uni`: The uniform distribution has quantile function implemented as

$$Q(s; p_1, p_2) = s(p_2 - p_1) + p_1$$

for  $p_1 < s < p_2$ .

- **cauchy**: The cauchy distribution has quantile function implemented as

$$Q(s) = p_1 + p_2 \tan \left\{ \pi \left( s - \frac{1}{2} \right) \right\}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- **fnorm**: The half normal distribution has quantile function implemented as

$$Q(s) = p_1 \sqrt{2} \operatorname{erf}^{-1}(s)$$

where and  $p_1$  is the standard deviation of the distribution.

- **normmix**: The quantile function normal mixture distribution is estimated numerically, based on the built in quantile function.
- **skewnorm**: There is no closed form expression for the quantile function of the skew normal distribution. For this reason, the quantiles are calculated through the `qsn` function of the `sn` package.
- **fas**: There is no closed form expression for the quantile function of the Fernandez and Steel distribution. For this reason, the quantiles are calculated through the `qskt` function of the `skewt` package.
- **shash**: There is no closed form expression for the quantile function of the Sinh-Arcsinh distribution. For this reason, the quantiles are calculated through the `qSHAShO` function of the `gamlss` package.

### Value

A vector containing the quantile values at the user specified points `s`.

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

### See Also

[r.sample](#), [d.sample](#), [p.sample](#)

### Examples

```
selected.q <- "norm" #select Normal as the distribution
shape <- 2 # specify shape parameter
scale <- 2 # specify scale parameter
xout <- seq(0.1,1,length=50) #design point where the quantile function is evaluated
q.sample(xout,selected.q,shape,scale) # calculate quantiles at xout
```

---

r.sample                      *Switch between a range of available random number generators.*

---

### Description

Generate a random sample of size  $n$  out of a range of available distributions.

### Usage

```
r.sample(s, dist, p1=0, p2=1)
```

### Arguments

**s**                      A scalar which specifies the size of the random sample drawn.

**dist**                  Character string, used as a switch to the user selected distribution function (see details below).

**p1**                    A scalar. Parameter 1 (vector or object) of the selected distribution.

**p2**                    A scalar. Parameter 2 (vector or object) of the selected distribution.

### Details

Based on user-specified argument `dist`, the function returns a random sample of size  $s$  from the corresponding distribution.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- cauchy: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- fnorm: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- normmixt: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(\text{mean1}, \text{sd1}, \text{mean2}, \text{sd2})$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ f_t(s/p_1; p_2) I_{\{s \geq 0\}} + f_t(p_1 s; p_2) I_{\{s < 0\}} \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector of random values at the user specified points  $s$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

**See Also**

[d.sample](#), [q.sample](#), [p.sample](#)

**Examples**

```
selected.r <- "norm" #select Normal as the distribution
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
r.sample(n,selected.r,shape,scale) # calculate CDF at the designated point
```

---

Rho.p	<i>Calculates <math>\rho_p</math>, used in the implementation of the strong asymmetry measure <math>\eta(X)</math>.</i>
-------	---

---

**Description**

Estimates  $\rho_p$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

**Usage**

```
Rho.p(xin, p.param, dist, p1=0, p2=1)
```

**Arguments**

xin	A vector of data points - the available sample.
p.param	A parameter with the value greater than or equal to 1/2 and less than 1.
dist	Character string, specifies selected distribution function.
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

**Details**

Implements the quantity:

$$\frac{2\sqrt{3} - \int_{-\infty}^{\xi_p} f^2(x)F(x) dx - \frac{p}{2} \int_{-\infty}^{\xi_p} f^2(x) dx}{p \left\{ p \int_{-\infty}^{\xi_p} f^3(x) dx - \left( \int_{-\infty}^{\xi_p} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 Patil, Bagkavos and Wood, see also (4) in Bagkavos, Patil and Wood . Estimation of the p.d.f. and c.d.f. functions is currently performed by maximum likelihood as e.g. kernel estimates inherit large amount of variance to  $\rho_p$ .

**Value**

Returns a scalar, the value of  $\rho_p$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., González Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

**See Also**

[Rho.p.exact](#), [Rhostar.p](#), [Rhostar.p.exact](#)

**Examples**

```
set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rho.p(xout,param,selected.r,shape,scale) # calculate Rho.p
#-0.06665222 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
```

```

param <- 0.9 # specify parameter
xout <- r.sample(n,selected.r2,mean,sd) # specify sample
Rho.p(xout,param,selected.r2,mean,sd) # calculate Rho.p
#-0.1005591 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
Rho.p(xout,param,selected.r3,location,scale) # calculate Rho.p
#-0.0580943 # returns the result

```

---

Rho.p.exact	<i>Calculates the exact value <math>\rho_p</math>, used in the implementation of the strong asymmetry measure <math>\eta(X)</math>.</i>
-------------	---

---

### Description

Returns  $\rho_p$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

### Usage

```
Rho.p.exact(xin, p.param, dist, p1=0, p2=1)
```

### Arguments

xin	A vector of data points - the available sample.
p.param	A parameter with the value greater than or equal to 1/2 and less than 1.
dist	Character string, specifies selected distribution function.
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

### Details

Implements the quantity:

$$\frac{2\sqrt{3} - \int_{-\infty}^{\xi_p} f^2(x)F(x) dx - \frac{p}{2} \int_{-\infty}^{\xi_p} f^2(x) dx}{p \left\{ p \int_{-\infty}^{\xi_p} f^3(x) dx - \left( \int_{-\infty}^{\xi_p} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood](#), see also (4) in [Bagkavos, Patil and Wood](#). This implementation uses exact calculation of the functionals in the definition of  $\rho_p$ .

**Value**

Returns a scalar, the exact value of  $\rho_p$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

**See Also**

[Rho.p](#), [Rhostar.p](#), [Rhostar.p.exact](#)

**Examples**

```
set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rho.p.exact(xout,param,selected.r,shape,scale) # calculate Rho.p.exact
#-0.06665222 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
param <- 0.9 # specify parameter
xout <-r.sample(n,selected.r2,mean,sd) # specify sample
Rho.p.exact(xout,param,selected.r2,mean,sd) # calculate Rho.p.exact
#-0.2384271 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
```

```
Rho.p.exact(xout,param,selected.r3,location,scale) # calculate Rho.p.exact
#-0.02340374 # returns the result
```

---

Rhostar.p	<i>Calculates <math>\rho_p^*</math>, used in the implementation of the strong asymmetry measure <math>\eta(X)</math>.</i>
-----------	---

---

### Description

Estimates  $\rho_p^*$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

### Usage

```
Rhostar.p(xin, p.param, dist, p1, p2)
```

### Arguments

xin	A vector of data points - the available sample.
p.param	A parameter with the value greater than or equal to 1/2 and less than 1.
dist	Character string, specifies selected distribution function.
p1	A scalar. Parameter 1 (vector or object) of the selected distribution.
p2	A scalar. Parameter 2 (vector or object) of the selected distribution.

### Details

Implements the quantity

$$\frac{2\sqrt{3} - \int_{\xi_{1-p}}^{\infty} f^2(x)(1 - F(x)) dx + \frac{p}{2} \int_{\xi_{1-p}}^{\infty} f^2(x) dx}{p \left\{ p \int_{\xi_{1-p}}^{\infty} f^3(x) dx - \left( \int_{\xi_{1-p}}^{\infty} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood \(2014\)](#), see also (5) in [Bagkavos, Patil and Wood \(2016\)](#). Estimation of the p.d.f. and c.d.f. functions is currently performed by maximum likelihood as e.g. kernel estimates inherit large amount of variance to  $\rho_p^*$ .

### Value

Returns a scalar, the value of  $\rho_p^*$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

## See Also

[Rho.p](#), [Rhostar.p.exact](#), [Rho.p.exact](#)

## Examples

```
set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rhostar.p(xout,param,selected.r,shape,scale) # calculate Rhostar.p
#-0.08936363 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
param <- 0.9 # specify parameter
xout <-r.sample(n,selected.r2,mean,sd) # specify sample
Rhostar.p(xout,param,selected.r2,mean,sd) # calculate Rhostar.p
#-0.02302223 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
Rhostar.p(xout,param,selected.r3,location,scale) # calculate Rhostar.p
#0.02043852 # returns the result
```

---

Rhostar.p.exact

*Calculates the exact value of  $p_{-p}^*$ , used in the implementation of the strong asymmetry measure  $\eta(X)$ .*

---

**Description**

Returns  $\rho_p^*$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

**Usage**

```
Rhostar.p.exact(xin, p.param, dist, p1, p2)
```

**Arguments**

<code>xin</code>	A vector of data points - the available sample.
<code>p.param</code>	A parameter with the value greater than or equal to 1/2 and less than 1.
<code>dist</code>	Character string, specifies selected distribution function.
<code>p1</code>	A scalar. Parameter 1 (vector or object) of the selected distribution.
<code>p2</code>	A scalar. Parameter 2 (vector or object) of the selected distribution.

**Details**

Implements the quantity

$$\frac{2\sqrt{3} - \int_{\xi_{1-p}}^{\infty} f^2(x)(1 - F(x)) dx + \frac{p}{2} \int_{\xi_{1-p}}^{\infty} f^2(x) dx}{p \left\{ p \int_{\xi_{1-p}}^{\infty} f^3(x) dx - \left( \int_{\xi_{1-p}}^{\infty} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood \(2014\)](#), see also (5) in [Bagkavos, Patil and Wood \(2016\)](#). This implementation uses exact calculation of the functionals in the definition of  $\rho_p^*$ .

**Value**

Returns a scalar, the exact value of  $\rho_p^*$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- [Patil P.N., Bagkavos D. and Wood A.T.A., \(2014\). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, Sankhya A, 76, 123–145.](#)
- [Bagkavos D., Patil P.N., Wood A.T.A. \(2016\), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. \(eds\), Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.](#)

**See Also**

[Rho.p](#), [Rhostar.p](#), [Rho.p.exact](#)

**Examples**

```

set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rhostar.p.exact(xout,param,selected.r,shape,scale) # calculate Rhostar.p.exact
#-0.05206678 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
param <- 0.9 # specify parameter
xout <-r.sample(n,selected.r2,mean,sd) # specify sample
Rhostar.p.exact(xout,param,selected.r2,mean,sd) # calculate Rhostar.p.exact
#-0.008687447 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
Rhostar.p.exact(xout,param,selected.r3,location,scale) # calculate Rhostar.p.exact
#0.0280602 # returns the result

```

---

SimpsonInt

*Simpson integration*


---

**Description**

Implements simpson's extended integration rule.

**Usage**

```
SimpsonInt(xin,h)
```

**Arguments**

<code>xin</code>	A vector of design points where the integral will be evaluated.
<code>h</code>	Assuming $a < b$ and $n$ is a positive integer. $h = (b - a)/n$ .

**Details**

Simpson's extended numerical integration rule is implemented for  $n+1$  equally spaced subdivisions (where  $n$  is even) of  $[a, b]$  as

$$\int_a^b f(x) dx = \frac{h}{3} \{f(a) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(b)\}$$

where  $hx = (b - a)/n$  and  $x_i = a + ihx$ . Simpson's rule will return an exact result when the polynomial in question has a degree of three or less. For other functions, Simpson's Rule only gives an approximation.

**Value**

A scalar, the approximate value of the integral.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

[Simpson's Rule](#)

**Examples**

```
x.in<- seq(0,pi/4,length=5)
h.out <- pi/8
SimpsonInt(x.in,h.out)
```

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