Package ‘copulaSQM’

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Type Package

Title Copula Based Stochastic Frontier Quantile Model

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Description Provides estimation procedures for copula-based stochastic frontier quantile models for cross-sectional data. The package implements maximum likelihood estimation of quantile regression models allowing flexible dependence structures between error components through various copula families (e.g., Gaussian and Student-t). It enables estimation of conditional quantile effects, dependence parameters, log-likelihood values, and information criteria (AIC and BIC). The framework combines quantile regression methodology introduced by Koenker and Bassett (1978) <doi:10.2307/1913643> with copula theory described in Joe (2014, ISBN:9781466583221). This approach allows modeling heterogeneous effects across quantiles while capturing nonlinear dependence structures between variables.

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Contents

copSQM *Copula-Based Stochastic Frontier Quantile Model*

Description

Estimates a copula-based stochastic frontier quantile model using simulated maximum likelihood. Dependence between the two-sided noise component and the one-sided inefficiency component is captured via a bivariate copula from the `VineCopula` package (Nagler et al., 2025).

Usage

```
copSQM(Y, X, family, tau,
       RHO, LB, UB,
       RHO2 = NULL, LB2 = NULL, UB2 = NULL,
       nSim = 50,
       seed = NULL,
       maxit = 10000)
```

Arguments

`Y`	A numeric vector of dependent variable observations.
`X`	A numeric matrix (or object coercible to a matrix) of explanatory variables. The intercept is added internally.
`family`	Integer specifying the bivariate copula family (see `VineCopula::BiCopPDF`). Examples include 1 = `Gaussian`, 2 = `Student-t`. Some copula families require a second parameter.
`tau`	Quantile level in $(0, 1)$ for the asymmetric Laplace distribution (ALD).
`RHO`	Initial value of the first copula parameter (passed to `par` in `VineCopula`). Note that this parameter is not always a correlation; its interpretation depends on the chosen copula family.
`LB`	Lower bound for the first copula parameter `RHO`.
`UB`	Upper bound for the first copula parameter `RHO`.
`RHO2`	Optional initial value of the second copula parameter (passed to `par2` in `VineCopula`). This is required for copula families that have two parameters (e.g. Student-t where `RHO2` represents degrees of freedom, and BB families where `RHO2` is a shape parameter). If `NULL`, a one-parameter copula is assumed.
`LB2`	Optional lower bound for the second copula parameter `RHO2`. Must be supplied when `RHO2` is not `NULL`.
`UB2`	Optional upper bound for the second copula parameter `RHO2`. Must be supplied when `RHO2` is not `NULL`.
`nSim`	Number of Monte Carlo draws used to approximate the likelihood integral for each observation. Larger values improve accuracy but increase computation time.
`seed`	Optional integer seed for reproducibility of the simulation draws used in the likelihood. If `NULL`, no seed is set.
`maxit`	Maximum number of iterations for the optimizer (`stats::optim` with `"L-BFGS-B"`).

Details

The model follows the stochastic frontier decomposition

$$Y_i = x_i^\top \beta + v_i - u_i,$$

where v_i is a two-sided noise term and $u_i \geq 0$ is the inefficiency term. Both components are modeled using the asymmetric Laplace distribution (ALD) at quantile level `tau`, with u_i truncated to $(0, \infty)$.

Dependence between u_i and v_i is introduced through a bivariate copula density $c(\cdot, \cdot)$ from the `VineCopula` package. The log-likelihood is evaluated by simulating draws of u_i from the truncated ALD and approximating the likelihood integral by Monte Carlo averaging (`nSim` draws per observation).

When a two-parameter copula family is used, `RHO2` must be provided along with bounds `LB2` and `UB2`.

The following copula families are supported, together with their parameter bounds:

```
1  = Gaussian copula (par: (LB = -0.99, UB = 0.99))
2  = Student t copula (par: (LB = -0.99, UB = 0.99); par2: (LB = 0, UB = Inf))
3  = Clayton copula (par: (LB = 0.1, UB = Inf))
4  = Gumbel copula (par: [LB = 0.99, UB = Inf))
5  = Frank copula (par: (LB = -Inf, UB =  0) U (LB = 0, UB = Inf))
6  = Joe copula (par: (LB = 0.99, UB = Inf))
7  = BB1 (Clayton-Gumbel) copula (par: (LB = 0, UB = Inf); par2: [LB = 0.99, UB = Inf))
8  = BB6 copula (par: (LB = 0.99, UB = Inf); par2: (LB = 0, UB = Inf))
9  = BB7 (Joe-Clayton) copula (par: [LB = 0.99, UB = Inf); par2: (LB = 0, UB = Inf))
10 = BB8 copula (par: (LB = 0, UB = 1); par2: (LB = 0, UB =  Inf))
13 = Survival Clayton (180 degrees rotation; par: (LB = 0, UB = Inf))
14 = Survival Gumbel (180 degrees rotation; par: [LB = 0.99, UB = Inf))
16 = Survival Joe (180 degrees rotation; par: (LB = 0.99, UB = Inf))
17 = Survival BB1 (par: (LB = 0, UB = Inf); par2: [LB = 0.99, UB = Inf))
18 = Survival BB6 (par: (LB = 0.99, UB = Inf); par2: (0, UB = Inf))
19 = Survival BB7 (par: [LB = 0.99, UB = Inf); par2: (0, UB = Inf))
20 = Survival BB8 (par: (LB = 0, 0.99); par2: (LB = 0, UB = Inf))
23 = Rotated Clayton (90 degrees; par: (LB = -Inf, UB = 0))
24 = Rotated Gumbel (90 degrees; par: (LB = -Inf, UB = -0.99])
26 = Rotated Joe (90 degrees; par: (LB = -Inf, UB = -0.99))
27 = Rotated BB1 (90 degrees; par: (LB = -Inf, UB = 0); par2: [LB = 0.99, UB = Inf))
28 = Rotated BB6 (90 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
29 = Rotated BB7 (90 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
30 = Rotated BB8 (90 degrees; par: (LB = -0.99, UB = 0); par2: (LB = 0, UB = Inf))
33 = Rotated Clayton (270 degrees; par: (LB = -Inf, UB = 0))
34 = Rotated Gumbel (270 degrees; par: (LB = -Inf, UB = -0.99])
36 = Rotated Joe (270 degrees; par: (LB = -Inf, UB = -0.99))
37 = Rotated BB1 (270 degrees; par: (LB = -Inf, UB = 0); par2: [LB = 0.99, UB = Inf))
38 = Rotated BB6 (270 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
39 = Rotated BB7 (270 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
40 = Rotated BB8 (270 degrees; par: (LB = -0.99, UB = 0); par2: (LB = 0, UB = Inf))
```

See the **VineCopula** package (Nagler et al., 2025) for additional details on copula parameterization.

Value

A list with components:

`result`	A matrix of parameter estimates, standard errors, z-values, and p-values.
`AIC`	Akaike Information Criterion.
`BIC`	Bayesian Information Criterion.
`Loglikelihood`	Maximized log-likelihood value.
`convergence`	Convergence code returned by `stats::optim`. A value of 0 indicates successful convergence.

Author(s)

Woraphon Yamaka, Paravee Maneejuk and Nuttaphong Kaewtathip

References

Pipitpojanakarn, V., Maneejuk, P., Yamaka, W., & Sriboonchitta, S. (2016). Analysis of agricultural production in Asia and measurement of technical efficiency using copula-based stochastic frontier quantile model. In *International Symposium on Integrated Uncertainty in Knowledge Modelling and Decision Making* (pp. 701–714). Springer.

Pipitpojanakarn, V., Yamaka, W., Sriboonchitta, S., & Maneejuk, P. (2017). Frontier Quantile Model Using a Generalized Class of Skewed Distributions. *Advanced Science Letters*, 23(11), 10737–10742.

Nagler, T., Schepsmeier, U., Stoeber, J., Brechmann, E. C., Graeler, B., Erhardt, T., ... & Killiches, M. (2025, July). Package 'VineCopula'.

Examples

```
set.seed(123)

sim_data <- sfa_simu_quantile(
  n = 50,
  beta = c(1, 0.5, -0.3),
  sigV = 1,
  sigU = 1,
  tau = 0.5,
  family = 1,
  rho = 0.3,
  seed = 123
)

model <- copSQM(
  Y = sim_data$Y,
  X = sim_data$X,
  family = 1,
  tau = 0.5,
  RHO = 0.3,
  LB = -0.99,
  UB = 0.99,
```

```
  nSim = 50,
  seed = 123
)
model

## Example with a two-parameter copula (e.g., Student-t: family = 2)
## Here RHO2 typically represents degrees of freedom and must be bounded.
model_t <- copSQM(
  Y = sim_data$Y,
  X = sim_data$X,
  family = 2,
  tau = 0.5,
  RHO = 0.3,
  LB = -0.99,
  UB = 0.99,
  RHO2 = 4,
  LB2 = 2.01,
  UB2 = 50,
  nSim = 50,
  seed = 123
)
model_t
```

`sfa_simu_quantile` *Simulation of Copula-Based Stochastic Frontier Quantile Model*

Description

Generates simulated data from a copula-based stochastic frontier quantile model with asymmetric Laplace noise and a truncated asymmetric Laplace inefficiency term.

Usage

```
sfa_simu_quantile(n,
                  beta,
                  sigV,
                  sigU,
                  tau = 0.5,
                  family = 1,
                  rho = 0.5,
                  rho2 = NULL,
                  seed = NULL)
```

Arguments

`n`	Integer. Number of observations to simulate.

- `beta` Numeric vector of regression coefficients including an intercept. Its length must match the number of regressors plus one (intercept). This simulator generates two regressors, so `length(beta)` must be 3.
- `sigV` Positive numeric value. Scale parameter of the two-sided noise term V.
- `sigU` Positive numeric value. Scale parameter of the one-sided inefficiency term U.
- `tau` Quantile level in $(0, 1)$. Default is `0.5`.
- `family` Integer specifying the bivariate copula family (see `VineCopula::BiCopSim`). Examples include `1` (Gaussian) and `2` (Student-t). Some families require a second parameter `rho2`.
- `rho` First copula parameter (passed to `par` in `VineCopula`). Interpretation depends on the copula family and is not always a correlation.
- `rho2` Optional second copula parameter (passed to `par2` in `VineCopula`). This must be provided for copula families that require a second parameter (e.g., degrees of freedom for the Student-t copula, or a shape parameter for BB copulas).
- `seed` Optional integer seed for reproducibility. If `NULL`, no seed is set.

Details

This function simulates data from the stochastic frontier model:

$$Y = X\beta + V - U,$$

where:

- V follows an asymmetric Laplace distribution (ALD) with location 0, scale `sigV`, and quantile level `tau`.
- U is a non-negative inefficiency term generated from an ALD distribution with location 0, scale `sigU`, and quantile level `tau`, truncated to $(0, \infty)$.
- Dependence between V and U is introduced via a bivariate copula using `VineCopula::BiCopSim`.

The simulator constructs two regressors internally (denoted x_1 and x_2).

The following copula families are supported, together with their parameter bounds:

```
1  = Gaussian copula (par: (LB = -0.99, UB = 0.99))
2  = Student t copula (par: (LB = -0.99, UB = 0.99); par2: (LB = 0, UB = Inf))
3  = Clayton copula (par: (LB = 0.1, UB = Inf))
4  = Gumbel copula (par: [LB = 0.99, UB = Inf))
5  = Frank copula (par: (LB = -Inf, UB =  0) U (LB = 0, UB = Inf))
6  = Joe copula (par: (LB = 0.99, UB = Inf))
7 = BB1 (Clayton-Gumbel) copula (par: (LB = 0, UB = Inf); par2: [LB = 0.99, UB = Inf))
8  = BB6 copula (par: (LB = 0.99, UB = Inf); par2: (LB = 0, UB = Inf))
9 = BB7 (Joe-Clayton) copula (par: [LB = 0.99, UB = Inf); par2: (LB = 0, UB = Inf))
10 = BB8 copula (par: (LB = 0, UB = 1); par2: (LB = 0, UB =  Inf))
13 = Survival Clayton (180 degrees rotation; par: (LB = 0, UB = Inf))
14 = Survival Gumbel (180 degrees rotation; par: [LB = 0.99, UB = Inf))
16 = Survival Joe (180 degrees rotation; par: (LB = 0.99, UB = Inf))
```

```
17 = Survival BB1 (par: (LB = 0, UB = Inf); par2: [LB = 0.99, UB = Inf))
18 = Survival BB6 (par: (LB = 0.99, UB = Inf); par2: (0, UB = Inf))
19 = Survival BB7 (par: [LB = 0.99, UB = Inf); par2: (0, UB = Inf))
20 = Survival BB8 (par: (LB = 0, 0.99); par2: (LB = 0, UB = Inf))
23 = Rotated Clayton (90 degrees; par: (LB = -Inf, UB = 0))
24 = Rotated Gumbel (90 degrees; par: (LB = -Inf, UB = -0.99])
26 = Rotated Joe (90 degrees; par: (LB = -Inf, UB = -0.99))
27 = Rotated BB1 (90 degrees; par: (LB = -Inf, UB = 0); par2: [LB = 0.99, UB = Inf))
28 = Rotated BB6 (90 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
29 = Rotated BB7 (90 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
30 = Rotated BB8 (90 degrees; par: (LB = -0.99, UB = 0); par2: (LB = 0, UB = Inf))
33 = Rotated Clayton (270 degrees; par: (LB = -Inf, UB = 0))
34 = Rotated Gumbel (270 degrees; par: (LB = -Inf, UB = -0.99])
36 = Rotated Joe (270 degrees; par: (LB = -Inf, UB = -0.99))
37 = Rotated BB1 (270 degrees; par: (LB = -Inf, UB = 0); par2: [LB = 0.99, UB = Inf))
38 = Rotated BB6 (270 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
39 = Rotated BB7 (270 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
40 = Rotated BB8 (270 degrees; par: (LB = -0.99, UB = 0); par2: (LB = 0, UB = Inf))
```

See the **VineCopula** package (Nagler et al., 2025) for additional details on copula parameterization.

Value

A list containing:

- `Y` — Numeric vector of simulated dependent variable values.
- `X` — Matrix of simulated independent variables (without intercept column).
- `V` — Simulated two-sided noise term.
- `U` — Simulated non-negative inefficiency term.
- `copula_uniforms` — The simulated dependent uniforms from the copula (two columns).

Author(s)

Woraphon Yamaka, Paravee Maneejuk and Nuttaphong Kaewtathip

References

Koenker, R. (2005). *Quantile Regression*. Cambridge University Press.

Joe, H. (2014). *Dependence Modeling with Copulas*. Chapman & Hall/CRC.

Nagler, T., Schepsmeier, U., Stoeber, J., Brechmann, E. C., Graeler, B., Erhardt, T., ... & Killiches, M. (2025, July). Package 'VineCopula'.

Examples

```
sim_data <- sfa_simu_quantile(
  n = 100,
  beta = c(1, 0.5, -0.3),
```

```
  sigV = 1,
  sigU = 1,
  tau = 0.5,
  family = 1,
  rho = 0.3,
  seed = 123
)
str(sim_data)
```

TE *Technical Efficiency Measure*

Description

Computes technical efficiency for a copula-based stochastic frontier quantile model using simulation-based conditional expectations.

Usage

```
TE(theta, Y, X, family, tau,
   nSim = 200,
   rho2 = NULL,
   seed = NULL,
   plot = FALSE)
```

Arguments

`theta`	Numeric vector of estimated model parameters. The expected ordering is: regression coefficients (including intercept), σ_v, σ_u, copula parameter `rho`, and optionally `rho2` if a two-parameter copula is used.
`Y`	Numeric vector of dependent variable observations.
`X`	Numeric matrix (or object coercible to a matrix) of independent variables (without intercept column).
`family`	Integer specifying the copula family (see `VineCopula::BiCopPDF`). Some families require a second parameter `rho2`.
`tau`	Quantile level in $(0, 1)$ for the asymmetric Laplace distribution.
`nSim`	Number of Monte Carlo draws used to approximate the conditional expectation. Larger values reduce simulation noise but increase computation time.
`rho2`	Optional second copula parameter. If `NULL` and `theta` contains an additional element beyond `rho`, the function will use that element as `rho2`.
`seed`	Optional integer seed for reproducibility in simulation-based computation. If `NULL`, no seed is set.
`plot`	Logical. If `TRUE`, produces a plot of sorted technical efficiency values.

Details

Technical efficiency is computed as a simulated conditional expectation:

$$TE_i = E[\exp(-U_i) \mid w_i],$$

where $w_i = Y_i - x_i^\top \beta$ and the weights are constructed using the asymmetric Laplace density for the noise term and the copula density capturing dependence between the inefficiency component and the noise component.

The inefficiency term U is generated from a truncated asymmetric Laplace distribution on $(0, \infty)$.

If `plot = TRUE`, the function produces a line plot of sorted technical efficiency values.

The following copula families are supported, together with their parameter bounds:

```
1  = Gaussian copula (par: (LB = -0.99, UB = 0.99))
2  = Student t copula (par: (LB = -0.99, UB = 0.99); par2: (LB = 0, UB = Inf))
3  = Clayton copula (par: (LB = 0.1, UB = Inf))
4  = Gumbel copula (par: [LB = 0.99, UB = Inf))
5  = Frank copula (par: (LB = -Inf, UB =  0) U (LB = 0, UB = Inf))
6  = Joe copula (par: (LB = 0.99, UB = Inf))
7  = BB1 (Clayton-Gumbel) copula (par: (LB = 0, UB = Inf); par2: [LB = 0.99, UB = Inf))
8  = BB6 copula (par: (LB = 0.99, UB = Inf); par2: (LB = 0, UB = Inf))
9  = BB7 (Joe-Clayton) copula (par: [LB = 0.99, UB = Inf); par2: (LB = 0, UB = Inf))
10 = BB8 copula (par: (LB = 0, UB = 1); par2: (LB = 0, UB =  Inf))
13 = Survival Clayton (180 degrees rotation; par: (LB = 0, UB = Inf))
14 = Survival Gumbel (180 degrees rotation; par: [LB = 0.99, UB = Inf))
16 = Survival Joe (180 degrees rotation; par: (LB = 0.99, UB = Inf))
17 = Survival BB1 (par: (LB = 0, UB = Inf); par2: [LB = 0.99, UB = Inf))
18 = Survival BB6 (par: (LB = 0.99, UB = Inf); par2: (0, UB = Inf))
19 = Survival BB7 (par: [LB = 0.99, UB = Inf); par2: (0, UB = Inf))
20 = Survival BB8 (par: (LB = 0, 0.99); par2: (LB = 0, UB = Inf))
23 = Rotated Clayton (90 degrees; par: (LB = -Inf, UB = 0))
24 = Rotated Gumbel (90 degrees; par: (LB = -Inf, UB = -0.99])
26 = Rotated Joe (90 degrees; par: (LB = -Inf, UB = -0.99))
27 = Rotated BB1 (90 degrees; par: (LB = -Inf, UB = 0); par2: [LB = 0.99, UB = Inf))
28 = Rotated BB6 (90 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
29 = Rotated BB7 (90 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
30 = Rotated BB8 (90 degrees; par: (LB = -0.99, UB = 0); par2: (LB = 0, UB = Inf))
33 = Rotated Clayton (270 degrees; par: (LB = -Inf, UB = 0))
34 = Rotated Gumbel (270 degrees; par: (LB = -Inf, UB = -0.99])
36 = Rotated Joe (270 degrees; par: (LB = -Inf, UB = -0.99))
37 = Rotated BB1 (270 degrees; par: (LB = -Inf, UB = 0); par2: [LB = 0.99, UB = Inf))
38 = Rotated BB6 (270 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
39 = Rotated BB7 (270 degrees; par: (LB = -Inf, UB = -0.99]; par2: (LB = 0, UB = Inf))
40 = Rotated BB8 (270 degrees; par: (LB = -0.99, UB = 0); par2: (LB = 0, UB = Inf))
```

See the **VineCopula** package (Nagler et al., 2025) for additional details on copula parameterization.

Value

A numeric vector of technical efficiency values (one per observation).

Author(s)

Woraphon Yamaka, Paravee Maneejuk, and Nuttaphong Kaewtathip

References

Pipitpojanakarn, V., Maneejuk, P., Yamaka, W., & Sriboonchitta, S. (2016). Analysis of agricultural production in Asia and measurement of technical efficiency using copula-based stochastic frontier quantile model. In *International Symposium on Integrated Uncertainty in Knowledge Modelling and Decision Making* (pp. 701–714). Springer.

Nagler, T., Schepsmeier, U., Stoeber, J., Brechmann, E. C., Graeler, B., Erhardt, T., ... & Killiches, M. (2025, July). Package 'VineCopula'.

Examples

```
sim_data <- sfa_simu_quantile(
  n = 100,
  beta = c(1, 0.5, -0.3),
  sigV = 1,
  sigU = 1,
  tau = 0.5,
  family = 1,
  rho = 0.3,
  seed = 123
)

model <- copSQM(
  Y = sim_data$Y,
  X = sim_data$X,
  family = 1,
  tau = 0.5,
  RHO = 0.3,
  LB = -0.99,
  UB = 0.99,
  nSim = 50,
  seed = 123
)

te_values <- TE(
  theta = model$result[, "Estimate"],
  Y = sim_data$Y,
  X = sim_data$X,
  family = 1,
  tau = 0.5,
  nSim = 200,
  seed = 123,
  plot = TRUE
)

head(te_values)
```

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