

# Package ‘benford.analysis’

May 7, 2026

**Type** Package

**Title** Benford Analysis for Data Validation and Forensic Analytics

**Version** 0.1.5

**Author** Carlos Cinelli

**Maintainer** Carlos Cinelli <carloscinelli@hotmail.com>

**Description** Provides tools that make it easier to validate data using Benford's Law.

**Depends** R (>= 3.0.0)

**Imports** data.table

**License** GPL-3

**Suggests** testthat

**URL** <http://github.com/carloscinelli/benford.analysis>

**BugReports** <http://github.com/carloscinelli/benford.analysis/issues>

**RoxygenNote** 6.1.1

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2018-12-21 09:20:03 UTC

## Contents

benford . . . . .	2
benford.analysis . . . . .	4
census.2000_2010 . . . . .	5
census.2009 . . . . .	6
chisq . . . . .	6
corporate.payment . . . . .	7
dfactor . . . . .	7
duplicatesTable . . . . .	8
extract.digits . . . . .	8
getBfd . . . . .	9
getData . . . . .	10

getDigits . . . . .	10
getDuplicates . . . . .	11
getSuspects . . . . .	12
lakes.perimeter . . . . .	12
MAD . . . . .	13
MAD.conformity . . . . .	13
mantissa . . . . .	14
marc . . . . .	14
p.these.digits . . . . .	15
p.this.digit.at.n . . . . .	15
plot.Benford . . . . .	16
print.Benford . . . . .	17
sino.forest . . . . .	17
suspectsTable . . . . .	18
taxable.incomes.1978 . . . . .	18

## Index 19

---

benford	<i>Benford Analysis of a dataset</i>
---------	--------------------------------------

---

### Description

This function validates a dataset using Benford's Law. Its main purposes are to find out where the dataset deviates from Benford's Law and to identify suspicious data that need further verification.

For a more complete example, see the package help at [benford.analysis](#).

### Usage

```
benford(data, number.of.digits = 2, sign = "positive", discrete=TRUE, round=3)
```

### Arguments

data	a numeric vector.
number.of.digits	how many first digits to analyse .
sign	The default value for sign is "positive" and it analyzes only data greater than zero. There are also the options "negative" and "both" that will analyze only negative values or both positive and negative values of the data, respectively. For large datasets with both positive and negative numbers, it is usually recommended to perform a separate analysis for each group, for the incentives to manipulate the numbers are usually different.
discrete	most real data - like population numbers or accounting data - are discrete, so the default is TRUE. This parameter sets rounding to the differences of the ordered data to avoid floating point number errors in the second order distribution, that usually occurs when data is discrete and the ordered numbers are very close to each other. If your data is continuous (like a simulated lognormal) you should run with discrete = FALSE.

round            it defines the number of digits that the rounding will use if discrete = TRUE.

### Value

An object of class Benford containing the results of the analysis. It is a list of eight objects, namely:

info	<p>general information, including</p> <ul style="list-style-type: none"> <li>• data.name: the name of the data used.</li> <li>• n: the number of observations used.</li> <li>• n.second.order: the number of observations used for second order analysis.</li> <li>• number.of.digits: the number of first digits analysed.</li> </ul>
data	<p>a data frame with:</p> <ul style="list-style-type: none"> <li>• lines.used: the original lines of the dataset.</li> <li>• data.used: the data used.</li> <li>• data.mantissa: the log data's mantissa.</li> <li>• data.digits: the first digits of the data.</li> </ul>
s.o.data	<p>a data frame with:</p> <ul style="list-style-type: none"> <li>• data.second.order: the differences of the ordered data.</li> <li>• data.second.order.digits: the first digits of the second order analysis.</li> </ul>
bfd	<p>a data frame with:</p> <ul style="list-style-type: none"> <li>• digits: the groups of digits analysed.</li> <li>• data.dist: the distribution of the first digits of the data.</li> <li>• data.second.order.dist: the distribution of the first digits of the second order analysis.</li> <li>• benford.dist: the theoretical benford distribution.</li> <li>• data.second.order.dist.freq: the frequency distribution of the first digits of the second order analysis.</li> <li>• data.dist.freq: the frequency distribution of the first digits of the data.</li> <li>• benford.dist.freq: the theoretical benford frequency distribution.</li> <li>• benford.so.dist.freq: the theoretical benford frequency distribution of the second order analysis.</li> <li>• data.summation: the summation of the data values grouped by first digits.</li> <li>• abs.excess.summation: the absolute excess summation of the data values grouped by first digits.</li> <li>• difference: the difference between the data and benford frequencies.</li> <li>• squared.diff: the chi-squared difference between data and benford frequencies.</li> <li>• absolute.diff: the absolute difference between data and benford frequencies.</li> </ul>
mantissa	<p>a data frame with:</p> <ul style="list-style-type: none"> <li>• mean.mantissa: the mean of the mantissa.</li> <li>• var.mantissa: the variance of the mantissa.</li> <li>• ek.mantissa: the excess kurtosis of the mantissa.</li> <li>• sk.mantissa: the skewness of the mantissa.</li> </ul>

MAD                    the mean absolute deviation.  
distortion.factor     the distortion factor

stats                  list of "htest" class statistics:

- chisq: Pearson's Chi-squared test.
- mantissa.arc.test: Mantissa Arc Test.

### Examples

```
data(corporate.payment) #loads data
bfd.cp <- benford(corporate.payment$Amount) #generates benford object
bfd.cp #prints
plot(bfd.cp) #plots
```

---

benford.analysis            *Benford Analysis for data validation and forensic analytics*

---

### Description

The Benford Analysis package provides tools that make it easier to validate data using Benford's Law. The main purpose of the package is to identify suspicious data that need further verification.

### Details

More information can be found on its help documentation.

The main function is `benford`. It generates a Benford S3 object.

The package defines S3 methods for plotting and printing Benford type objects.

After running `benford` you can easily get the "suspicious" data by using the functions: `suspectsTable`, `getSuspects`, `duplicatesTable` and `getDuplicates`. See help documentation and examples for further details.

The package also includes 6 real datasets for illustration purposes.

### References

Alexander, J. (2009). Remarks on the use of Benford's Law. Working Paper, Case Western Reserve University, Department of Mathematics and Cognitive Science.

Berger, A. and Hill, T. (2011). A basic theory of Benford's Law. *Probability Surveys*, 8, 1-126.

Hill, T. (1995). A statistical derivation of the significant-digit law. *Statistical Science*, 10(4), 354-363.

Nigrini, M. J. (2012). *Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection*. Wiley and Sons: New Jersey.

Nigrini, M. J. (2011). *Forensic Analyticis: Methods and Techniques for Forensic Accounting Investigations*. Wiley and Sons: New Jersey.

### Examples

```
data(corporate.payment) #gets data
cp <- benford(corporate.payment$Amount, 2, sign="both") #generates benford object
cp #prints
plot(cp) #plots

head(suspectsTable(cp),10) #prints the digits by decreasing order of discrepancies

#gets observations of the 2 most suspicious groups
suspects <- getSuspects(cp, corporate.payment, how.many=2)

duplicatesTable(cp) #prints the duplicates by decreasing order

#gets the observations of the 2 values with most duplicates
duplicates <- getDuplicates(cp, corporate.payment,how.many=2)

MAD(cp) #gets the Mean Absolute Deviation

chisq(cp) #gets the Chi-squared test

#gets observations starting with 50 or 99
digits_50_and_99 <- getDigits(cp, corporate.payment, digits=c(50, 99))
```

---

census.2000\_2010

*Population data - US - 2000 and 2010*

---

### Description

A dataset containing population data of the United States - 2000 and 2010.

### Format

A data frame with 3143 rows and 5 variables

### References

Nigrini, M. J. (2012). *Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection*. Wiley and Sons: New Jersey.

---

`census.2009`*Population data of Towns and Cities of the US - 2009*

---

**Description**

A dataset containing the population of towns and cities of the United States, as of July of 2009.

**Format**

A data frame with 19509 rows and 3 variables

**References**

Nigrini, M. J. (2012). *Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection*. Wiley and Sons: New Jersey.

---

`chisq`*Gets the Chi-squared test of a Benford object*

---

**Description**

It gets the Chi-squared test for a Benford object. See the section value of [benford](#).

**Usage**

```
chisq(bfd)
```

**Arguments**

`bfd` an object of class "Benford". See [benford](#).

**Value**

A list with class "htest" containing the results of the Chi-squared test.

**Examples**

```
data(census.2009) #gets data
c2009 <- benford(census.2009$pop.2009) #generates benford object
chisq(c2009) # equivalent to c2009$stats$chisq
```

---

corporate.payment	<i>Corporate payments of a West Coast utility company - 2010</i>
-------------------	------------------------------------------------------------------

---

**Description**

A dataset of the 2010's payments data of a division of a West Coast utility company.

**Format**

A data frame with 189470 rows and 4 variables

**References**

Nigrini, M. J. (2012). *Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection*. Wiley and Sons: New Jersey.

---

dfactor	<i>Gets the Distortion Factor of a Benford object</i>
---------	-------------------------------------------------------

---

**Description**

It gets the Distortion Factor of a Benford object. See the section value of [benford](#).

**Usage**

```
dfactor(bfd)
```

**Arguments**

bfd                    an object of class "Benford". See [benford](#).

**Value**

The distortion factor.

**Examples**

```
data(corporate.payment) #gets data
cp <- benford(corporate.payment$Amount) #generates benford object
dfactor(cp) # equivalent to cp$distortion.factor
```

---

duplicatesTable      *Shows the duplicates of the data*

---

### Description

It creates a data frame with the duplicates in decreasing order.

### Usage

```
duplicatesTable(bfd)
```

### Arguments

bfd                    an object of class "Benford". See [benford](#).

### Value

A data frame with 2 variables: number and duplicates.

### Examples

```
data(census.2009) #gets data
c2009 <- benford(census.2009$pop.2009) #generates benford object
duplicatesTable(c2009)
```

---

extract.digits      *Extracts the leading digits from the data*

---

### Description

It extracts the leading digits from the data.

This function is used by the main function of the package [benford](#) to extract the leading digits of the data.

### Usage

```
extract.digits(data, number.of.digits = 2,
               sign="positive", second.order = FALSE, discrete=TRUE, round=3)
```

**Arguments**

<code>data</code>	a numeric vector.
<code>number.of.digits</code>	how many first digits to analyse .
<code>sign</code>	The default value for sign is "positive" and it analyzes only data greater than zero. There are also the options "negative" and "both" that will analyze only negative values or both positive and negative values of the data, respectively. For large datasets with both positive and negative numbers, it is usually recommended to perform a separate analysis for each group, for the incentives to manipulate the numbers are usually different.
<code>second.order</code>	If TRUE, the function will extract the first digits of the second order distribution.
<code>discrete</code>	Most real data - like population numbers or accounting data - are discrete, so the default is TRUE. This parameter sets rounding to the differences of the ordered data to avoid floating point number errors in the second order distribution, that usually occurs when data is discrete and the ordered numbers are very close to each other. If your data is continuous (like a simulated lognormal) you should run with discrete = FALSE.
<code>round</code>	it defines the number of digits that the rounding will use if discrete = TRUE and second.order = TRUE.

**Value**

A data.frame with the data and the first digits.

---

<code>getBfd</code>	<i>Gets the the statistics of the first Digits of a benford object</i>
---------------------	------------------------------------------------------------------------

---

**Description**

It gets the statistics of the first digits (Frequencies, Squared Differences, Absolute Differences etc). See the section value of [benford](#).

**Usage**

```
getBfd(bfd)
```

**Arguments**

`bfd` an object of class "Benford". See [benford](#).

**Value**

A data.frame with first digits and their statistics.

**Examples**

```
data(corporate.payment)
cp <- benford(corporate.payment$Amount) #generates benford object
getBfd(cp) # equivalent to cp$bfd
```

---

getData	<i>Gets the data used of a Benford object</i>
---------	-----------------------------------------------

---

**Description**

It gets the lines, values, mantissa and first digits of the data used of a Benford object . See the section value of [benford](#).

**Usage**

```
getData(bfd)
```

**Arguments**

bfd                    an object of class "Benford". See [benford](#).

**Value**

A data.frame with the lines, values, mantissa and first digits of the data.

**Examples**

```
data(corporate.payment)
cp <- benford(corporate.payment$Amount) #generates benford object
getData(cp) # equivalent to cp$data
```

---

getDigits	<i>Gets the data starting with some specific digits</i>
-----------	---------------------------------------------------------

---

**Description**

It subsets the original data according to the leading digits.

**Usage**

```
getDigits(bfd, data, digits)
```

**Arguments**

bfd                    an object of class "Benford". See [benford](#).  
data                    the original data of the analysis.  
digits                  the first digits to get.

**Value**

The the original data starting only with the leading digits.

**Examples**

```
data(census.2000_2010) #gets data

#generates benford object
c2010 <- benford(census.2000_2010$pop.2010)

#subsets data starting with digits 10 and 25
digits.10.25 <- getDigits(c2010, census.2000_2010, c(10,25))
```

---

getDuplicates	<i>Gets the duplicates from data</i>
---------------	--------------------------------------

---

**Description**

It gets the duplicates from the original data.

**Usage**

```
getDuplicates(bfd, data, how.many=2)
```

**Arguments**

bfd	an object of class "Benford". See <a href="#">benford</a> .
data	the original data used for the benford analysis.
how.many	how many groups of duplicates to get.

**Value**

The duplicates from the original data.

**Examples**

```
data(census.2000_2010) #gets data
c2010 <- benford(census.2000_2010$pop.2010) #generates benford object
duplicates <- getDuplicates(c2010, census.2000_2010)
```

---

getSuspects	<i>Gets the 'suspicious' observations according to Benford's Law</i>
-------------	----------------------------------------------------------------------

---

**Description**

It gets the original data from the 'suspicious' digits groups according to benford analysis.

**Usage**

```
getSuspects(bfd, data, by="absolute.diff", how.many=2)
```

**Arguments**

bfd	an object of class "Benford". See <a href="#">benford</a> .
data	the original data used for the benford analysis.
by	a character string selecting how to order the digits. It can be 'abs.excess.summation', 'difference', 'squared.difference', or 'absolute.diff'.
how.many	how many groups of digits to get.

**Value**

The 'suspicious' observations from the original data.

**Examples**

```
data(lakes.perimeter) #gets data
lk <- benford(lakes.perimeter[,1]) #generates benford object
suspects <- getSuspects(lk, lakes.perimeter)
```

---

lakes.perimeter	<i>Perimeter of lakes arround the world</i>
-----------------	---------------------------------------------

---

**Description**

A dataset of the perimeter of the lakes around the water from the global lakes and wetlands database (GLWD) <<http://www.worldwildlife.org/pages/global-lakes-and-wetlands-database>>.

**Format**

A data frame with 248607 rows and 1 variable.

**References**

Lehner, B. and Doll, P. (2004). Development and validation of a global database of lakes, reservoirs and wetlands. *Journal of Hydrology*, 296(1), pp.1-22.

Nigrini, M. J. (2012). *Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection*. Wiley and Sons: New Jersey.

---

MAD	<i>Gets the MAD of a Benford object</i>
-----	-----------------------------------------

---

**Description**

It gets the Mean Absolute Deviation (MAD) of a Benford object. See the section value of [benford](#).

**Usage**

```
MAD(bfd)
```

**Arguments**

`bfd` an object of class "Benford". See [benford](#).

**Value**

The MAD.

**Examples**

```
data(census.2000_2010) #gets data
c2010 <- benford(census.2000_2010$pop.2010) #generates benford object
MAD(c2010) #equivalent to c2010$MAD
```

---

MAD.conformity	<i>MAD conformity to Benford's Law using the MAD</i>
----------------	------------------------------------------------------

---

**Description**

This function checks the MAD against the conformity criteria proposed by Nigrini (2012).

**Usage**

```
MAD.conformity(MAD = NULL, digits.used = c("First Digit",
      "Second Digit", "First-Two Digits", "First-Three Digits"))
```

**Arguments**

`MAD` The mean absolute deviation, as computed in the function [benford](#)  
`digits.used` How many digits used in the analysis.

**Value**

A list with the MAD, `digits.used` and the conformity level.

## References

Nigrini, M. J. (2012). Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection. Wiley and Sons: New Jersey.

---

<code>mantissa</code>	<i>Gets the main stats of the Mantissa of a Benford object</i>
-----------------------	----------------------------------------------------------------

---

## Description

It gets the Mean, Variance, Excess Kurtosis and Skewness of the Mantissa. See the section value of [benford](#).

## Usage

```
mantissa(bfd)
```

## Arguments

`bfd` an object of class "Benford". See [benford](#).

## Value

A data.frame with the main stats of the Mantissa.

## Examples

```
data(corporate.payment) #gets data
cp <- benford(corporate.payment$Amount) #generates benford object
mantissa(cp) # equivalent to cp$mantissa
```

---

<code>marc</code>	<i>Gets the Mantissa Arc test of a Benford object</i>
-------------------	-------------------------------------------------------

---

## Description

It gets the Mantissa Arc Test of a Benford object. See the section value of [benford](#).

## Usage

```
marc(bfd)
```

## Arguments

`bfd` an object of class "Benford". See [benford](#).

**Value**

A list with class "htest" containing the results of the Matissa Arc test.

**Examples**

```
data(corporate.payment) #gets data
cp <- benford(corporate.payment$Amount) #generates benford object
marc(cp) # equivalent to cp$stats$mantissa.arc.test
```

---

*p.these.digits*                      *Probability of a digit sequence*

---

**Description**

It calculates the probability of a digit sequence "d".

**Usage**

```
p.these.digits(d)
```

**Arguments**

d                      a digit sequence, like 1234 ou 999999.

**Value**

The probability of the sequence d.

**Examples**

```
p.these.digits(1) # 0.30103
p.these.digits(11) # 0.03778856
p.these.digits(999999) # 4.342947e-07
```

---

*p.this.digit.at.n*                      *Probability of a digit at the nth position*

---

**Description**

It calculates the probability of digit "d" at the "n"th position.

**Usage**

```
p.this.digit.at.n(d,n)
```

**Arguments**

d a digit from 0 to 9 (except at position n=1, where d cannot be 0, it will give you NA).

n the nth position.

**Value**

The probability of d at position n.

**Examples**

```
p.this.digit.at.n(1,1) # 0.30103
p.this.digit.at.n(1,2) # 0.1138901
p.this.digit.at.n(9,3) # 0.09826716
matrix <- as.data.frame(round(sapply(1:4, function(x) sapply(0:9,p.this.digit.at.n,n=x)),5))
names(matrix) <- paste0("n=",1:4)
rownames(matrix) <- paste0("d=",0:9)
matrix # a table with the probabilities of digits 0 to 9 in positions 1 to 4.
```

---

plot.Benford

*Plot method for Benford Analysis*

---

**Description**

The plot method for "Benford" objects.

**Usage**

```
## S3 method for class 'Benford'
plot(x,except=c("mantissa","abs diff"), multiple=TRUE, ...)
```

**Arguments**

x a "Benford" object

except it specifies which plots are not going to be plotted. Currently, you can choose from 7 plots: "digits", "second order", "summation", "mantissa", "chi square", "abs diff", "ex summation". If you want to plot all, just put except = "none". The default is not to plot the "mantissa" and "abs diff".

multiple if TRUE, all plots are grouped in the same window.

... arguments to be passed to generic plot functions,

**Value**

Plots the Benford object.

---

print.Benford	<i>Print method for Benford Analysis</i>
---------------	------------------------------------------

---

**Description**

The print method for "Benford" objects.

**Usage**

```
## S3 method for class 'Benford'  
print(x, how.many=5, ...)
```

**Arguments**

x	a "Benford" object.
how.many	a number that defines how many of the biggest absolute differences to show.
...	arguments to be passed to generic print functions.

**Value**

Prints the Benford object.

---

sino.forest	<i>Financial Statemens of Sino Forest Corporation's 2010 Report</i>
-------------	---------------------------------------------------------------------

---

**Description**

Financial Statemens numbers of Sino Forest Corporation's 2010 Report.

**Format**

A data frame with 772 rows and 1 variable.

**References**

Nigrini, M. J. (2012). Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection. Wiley and Sons: New Jersey.

---

suspectsTable	<i>Shows the first digits ordered by the mains discrepancies from Benford's Law</i>
---------------	-------------------------------------------------------------------------------------

---

**Description**

It creates a data frame with the first digits and the differences from Benford's Law in decreasing order.

**Usage**

```
suspectsTable(bfd, by="absolute.diff")
```

**Arguments**

bfd	an object of class "Benford". See <a href="#">benford</a> .
by	a character string selecting how to order the digits. It can be 'abs.excess.summation', 'difference', 'squared.difference', or 'absolute.diff'.

**Value**

A data frame with 2 variables: digits and the group chosen in by.

**Examples**

```
data(corporate.payment) #gets data
cp <- benford(corporate.payment$Amount) #generates benford object
suspectsTable(cp)
```

---

taxable.incomes.1978	<i>Taxable Income 1978</i>
----------------------	----------------------------

---

**Description**

Taxable Incomes of the 1978 Individual Tax Model File (ITMF).

**Format**

A data frame with 157518 rows and 1 variable.

**References**

Nigrini, M. J. (2012). Benford's Law: Application for Forensic Accounting, Auditing and Fraud Detection. Wiley and Sons: New Jersey.

# Index

## \* dataset

- census.2000\_2010, 5
- census.2009, 6
- corporate.payment, 7
- lakes.perimeter, 12
- sino.forest, 17
- taxable.incomes.1978, 18

- benford, 2, 4, 6–14, 18
- benford.analysis, 2, 4
- benford.analysis-package
  - (benford.analysis), 4

- census.2000\_2010, 5
- census.2009, 6
- chisq, 6
- corporate.payment, 7

- dfactor, 7
- duplicatesTable, 4, 8

- extract.digits, 8

- getBfd, 9
- getData, 10
- getDigits, 10
- getDuplicates, 4, 11
- getSuspects, 4, 12

- lakes.perimeter, 12

- MAD, 13
- MAD.conformity, 13
- mantissa, 14
- marc, 14

- p.these.digits, 15
- p.this.digit.at.n, 15
- plot.Benford, 16
- print.Benford, 17

- sino.forest, 17

- suspectsTable, 4, 18

- taxable.incomes.1978, 18