

# Package ‘cpd’

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**Title** Complex Pearson Distributions

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**Maintainer** Silverio Vilchez-Lopez <svilchez@ujaen.es>

**Description** Probability mass function, distribution function, quantile function and random generation for the Complex Triparametric Pearson (CTP) and Complex Biparametric Pearson (CBP) distributions developed by Rodriguez-Avi et al (2003) <[doi:10.1007/s00362-002-0134-7](https://doi.org/10.1007/s00362-002-0134-7)>, Rodriguez-Avi et al (2004) <[doi:10.1007/BF02778271](https://doi.org/10.1007/BF02778271)> and Olmo-Jimenez et al (2018) <[doi:10.1080/00949655.2018.1482897](https://doi.org/10.1080/00949655.2018.1482897)>. The package also contains maximum-likelihood fitting functions for these models.

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**Author** Silverio Vilchez-Lopez [aut, cre],  
Maria Jose Olmo-Jimenez [aut],  
Jose Rodriguez-Avi [aut]

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cbp *The Complex Biparametric Pearson (CBP) Distribution*

---

### Description

Probability mass function, distribution function, quantile function and random generation for the Complex Biparametric Pearson (CBP) distribution with parameters  $b$  and  $\gamma$ .

### Usage

```

dcbp(x, b, gamma)

pcbp(q, b, gamma, lower.tail = TRUE)

qcbp(p, b, gamma, lower.tail = TRUE)

rcbp(n, b, gamma)

```

### Arguments

x	vector of (non-negative integer) quantiles.
b	parameter b (real)
gamma	parameter gamma (positive)
q	vector of quantiles.
lower.tail	if TRUE (default), probabilities are $P(X \leq x)$ , otherwise, $P(X > x)$ .
p	vector of probabilities.
n	number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.

### Details

The CBP distribution with parameters  $b$  and  $\gamma$  has pmf

$$f(x|b, \gamma) = C\Gamma(ib + x)\Gamma(-ib + x)/(\Gamma(\gamma + x)x!), x = 0, 1, 2, \dots$$

where  $i$  is the imaginary unit,  $\Gamma()$  the gamma function and

$$C = \Gamma(\gamma - ib)\Gamma(\gamma + ib)/(\Gamma(\gamma)\Gamma(ib)\Gamma(-ib))$$

the normalizing constant.

The CBP is a particular case of the CTP when  $a = 0$ .

The mean and the variance of the CBP distribution are  $E(X) = \mu = b^2/(\gamma - 1)$  and  $Var(X) = \mu(\mu + \gamma - 1)/(\gamma - 2)$  so  $\gamma > 2$ .

It is always overdispersed.

### Value

dcbp gives the pmf, pcbp gives the distribution function, qcbp gives the quantile function and rcbp generates random values.

### References

Jose Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ (2003). "A new class of discrete distributions with complex parameters." *Stat. Pap.*, **44**, 67–88. doi:10.1007/s0036200201347.

### See Also

Probability mass function, distribution function, quantile function and random generation for the CTP distribution: [dctp](#), [pctp](#), [qctp](#) and [rctp](#). Functions for maximum-likelihood fitting of the CBP distribution: [fitcbp](#).

### Examples

```
# Examples for the function dcbp
dcbp(3,2,5)
dcbp(c(3,4),2,5)

# Examples for the function pcbp
pcbp(3,2,3)
pcbp(c(3,4),2,3)

# Examples for the function qcbp
qcbp(0.5,2,3)
qcbp(c(.8,.9),2,3)

# Examples for the function rcbp
rcbp(10,1,3)
```

---

chisq.test2

*Pearson's Chi-squared Test for Count Data*

---

### Description

chisq.test2 performs Pearson chi-squared goodness-of-fit test for count data

**Usage**

```
chisq.test2(obs, p.esp, npar = NULL, grouping = FALSE)
```

**Arguments**

obs	a numeric vector with the counts
p.esp	a numeric vector with the expected probabilities of the same length of obs. They must sum 1.
npar	an integer specifying the number of parameters of the model. By default npar is NULL, so the degrees of freedom of the chi-squared statistics are the number of classes minus 1.
grouping	a logical indicating whether to group in classes with expected frequency greater than or equal to 5. By default grouping is FALSE.

**Value**

A list with class "htest" containing the following components:

- `statistic`: the value of the chi-squared test statistic.
- `df`: the degrees of freedom of the approximate chi-squared distribution.
- `p.value`: the p-value for the test.
- `observed`: the observed counts.
- `observed.grouped`: the observed counts grouped in classes with expected frequency greater or equal to 5.
- `expected`: the expected counts under the null hypothesis.
- `expected.grouped`: the expected counts under the null hypothesis grouped in classes with expected frequency greater or equal to 5.
- `residuals`: the Pearson residuals computed as  $(\text{observed} - \text{expected}) / \sqrt{\text{expected}}$ .

**Examples**

```
set.seed(123)
x <- rctp(500, -1.5, 1, 2)
table(x)
obs <- c(172, 264, 57, 6, 0, 1)
fit <- fitctp(x)
p.esp <- c(dctp(0:(length(obs)-1), fit$coefficients[1], fit$coefficients[2],
             fit$coefficients[3])[1:(length(obs)-1)], 1-sum(dctp(0:(length(obs)-1),
             fit$coefficients[1], fit$coefficients[2], fit$coefficients[3])[1:(length(obs)-1])))
chisq.test2(obs, p.esp)
chisq.test2(obs, p.esp, grouping = TRUE)
chisq.test2(obs, npar= 3, p.esp)
```

**Description**

Probability mass function, distribution function, quantile function and random generation for the Complex Triparametric Pearson (CTP) distribution with parameters  $a$ ,  $b$  and  $\gamma$ .

**Usage**

dctp(x, a, b, gamma)

pctp(q, a, b, gamma, lower.tail = TRUE)

qctp(p, a, b, gamma, lower.tail = TRUE)

rctp(n, a, b, gamma)

**Arguments**

x	vector of (non-negative integer) quantiles.
a	parameter a (real)
b	parameter b (real)
gamma	parameter $\gamma$ (positive)
q	vector of quantiles.
lower.tail	if TRUE (default), probabilities are $P(X \leq x)$ , otherwise, $P(X > x)$ .
p	vector of probabilities.
n	number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.

**Details**

The CTP distribution with parameters  $a$ ,  $b$  and  $\gamma$  has pmf

$$f(x|a, b, \gamma) = C\Gamma(a + ib + x)\Gamma(a - ib + x)/(\Gamma(\gamma + x)x!), x = 0, 1, 2, \dots$$

where  $i$  is the imaginary unit,  $\Gamma()$  the gamma function and

$$C = \Gamma(\gamma - a - ib)\Gamma(\gamma - a + ib)/(\Gamma(\gamma - 2a)\Gamma(a + ib)\Gamma(a - ib))$$

the normalizing constant.

If  $a = 0$  the CTP is a Complex Biparametric Pearson (CBP) distribution, so the pmf of the CBP distribution is obtained. If  $b = 0$  the CTP is an Extended Biparametric Waring (EBW) distribution, so the pmf of the EBW distribution is obtained.

The mean and the variance of the CTP distribution are  $E(X) = \mu = (a^2 + b^2)/(\gamma - 2a - 1)$  and  $Var(X) = \mu(\mu + \gamma - 1)/(\gamma - 2a - 2)$  so  $\gamma > 2a + 2$ .

It is underdispersed if  $a < -(\mu + 1)/2$ , equidispersed if  $a = -(\mu + 1)/2$  or overdispersed if  $a > -(\mu + 1)/2$ . In particular, if  $a \geq 0$  the CTP is always overdispersed.

## Value

dctp gives the pmf, pctp gives the distribution function, qctp gives the quantile function and rctp generates random values.

If  $a = 0$  the probability mass function, distribution function, quantile function and random generation function for the CBP distribution arise. If  $b = 0$  the probability mass function, distribution function, quantile function and random generation function for the EBW distribution arise.

## References

Jose Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ (2003). “A new class of discrete distributions with complex parameters.” *Stat. Pap.*, **44**, 67–88. doi:10.1007/s0036200201347.

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2004). “A triparametric discrete distribution with complex parameters.” *Stat. Pap.*, **45**, 81-95. doi:10.1007/BF02778271.

Olmo-Jimenez MJ, Rodriguez-Avi J, Cueva-Lopez V (2018). “A review of the CTP distribution: a comparison with other over- and underdispersed count data models.” *Journal of Statistical Computation and Simulation*, **88**(14), 2684-2706. doi:10.1080/00949655.2018.1482897.

Cueva-Lopez V, Olmo-Jimenez MJ, Rodriguez-Avi J (2021). “An Over and Underdispersed Biparametric Extension of the Waring Distribution.” *Mathematics*, **9**(170), 1-15. doi:10.3390/math9020170.

## See Also

Functions for maximum-likelihood fitting of the CTP, CBP and EBW distributions: [fitctp](#), [fitchbp](#) and [fitebw](#).

## Examples

```
# Examples for the function dctp
dctp(3,1,2,5)
dctp(c(3,4),1,2,5)
```

```
# Examples for the function pctp
pctp(3,1,2,3)
pctp(c(3,4),1,2,3)
```

```
# Examples for the function qctp
qctp(0.5,1,2,3)
qctp(c(.8,.9),1,2,3)
```

```
# Examples for the function rctp
rctp(10,1,1,3)
```

**Description**

Probability mass function, distribution function, quantile function and random generation for the Extended Biparametric Waring (EBW) distribution with parameters  $\alpha$  and  $\gamma$  (or  $\rho$ ).

**Usage**

```
debw(x, alpha, gamma, rho)
pebw(q, alpha, gamma, rho, lower.tail = TRUE)
qebw(p, alpha, gamma, rho, lower.tail = TRUE)
rebw(n, alpha, gamma, rho, lower.tail = TRUE)
```

**Arguments**

x	vector of (non-negative integer) quantiles.
alpha	parameter alpha (real)
gamma	parameter $\gamma$ (positive)
rho	parameter rho (positive)
q	vector of quantiles.
lower.tail	if TRUE (default), probabilities are $P(X \leq x)$ , otherwise, $P(X > x)$ .
p	vector of probabilities.
n	number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.

**Details**

The EBW distribution with parameters  $\alpha$  and  $\gamma$  has pmf

$$f(x|a, \alpha, \gamma) = C\Gamma(\alpha + x)^2 / (\Gamma(\gamma + x)x!), x = 0, 1, 2, \dots$$

where  $\Gamma(\cdot)$  is the gamma function and

$$C = \Gamma(\gamma - \alpha^2) / (\Gamma(\alpha)^2 \Gamma(\gamma - 2\alpha))$$

the normalizing constant.

There is an alternative parametrization in terms of  $\alpha$  and  $\rho = \gamma - 2\alpha > 0$  when  $\alpha > 0$ . So, introduce only  $\alpha$  and  $\gamma$  or  $\alpha$  and  $\rho$ , depending on the parametrization you wish to use.

The mean and the variance of the EBW distribution are  $E(X) = \mu = \alpha^2 / (\gamma - 2\alpha - 1)$  and  $Var(X) = \mu(\mu + \gamma - 1) / (\gamma - 2\alpha - 2)$  so  $\gamma > 2\alpha + 2$ .

It is underdispersed if  $\alpha < -(\mu + 1)/2$ , equidispersed if  $\alpha = -(\mu + 1)/2$  or overdispersed if  $\alpha > -(\mu + 1)/2$ . In particular, if  $\alpha \geq -0.5$  the EBW is overdispersed, whereas if  $\alpha < -1$  the EBW is underdispersed. In the case  $-1 < \alpha \leq -0.5$ , the EBW may be under-, equi- or overdispersed depending on the value of  $\gamma$ .

### Value

debw gives the pmf, pebw gives the distribution function, qebw gives the quantile function and rebw generates random values.

If  $\alpha > 0$  the probability mass function, distribution function, quantile function and random generation function for the UGW( $\alpha, \alpha, \rho$ ) distribution arise.

If  $\alpha < 0$  the probability mass function, distribution function, quantile function and random generation function for the CTP( $\alpha, 0, \gamma$ ) distribution arise.

### References

Jose Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ (2003). "A new class of discrete distributions with complex parameters." *Stat. Pap.*, **44**, 67–88. doi:10.1007/s0036200201347.

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2004). "A triparametric discrete distribution with complex parameters." *Stat. Pap.*, **45**, 81-95. doi:10.1007/BF02778271.

Olmo-Jimenez MJ, Rodriguez-Avi J, Cueva-Lopez V (2018). "A review of the CTP distribution: a comparison with other over- and underdispersed count data models." *Journal of Statistical Computation and Simulation*, **88**(14), 2684-2706. doi:10.1080/00949655.2018.1482897.

### See Also

Functions for maximum-likelihood fitting of the CTP and CBP distributions: [fitctp](#) and [fitcbp](#).

### Examples

```
# Examples for the function dctp
debw(3,1,rho=5)
debw(c(3,4),2,rho=5)

# Examples for the function pebw
pebw(3,2,rho=5)
pebw(c(3,4),2,rho=5)

# Examples for the function qebw
qebw(0.5,-2.1,gamma=0.1)
qebw(c(.8,.9),-2.1,gamma=0.1)
qebw(0.5,2,rho=5)
qebw(c(.8,.9),2,rho=5)

# Examples for the function rebw
rebw(10,2,rho=5)
rebw(10,-2.1,gamma=5)
```

---

fitcbp

*Maximum-likelihood fitting of the CBP distribution*


---

### Description

Maximum-likelihood fitting of the Complex Biparametric Pearson (CBP) distribution with parameters  $b$  and  $\gamma$ . Generic methods are `print`, `summary`, `coef`, `logLik`, `AIC`, `BIC` and `plot`.

### Usage

```
fitcbp(x, bstart = NULL, gammastart = NULL, method = "L-BFGS-B", control = list(), ...)
```

### Arguments

<code>x</code>	A numeric vector of length at least one containing only finite values.
<code>bstart</code>	A starting value for the parameter $b$ ; by default <code>NULL</code> .
<code>gammastart</code>	A starting value for the parameter $\gamma > 0$ ; by default <code>NULL</code> .
<code>method</code>	The method to be used in fitting the model. See 'Details'.
<code>control</code>	A list of parameters for controlling the fitting process.
<code>...</code>	Additional parameters.

### Details

If the starting values of the parameters  $b$  and  $\gamma$  are omitted (default option), they are computed by the method of moments (if possible; otherwise they must be entered).

The default method is "L-BFGS-B" (see details in `optim` function), but non-linear minimization (`nlm`) or those included in the `optim` function ("Nelder-Mead", "BFGS", "CG" and "SANN") may be used.

Standard error (SE) estimates for  $b$  and  $\gamma$  are provided by the default method; otherwise, SE for  $\gamma_0$  where  $\gamma = \exp(\gamma_0)$  is computed.

### Value

An object of class 'fitCBP' is a list containing the following components:

- `n`, the number of observations,
- `initialValues`, a vector with the starting values used,
- `coefficients`, the parameter ML estimates of the CTP distribution,
- `se`, a vector of the standard error estimates,
- `hessian`, a symmetric matrix giving an estimate of the Hessian at the solution found in the optimization of the log-likelihood function,
- `cov`, an estimate of the covariance matrix of the model coefficients,
- `corr`, an estimate of the correlation matrix of the model estimates,

- `loglik`, the maximized log-likelihood,
- `aic`, Akaike Information Criterion, minus twice the maximized log-likelihood plus twice the number of parameters,
- `bic`, Bayesian Information Criterion, minus twice the maximized log-likelihood plus twice the number of parameters,
- `code`, a code that indicates successful convergence of the fitter function used (see `nlm` and `optim` helps),
- `converged`, logical value that indicates if the optimization algorithms successful,
- `method`, the name of the fitter function used.

Generic functions:

- `print`: The print of a 'fitCBP' object shows the ML parameter estimates and their standard errors in parenthesis.
- `summary`: The summary provides the ML parameter estimates, their standard errors and the statistic and p-value of the Wald test to check if the parameters are significant. This summary also shows the loglikelihood, AIC and BIC values, as well as the results for the chi-squared goodness-of-fit test and the Kolmogorov-Smirnov test for discrete variables. Finally, the correlation matrix between parameter estimates appears.
- `coef`: It extracts the fitted coefficients from a 'fitCBP' object.
- `logLik`: It extracts the estimated log-likelihood from a 'fitCBP' object.
- `AIC`: It extracts the value of the Akaike Information Criterion from a 'fitCBP' object.
- `BIC`: It extracts the value of the Bayesian Information Criterion from a 'fitCBP' object.
- `plot`: It shows the plot of a 'fitCBP' object. Observed and theoretical probabilities, empirical and theoretical cumulative distribution functions or empirical cumulative probabilities against theoretical cumulative probabilities are the three plot types.

## References

Jose Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ (2003). "A new class of discrete distributions with complex parameters." *Stat. Pap.*, **44**, 67–88. doi:10.1007/s0036200201347.

## See Also

Plot of observed and theoretical frequencies for a CBP fit: [plot.fitCBP](#)

Maximum-likelihood fitting for the CTP distribution: [fitctp](#).

Maximum-likelihood fitting for the EBW distribution: [fitebw](#).

## Examples

```
set.seed(123)
x <- rcbp(500, 1.75, 3.5)
fitcbp(x)
summary(fitcbp(x, bstart = 1.1, gammastart = 3))
```

---

fitctp	<i>Maximum-likelihood fitting of the CTP distribution</i>
--------	---

---

### Description

Maximum-likelihood fitting of the Complex Triparametric Pearson (CTP) distribution with parameters  $a$ ,  $b$  and  $\gamma$ . Generic methods are `print`, `summary`, `coef`, `logLik`, `AIC`, `BIC` and `plot`.

### Usage

```
fitctp(x, astart = NULL, bstart = NULL, gammastart = NULL,
       method = "L-BFGS-B", control = list(), ...)
```

### Arguments

<code>x</code>	A numeric vector of length at least one containing only finite values.
<code>astart</code>	A starting value for the parameter $a > 0$ ; by default NULL.
<code>bstart</code>	A starting value for the parameter $b$ ; by default NULL.
<code>gammastart</code>	A starting value for the parameter $\gamma > \max(0, 2a)$ ; by default NULL.
<code>method</code>	The method to be used in fitting the model. See 'Details'.
<code>control</code>	A list of parameters for controlling the fitting process.
<code>...</code>	Additional parameters.

### Details

If the starting values of the parameters  $a$ ,  $b$  and  $\gamma$  are omitted (default option), they are computed by the method of moments (if possible; otherwise they must be entered).

The default method is "L-BFGS-B" (see details in `optim` function), but non-linear minimization (`nlm`) or those included in the `optim` function ("Nelder-Mead", "BFGS", "CG" and "SANN") may be used.

Standard error (SE) estimates for  $a$ ,  $b$  and  $\gamma$  are provided by the default method; otherwise, SE for  $\gamma_0$  where  $\gamma = \exp(\gamma_0)$  is computed.

### Value

An object of class 'fitCTP' is a list containing the following components:

- `n`, the number of observations,
- `initialValues`, a vector with the starting values used,
- `coefficients`, the parameter ML estimates of the CTP distribution,
- `se`, a vector of the standard error estimates,
- `hessian`, a symmetric matrix giving an estimate of the Hessian at the solution found in the optimization of the log-likelihood function,

- `cov`, an estimate of the covariance matrix of the model coefficients,
- `corr`, an estimate of the correlation matrix of the model estimates,
- `loglik`, the maximized log-likelihood,
- `aic`, Akaike Information Criterion, minus twice the maximized log-likelihood plus twice the number of parameters,
- `bic`, Bayesian Information Criterion, minus twice the maximized log-likelihood plus twice the number of parameters,
- `code`, a code that indicates successful convergence of the fitter function used (see `nlm` and `optim` helps),
- `converged`, logical value that indicates if the optimization algorithms successful,
- `method`, the name of the fitter function used.

#### Generic functions:

- `print`: The print of a 'fitCTP' object shows the ML parameter estimates and their standard errors in parenthesis.
- `summary`: The summary provides the ML parameter estimates, their standard errors and the statistic and p-value of the Wald test to check if the parameters are significant. This summary also shows the loglikelihood, AIC and BIC values, as well as the results for the chi-squared goodness-of-fit test and the Kolmogorov-Smirnov test for discrete variables. Finally, the correlation matrix between parameter estimates appears.
- `coef`: It extracts the fitted coefficients from a 'fitCTP' object.
- `logLik`: It extracts the estimated log-likelihood from a 'fitCTP' object.
- `AIC`: It extracts the value of the Akaike Information Criterion from a 'fitCTP' object.
- `BIC`: It extracts the value of the Bayesian Information Criterion from a 'fitCTP' object.
- `plot`: It shows the plot of a 'fitCTP' object. Observed and theoretical probabilities, empirical and theoretical cumulative distribution functions or empirical cumulative probabilities against theoretical cumulative probabilities are the three plot types.

#### References

- Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2004). "A triparametric discrete distribution with complex parameters." *Stat. Pap.*, **45**, 81-95. doi:10.1007/BF02778271.
- Olmo-Jimenez MJ, Rodriguez-Avi J, Cueva-Lopez V (2018). "A review of the CTP distribution: a comparison with other over- and underdispersed count data models." *Journal of Statistical Computation and Simulation*, **88**(14), 2684-2706. doi:10.1080/00949655.2018.1482897.

#### See Also

- Plot of observed and theoretical frequencies for a CTP fit: [plot.fitCTP](#)
- Maximum-likelihood fitting for the CBP distribution: [fitcbp](#).
- Maximum-likelihood fitting for the EBW distribution: [fitebw](#).

**Examples**

```
set.seed(123)
x <- rctp(500, -0.5, 1, 2)
fitctp(x)
summary(fitctp(x, astart = 1, bstart = 1.1, gammastart = 3))
```

fitebw

*Maximum-likelihood fitting of the EBW distribution***Description**

Maximum-likelihood fitting of the Extended Biparametric Waring (EBW) distribution with parameters  $\alpha$ ,  $\rho$  and  $\gamma$ . Generic methods are `print`, `summary`, `coef`, `logLik`, `AIC`, `BIC` and `plot`. The method to be used in fitting the model is "L-BFGS-B" which allows constraints for each variable (see details in `optim` function).

**Usage**

```
fitebw(x, alphastart = NULL, rhostart = NULL, gammastart = NULL,
       method = "L-BFGS-B", control = list(),...)
```

**Arguments**

<code>x</code>	A numeric vector of length at least one containing only finite values.
<code>alphastart</code>	A starting value for the parameter $\alpha$ ; by default NULL.
<code>rhostart</code>	A starting value for the parameter $\rho > 0$ ; by default NULL.
<code>gammastart</code>	A starting value for the parameter $\gamma > \max(0, 2\alpha)$ ; by default NULL.
<code>method</code>	The method to be used in fitting the model. The default method is "L-BFGS-B" ( <code>optim</code> ).
<code>control</code>	A list of parameters for controlling the fitting process.
<code>...</code>	Additional parameters.

**Details**

If the starting value for  $\alpha$  is positive, the parameterization  $(\alpha, \rho)$  is used; otherwise, the parameterization  $(\alpha, \gamma)$  is used.

If the starting values of the parameters  $\alpha$ ,  $\gamma$  or  $\rho$  are omitted (default option), they are computed by the method of moments (if possible; otherwise they must be entered).

The default method is "L-BFGS-B" (see details in `optim` function), but non-linear minimization (`nlm`) or those included in the `optim` function ("Nelder-Mead", "BFGS", "CG" and "SANN") may be used.

Standard error (SE) estimates for  $\alpha$ ,  $\gamma$  or  $\rho$  are provided by the default method; otherwise, SE for  $\alpha_0$  and  $\gamma_0$  where  $\alpha = -\exp(\alpha_0)$  and  $\gamma = \exp(\gamma_0)$  (or for  $\alpha_0$  and  $\rho_0$  where  $\alpha = \exp(\alpha_0)$  and  $\rho = \exp(\rho_0)$ ) are computed.

## Value

An object of class 'fitEBW' is a list containing the following components:

- n, the number of observations,
- initialValues, a vector with the starting values used,
- coefficients, the parameter ML estimates of the CTP distribution,
- se, a vector of the standard error estimates,
- hessian, a symmetric matrix giving an estimate of the Hessian at the solution found in the optimization of the log-likelihood function,
- cov, an estimate of the covariance matrix of the model coefficients,
- corr, an estimate of the correlation matrix of the model estimates,
- loglik, the maximized log-likelihood,
- aic, Akaike Information Criterion, minus twice the maximized log-likelihood plus twice the number of parameters,
- bic, Bayesian Information Criterion, minus twice the maximized log-likelihood plus twice the number of parameters,
- code, a code that indicates successful convergence of the fitter function used (see nlm and optim helps),
- converged, logical value that indicates if the optimization algorithms successful.
- method, the name of the fitter function used.

Generic functions:

- print: The print of a 'fitEBW' object shows the ML parameter estimates and their standard errors in parenthesis.
- summary: The summary provides the ML parameter estimates, their standard errors and the statistic and p-value of the Wald test to check if the parameters are significant. This summary also shows the loglikelihood, AIC and BIC values, as well as the results for the chi-squared goodness-of-fit test and the Kolmogorov-Smirnov test for discrete variables. Finally, the correlation matrix between parameter estimates appears.
- coef: It extracts the fitted coefficients from a 'fitEBW' object.
- logLik: It extracts the estimated log-likelihood from a 'fitEBW' object.
- AIC: It extracts the value of the Akaike Information Criterion from a 'fitEBW' object.
- BIC: It extracts the value of the Bayesian Information Criterion from a 'fitEBW' object.
- plot: It shows the plot of a 'fitEBW' object. Observed and theoretical probabilities, empirical and theoretical cumulative distribution functions or empirical cumulative probabilities against theoretical cumulative probabilities are the three plot types.

## References

Cueva-Lopez V, Olmo-Jimenez MJ, Rodriguez-Avi J (2021). "An Over and Underdispersed Biparametric Extension of the Waring Distribution." *Mathematics*, **9**(170), 1-15. doi:10.3390/math9020170.

**See Also**

Plot of observed and theoretical frequencies for a EBW fit: [plot.fitEBW](#)

Maximum-likelihood fitting for the CTP distribution: [fitctp](#).

Maximum-likelihood fitting for the CBP distribution: [fitcbp](#).

**Examples**

```
set.seed(123)
x <- rebw(500, 2, rho = 5)
fitebw(x)
summary(fitebw(x, alphastart = 1, rhostart = 5))
```

---

plot.fitCBP

*Plot of observed and theoretical frequencies for a CBP fit*

---

**Description**

Plot of observed and theoretical frequencies for a CBP fit

**Usage**

```
## S3 method for class 'fitCBP'
plot(x, plty = "FREQ", maxValue = NULL, ...)
```

**Arguments**

x	An object of class 'fitCBP'
plty	Plot type to be shown. Default is "FREQ" which shows the observed and theoretical frequencies for each value of the variable; "CDF" and "PP" are also available for plotting the empirical and theoretical cumulative distribution functions or the theoretical cumulative probabilities against the empirical cumulative probabilities, respectively.
maxValue	maxValue you want to appear in the plot
...	Additional parameters.

**Examples**

```
set.seed(123)
x <- rcbp(500, 1.75, 3.5)
fit <- fitcbp(x)
plot(fit)
plot(fit, plty = "CDF")
plot(fit, plty = "PP")
```

---

plot.fitCTP                      *Plot of observed and theoretical frequencies for a CTP fit*

---

### Description

Plot of observed and theoretical frequencies for a CTP fit

### Usage

```
## S3 method for class 'fitCTP'
plot(x, plty = "FREQ", maxValue = NULL, ...)
```

### Arguments

x	An object of class 'fitCTP'
plty	Plot type to be shown. Default is "FREQ" which shows the observed and theoretical frequencies for each value of the variable; "CDF" and "PP" are also available for plotting the empirical and theoretical cumulative distribution functions or the theoretical cumulative probabilities against the empirical cumulative probabilities, respectively.
maxValue	maxValue you want to appear in the plot
...	Additional parameters.

### Examples

```
set.seed(123)
x <- rctp(500, -0.5, 1, 2)
fit <- fitctp(x)
plot(fit)
plot(fit, plty = "CDF")
plot(fit, plty = "PP")
```

---

plot.fitEBW                      *Plot of observed and theoretical frequencies for a EBW fit*

---

### Description

Plot of observed and theoretical frequencies for a EBW fit

### Usage

```
## S3 method for class 'fitEBW'
plot(x, plty = "FREQ", maxValue = NULL, ...)
```

**Arguments**

<code>x</code>	An object of class 'fitEBW'
<code>plty</code>	Plot type to be shown. Default is "FREQ" which shows the observed and theoretical frequencies for each value of the variable; "CDF" and "PP" are also available for plotting the empirical and theoretical cumulative distribution functions or the theoretical cumulative probabilities against the empirical cumulative probabilities, respectively.
<code>maxValue</code>	maxValue you want to appear in the plot
<code>...</code>	Additional parameters.

**Examples**

```
set.seed(123)
x <- rebw(500, -0.25, 1)
fit <- fitebw(x)
plot(fit)
plot(fit, plty = "CDF")
plot(fit, plty = "PP")
```

---

varcomp

*Variance decomposition for a EBW fit*


---

**Description**

One of the main drawbacks of the Univariate Generalized Waring (UGW) distribution with parameters  $a$ ,  $k$  and  $\rho$  is that the first two parameters are interchangeable, so it is not possible to distinguish the variance components 'liability' and 'proneness' without additional information. To solve this problem, an EBW distribution (where these components are uniquely identifiable) can be used since, given a UGW distribution, there always exists an EBW close to it.

**Usage**

```
varcomp(object, ...)
```

**Arguments**

<code>object</code>	An object of class 'fitEBW'
<code>...</code>	Additional parameters.

**Value**

A data frame with the variance components (randomness, liability and proneness) in absolute and relative terms.

**Examples**

```
set.seed(123)
x <- rebw(500, 2, rho = 5)
fit <- fitebw(x, alphastart = 1, rhostart = 5)
varcomp(fit)
```

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