

Package ‘bosfr’

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Title Computes Exact Bounds of Spearman's Footrule with Missing Data

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Author Yijin Zeng [aut, cre, cph]

Maintainer Yijin Zeng <yijinzeng98@gmail.com>

Description Computes exact bounds of Spearman's footrule in the presence of missing data, and performs independence test based on the bounds with controlled Type I error regardless of the values of missing data. Suitable only for distinct, univariate data where no ties is allowed.

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Encoding UTF-8

RoxygenNote 7.3.2

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Imports gtools, stats

Suggests testthat (>= 3.0.0)

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 boundsKendall

Bounds of Kendall's tau in the Presence of Missing Data

Description

Computes bounds of Kendall's tau in the presence of missing data. Suitable only for univariate distinct data where no ties is allowed.

Usage

```
boundsKendall(X, Y)
```

Arguments

`X, Y` Numeric vectors of data values with potential missing data. No ties in the data is allowed. Inf and -Inf values will be omitted.

Details

`boundsKendall()` computes bounds of Kendall's tau for partially observed univariate, distinct data. The bounds are computed by first calculating the bounds of Spearman's footrule (Zeng *et al.*, 2025), and then applying the combinatorial inequality between Kendall's tau and Spearman's footrule (Kendall, 1948). See Zeng *et al.*, 2025 for more details.

Let $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_n)$ be two vectors of univariate, distinct data. Kendall's tau is defined as the number of discordant pairs between X and Y :

$$\tau(X, Y) = \sum_{i < j} \{I(x_i < x_j)I(y_i > y_j) + I(x_i > x_j)I(y_i < y_j)\}.$$

Scaled Kendall's tau $\tau_{Scale}(X, Y) \in [0, 1]$ is defined as (Kendall, 1948):

$$\tau_{Scale}(X, Y) = 1 - 4\tau(X, Y)/(n(n - 1)).$$

Value

`bounds` bounds of Kendall's tau.
`bounds.scaled` bounds of scaled Kendall's tau.

References

- Zeng Y., Adams N.M., Bodenham D.A. Exact Bounds of Spearman's footrule in the Presence of Missing Data with Applications to Independence Testing. arXiv preprint arXiv:2501.11696. 2025 Jan 20.
- Kendall, M.G. (1948) Rank Correlation Methods. Charles Griffin, London.
- Diaconis, P. and Graham, R.L., 1977. Spearman's footrule as a measure of disarray. Journal of the Royal Statistical Society Series B: Statistical Methodology, 39(2), pp.262-268.

Examples

```
### compute bounds of Kendall's tau between incomplete ranked lists
X <- c(1, 2, NA, 4, 3)
Y <- c(3, NA, 4, 2, 1)
boundsKendall(X, Y)

### compute bounds of Kendall's tau between incomplete vectors of distinct data
X <- c(1.3, 2.6, NA, 4.2, 3.5)
Y <- c(5.5, NA, 6.5, 2.6, 1.1)
boundsKendall(X, Y)
```

boundsSFR *Exact bounds of Spearman's footrule in the Presence of Missing Data*

Description

Computes exact bounds of Spearman's footrule in the presence of missing data, and performs independence test based on the bounds with controlled Type I error regardless of the values of missing data. Suitable only for univariate distinct data where no ties is allowed.

Usage

```
boundsSFR(X, Y, pval = TRUE)
```

Arguments

X	Numeric vector of data values with potential missing data. No ties in the data is allowed. Inf and -Inf values will be omitted.
Y	Numeric vector of data values with potential missing data. No ties in the data is allowed. Inf and -Inf values will be omitted.
pval	Boolean for whether to compute the bounds of p-value or not.

Details

boundsSFR() computes exact bounds of Spearman's footrule for partially observed univariate, distinct data using the results and algorithms following Zeng *et al.*, 2025.

Let $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_n)$ be two vectors of univariate, distinct data, and denote the rank of x_i in X as $R(x_i, X)$, the rank of y_i in Y as $R(y_i, Y)$. Spearman's footrule is defined as the absolute distance between the ranked values of X and Y :

$$D(X, Y) = \sum_{i=1}^n |R(x_i, X) - R(y_i, Y)|.$$

Scaled Spearman's footrule is defined as:

$$D_{Scale}(X, Y) = 1 - 3D(X, Y)/(n^2 - 1).$$

When n is odd, $D_{Scale}(X, Y) \in [-0.5, 1]$, but when n is even, $D_{Scale}(X, Y) \in [-0.5\{1+3/(n^2-1)\}, 1]$ (Kendall, 1948).

The p-value of the independence test using Spearman's footrule, denoted as p , is computed using the normality approximation result in Diaconis, P., & Graham, R. L. (1977). If `pval = TRUE`, bounds of the p-value, p_l, p_u will be computed in the presence of missing data, such that $p \in [p_l, p_u]$. The independence test method proposed in Zeng et al., 2025 returns p_u as its p-value. This method controls the Type I error regardless of the values of missing data. See Zeng et al., 2025 for details.

Value

<code>bounds</code>	exact bounds of Spearman's footrule.
<code>bounds.scaled</code>	exact bounds of scaled Spearman's footrule.
<code>pvalue</code>	the p-value for the test. (Only present if argument <code>pval = TRUE</code> .)
<code>bounds.pvalue</code>	bounds of the p-value of independence test using Spearman's footrule. (Only present if argument <code>pval = TRUE</code> .)

References

- Zeng Y., Adams N.M., Bodenham D.A. Exact Bounds of Spearman's footrule in the Presence of Missing Data with Applications to Independence Testing. arXiv preprint arXiv:2501.11696. 2025 Jan 20.
- Kendall, M.G. (1948) Rank Correlation Methods. Charles Griffin, London.
- Diaconis, P. and Graham, R.L., 1977. Spearman's footrule as a measure of disarray. Journal of the Royal Statistical Society Series B: Statistical Methodology, 39(2), pp.262-268.

Examples

```
### compute exact bounds of Spearman's footrule between incomplete ranked lists
X <- c(1, 2, NA, 4, 3)
Y <- c(3, NA, 4, 2, 1)
boundsSFR(X, Y, pval=FALSE)

### compute exact bounds of Spearman's footrule between incomplete vectors of distinct data,
### and perform independence test
X <- c(1.3, 2.6, NA, 4.2, 3.5)
Y <- c(5.5, NA, 6.5, 2.6, 1.1)
boundsSFR(X, Y, pval=TRUE)
```

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