

# Package ‘decorrelate’

May 8, 2026

**Type** Package

**Title** Decorrelation Projection Scalable to High Dimensional Data

**Version** 0.1.6.4

**Date** 2025-06-18

**Description** Data whitening is a widely used preprocessing step to remove correlation structure since statistical models often assume independence. Here we use a probabilistic model of the observed data to apply a whitening transformation. This Gaussian Inverse Wishart Empirical Bayes model substantially reduces computational complexity, and regularizes the eigen-values of the sample covariance matrix to improve out-of-sample performance.

**License** Artistic-2.0

**Encoding** UTF-8

**URL** <https://gabrielhoffman.github.io/decorrelate/>

**BugReports** <https://github.com/GabrielHoffman/decorrelate/issues>

**Suggests** knitr, pander, whitening, CCA, yacca, mvtnorm, ggplot2, cowplot, colorRamps, RUnit, latex2exp, clusterGeneration, rmarkdown

**Depends** R (>= 4.2.0), methods

**Imports** Rfast, irlba, graphics, Rcpp, CholWishart, Matrix, utils, stats

**RoxygenNote** 7.3.2

**VignetteBuilder** knitr

**LinkingTo** Rcpp, RcppArmadillo

**NeedsCompilation** yes

**Author** Gabriel Hoffman [aut, cre] (ORCID:  
<<https://orcid.org/0000-0002-0957-0224>>)

**Maintainer** Gabriel Hoffman <gabriel.hoffman@mssm.edu>

**Repository** CRAN

**Date/Publication** 2025-07-18 13:10:03 UTC

## Contents

autocorr.mat . . . . .	2
averageCorr . . . . .	3
cov_transform . . . . .	5
decorrelate . . . . .	7
dmult . . . . .	8
eclairs . . . . .	9
eclairs-class . . . . .	11
eclairs_corMat . . . . .	12
eclairs_sq . . . . .	13
fastcca-class . . . . .	14
getCov . . . . .	15
getShrinkageParams . . . . .	16
getWhiteningMatrix . . . . .	17
kappa,eclairs-method . . . . .	18
lm_each_eclairs . . . . .	19
lm_eclairs . . . . .	21
logDet . . . . .	23
mahalanobisDistance . . . . .	24
mult_eclairs . . . . .	25
optimal_SVHT_coef . . . . .	26
plot,eclairs-method . . . . .	27
quadForm . . . . .	27
reform_decomp . . . . .	28
rmvnorm_eclairs . . . . .	29
sv_threshold . . . . .	30
whiten . . . . .	31
<b>Index</b>	<b>33</b>

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autocorr.mat	<i>Create auto-correlation matrix</i>
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### Description

Create auto-correlation matrix

### Usage

```
autocorr.mat(p, rho)
```

### Arguments

p	dimension of matrix
rho	autocorrelation value

**Value**

auto-matrix of size p with parameter rho

**Examples**

```
# Create 4x4 matrix with correlation between adjacent enties is 0.9
autocorr.mat(4, .9)
```

---

averageCorr	<i>Summarize correlation matrix</i>
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---

**Description**

Summarize correlation matrix as a scalar scalar value, given its SVD and shrinkage parameter. `averageCorr()` computes the average correlation, and `averageCorrSq()` computes the average squared correlation, where both exclude the diagonal terms. `sumInverseCorr()` computes the sum of entries in the inverse correlation matrix to give the 'effective number of independent features'. `effVariance()` evaluates effective variance of the correlation (or covariance) matrix. These values can be computed using the correlation matrix using standard MLE, or EB shrinkage.

**Usage**

```
averageCorr(ecl, method = c("EB", "MLE"))
averageCorrSq(ecl, method = c("EB", "MLE"))
sumInverseCorr(ecl, method = c("EB", "MLE"), absolute = TRUE)
effVariance(ecl, method = c("EB", "MLE"))
tr(ecl, method = c("EB", "MLE"))
```

**Arguments**

<code>ecl</code>	estimate of correlation matrix from <code>eclairs()</code> storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
<code>method</code>	compute average correlation for either the empirical Bayes (EB) shinken correlation matrix or the MLE correlation matrix
<code>absolute</code>	if TRUE (default) evaluate on absolute correlation matrix

**Details**

`tr()`: trace of the matrix. Sum of diagonals is the same as the sum of the eigen-values.

`averageCorr()`: The average correlation is computed by summing the off-diagonal values in the correlation matrix. The sum of all elements in a matrix is  $g = \sum_{i,j} C_{i,j} = 1^T C 1$ , where 1 is a vector of  $p$  elements with all entries 1. This last term is a quadratic form of the correlation matrix

that can be computed efficiently using the SVD and shrinkage parameter from `eclairs()`. Given the value of  $g$ , the average is computed by subtracting the diagonal values and dividing by the number of off-diagonal values:  $(g - p)/(p(p - 1))$ .

`averageCorrSq()`: The average squared correlation is computed using only the eigen-values. Surprisingly, this is a function of the variance of the eigen-values. This is reviewed by Watanabe (2022) and Durand and Le Roux (2017). Letting  $\lambda_i$  be the  $i^{\text{th}}$  sample or shrunk eigen-value, and  $\tilde{\lambda}$  be the mean eigen-value, then  $\sum_i (\lambda_i - \tilde{\lambda})^2 / p(p - 1) \tilde{\lambda}^2$ .

`sumInverseCorr()`: The 'effective number of independent features' is computed by summing the entires of the inverse covariance matrix. This has the form  $\sum_{i,j} C_{i,j}^{-1} = 1^T C^{-1} 1$ . This last term is a quadratic form of the correlation matrix that can be computed efficiently using the SVD and shrinkage parameter from `eclairs()` as described above.

`effVariance()`: Compute a metric of the amount of variation represented by a correlation (or covariance) matrix that is comparable across matrices of difference sizes. Proposed by Peña and Rodriguez (2003), the 'effective variance' is  $|C|^{\frac{1}{p}}$  where  $C$  is a correlation (or covariance matrix) between  $p$  variables. The effective variance is the mean of the log eigen-values.

## Value

value of summary statistic

## References

Durand, J. L., & Le Roux, B. (2017). Linkage index of variables and its relationship with variance of eigenvalues in PCA and MCA. *Statistica Applicata-Italian Journal of Applied Statistics*, (2-3), 123-135.

Peña, D., & Rodriguez, J. (2003). Descriptive measures of multivariate scatter and linear dependence. *Journal of Multivariate Analysis*, 85(2), 361-374.

Watanabe, J. (2022). Statistics of eigenvalue dispersion indices: Quantifying the magnitude of phenotypic integration. *Evolution*, 76(1), 4-28.

## Examples

```
library(Rfast)
n <- 200 # number of samples
p <- 800 # number of features

# create correlation matrix
Sigma <- matrix(.2, p, p)
diag(Sigma) <- 1

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# eclairs decomposition
ecl <- eclairs(Y, compute = "cor")

# Average correlation value
```

```

averageCorr(ecl)

# Average squared correlation value
averageCorrSq(ecl)

# Sum elements in inverse correlation matrix
# Gives the effective number of independent features
sumInverseCorr(ecl)

# Effective variance
effVariance(ecl)

```

---

cov\_transform

*Estimate covariance matrix after applying transformation*


---

### Description

Given covariance between features in the original data, estimate the covariance matrix after applying a transformation to each feature. Here we use the eclairs decomposition of the original covariance matrix, perform a parametric bootstrap and return the eclairs decomposition of the covariance matrix of the transformed data.

### Usage

```

cov_transform(
  ecl,
  f,
  n.boot,
  lambda = NULL,
  compute = c("covariance", "correlation"),
  seed = NULL
)

```

### Arguments

ecl	covariance/correlation matrix as an <a href="#">eclairs</a> object
f	function specifying the transformation.
n.boot	number of parametric bootstrap samples. Increasing n gives more precise estimates.
lambda	shrinkage parameter. If not specified, it is estimated from the data.
compute	evaluate either the "covariance" or "correlation" of X
seed	If you want the same to be generated again use a seed for the generator, an integer number

### Details

When the transformation is linear, these covariance matrices are the same.

**Value**

**eclairs** decomposition representing correlation/covariance on the transformed data

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# sample matrix from MVN with covariance Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma, seed = 1)

# perform eclairs decomposition
ecl <- eclairs(Y)

# Parametric bootstrap to estimate covariance
# after transformation

# transformation function
f <- function(x) log(x^2 + 1e-3)

# number of bootstrap samples
n_boot <- 5000

# Evaluate eclairs decomposition on bootstrap samples
ecl2 <- cov_transform(ecl, f = f, n_boot, lambda = 1e-4)

# Get full covariance matrix from eclairs decomposition
C1 <- getCov(ecl2)

# Parametric bootstrap samples directly from full covariance matrix
X <- rmvnorm(n_boot, rep(0, p), getCov(ecl))

# get covariance of transformed data
C2 <- cov(f(X))

# Evaluate differences
# small differences are due to Monte Carlo error from bootstrap sampling
range(C1 - C2)

# Plot entries from two covariance estimates
par(pty = "s")
plot(C1, C2, main = "Concordance between covariances")
abline(0, 1, col = "red")

# Same above but compute eclairs for correlation matrix
#-----

```

```

# Evaluate eclairs decomposition on bootstrap samples
ecl2 <- cov_transform(ecl, f = f, n_boot, compute = "correlation", lambda = 1e-4)

# Get full covariance matrix from eclairs decomposition
C1 <- getCor(ecl2)

# Parametric bootstrap samples directly from full covariance matrix
X <- rmvnorm(n_boot, rep(0, p), getCov(ecl))

# get correlation of transformed data
C2 <- cor(f(X))

# Evaluate differences
# small differences are due to Monte Carlo error from bootstrap sampling
range(C1 - C2)

# Plot entries from two correlation estimates
oldpar <- par(pty = "s")
plot(C1, C2, main = "Correlation between covariances")
abline(0, 1, col = "red")

par(oldpar)

```

decorrelate

*Decorrelation projection***Description**

Efficient decorrelation projection using [eclairs](#) decomposition

**Usage**

```
decorrelate(X, ecl, lambda, transpose = FALSE, alpha = -1/2)
```

**Arguments**

X	matrix to be transformed so <i>*columns*</i> are independent
ecl	estimate of covariance/correlation matrix from <a href="#">eclairs</a> storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
lambda	specify lambda and override value from ecl
transpose	logical, (default FALSE) indicating if X should be transposed first
alpha	default = -1/2. Exponent of eigen-values

**Details**

Apply a decorrelation transform using the implicit covariance approach to avoid directly evaluating the covariance matrix

**Value**

a matrix following the decorrelation transformation

**Examples**

```
library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)

# eclairs decomposition
ecl <- eclairs(Y)

# whitened Y
Y.transform <- decorrelate(Y, ecl)
#
```

---

dmult

---

*Multiply by diagonal matrix*


---

**Description**

Multiply by diagonal matrix using efficient algorithm

**Usage**

```
dmult(M, v, side = c("left", "right"))
```

**Arguments**

M	matrix
v	vector with entries forming a diagonal matrix matching the dimensions of M depending on the value of side
side	is the matrix M "left" or "right" multiplied by the diagonal matrix

**Details**

Naively multiplying by a diagonal matrix with  $p$  entries takes  $\mathcal{O}(p^2)$ , even though most values in the diagonal matrix are zero. R has built in syntax to scale *columns*, so `diag(v) %*% M` can be evaluated with `v * M`. This is often difficult to remember, hard to look up, and scaling *rows* requires `t(t(M) * v)`. This is hard to read and write, and requires two transpose operations.

Here, `dmult()` uses Rcpp to evaluate the right multiplication efficiently, and uses  $v * M$  for left multiplication. This gives good performance and readability.

In principle, the Rcpp code can be modified to use BLAS for better performance by treating a `NumericMatrix` as a C array. But this is not currently a bottleneck

### Value

matrix product

### Examples

```
# right multiply
# mat %*% diag(v)
n <- 30
p <- 20
mat <- matrix(n * p, n, p)
v <- rnorm(p)

A <- dmult(mat, v, side = "right")
B <- mat %*% diag(v)
range(A - B)

# left multiply
# diag(v) %*% mat
n <- 30
p <- 20
mat <- matrix(n * p, p, n)
v <- rnorm(p)

A <- dmult(mat, v, side = "left")
B <- diag(v) %*% mat
range(A - B)
```

---

eclairs

*Estimate covariance/correlation with low rank and shrinkage*

---

### Description

Estimate the covariance/correlation between columns as the weighted sum of a low rank matrix and a scaled identity matrix. The weight acts to shrink the sample correlation matrix towards the identity matrix or the sample covariance matrix towards a scaled identity matrix with constant variance. An estimate of this form is useful because it is fast, and enables fast operations downstream. The method is based on the Gaussian Inverse Wishart Empirical Bayes (GIW-EB) model.

### Usage

```
eclairs(
  X,
  k,
```

```

lambda = NULL,
compute = c("covariance", "correlation"),
n.samples = nrow(X)
)

```

### Arguments

<code>X</code>	data matrix with <code>n</code> samples as rows and <code>p</code> features as columns
<code>k</code>	the rank of the low rank component
<code>lambda</code>	shrinkage parameter. If not specified, it is estimated from the data.
<code>compute</code>	evaluate either the "covariance" or "correlation" of <code>X</code>
<code>n.samples</code>	number of samples data is from. Usually <code>nrow(X)</code> , but can be other values in special cases.

### Details

Compute  $U, d^2$  to approximate the correlation matrix between columns of data matrix  $X$  by  $U \text{diag}(d^2(1-\lambda))U^T + I\nu\lambda$ . When computing the covariance matrix, scale by the standard deviation of each feature.

### Value

`eclairs` object storing:

**U:** orthonormal matrix with `k` columns representing the low rank component

**dSq:** eigen-values so that  $U \text{diag}(d^2)U^T$  is the low rank component

**lambda:** shrinkage parameter  $\lambda$  for the scaled diagonal component

**sigma:** standard deviations of input columns

**nu:** diagonal value,  $\nu$ , of target matrix in shrinkage

**n:** number of samples (i.e. rows) in the original data

**p:** number of features (i.e. columns) in the original data

**k:** rank of low rank component

**rownames:** sample names from the original matrix

**colnames:** features names from the original matrix

**method:** method used for decomposition

**call:** the function call

### Examples

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

```

```

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# eclairs decomposition: covariance
ecl <- eclairs(Y, compute = "covariance")

ecl

# eclairs decomposition: correlation
ecl <- eclairs(Y, compute = "correlation")

ecl

```

---

eclairs-class

*Class eclairs*


---

## Description

Class eclairs

## Details

Object storing:

**U:** orthonormal matrix with k columns representing the low rank component

**dSq:** eigen-values so that  $U \text{diag}(d^2) U^T$  is the low rank component

**lambda:** shrinkage parameter  $\lambda$  for the scaled diagonal component

**sigma:** standard deviations of input columns

**nu:** diagonal value,  $\nu$ , of target matrix in shrinkage

**n:** number of samples (i.e. rows) in the original data

**p:** number of features (i.e. columns) in the original data

**k:** rank of low rank component

**rownames:** sample names from the original matrix

**colnames:** features names from the original matrix

**method:** method used for decomposition

**call:** the function call

---

eclairs_corMat	<i>Estimate covariance/correlation with low rank and shrinkage</i>
----------------	--

---

### Description

Estimate covariance/correlation with low rank and shrinkage from the correlation matrix

### Usage

```
eclairs_corMat(C, n, k = min(n, nrow(C)), lambda = NULL)
```

### Arguments

C	sample correlation matrix between features
n	number of samples used to estimate the sample correlation matrix
k	the rank of the low rank component. Defaults to min of sample size and feature number, $\min(n, p)$ .
lambda	shrinkage parameter. If not specified, it is estimated from the data.

### Value

[eclairs](#) object storing:

**U:** orthonormal matrix with k columns representing the low rank component

**dSq:** eigen-values so that  $U \text{diag}(d^2) U^T$  is the low rank component

**lambda:** shrinkage parameter  $\lambda$  for the scaled diagonal component

**sigma:** standard deviations of input columns

**nu:** diagonal value,  $\nu$ , of target matrix in shrinkage

**n:** number of samples (i.e. rows) in the original data

**p:** number of features (i.e. columns) in the original data

**k:** rank of low rank component

**rownames:** sample names from the original matrix

**colnames:** features names from the original matrix

**method:** method used for decomposition

**call:** the function call

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", 1:n)
colnames(Y) <- paste0("gene_", 1:p)

# eclairs decomposition
eclairs(Y, compute = "correlation")

# eclairs decomposition from correlation matrix
eclairs_corMat(cor(Y), n = n)

```

---

eclairs\_sq

*Compute eclairs decomp of squared correlation matrix*


---

**Description**

Given the eclairs decomp of  $C$ , compute the eclairs decomp of  $C^2$

**Usage**

```
eclairs_sq(ecl, rank1 = ecl$k, rank2 = Inf, varianceFraction = 1)
```

**Arguments**

ecl	estimate of correlation matrix from eclairs() storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
rank1	use the first 'rank' singular vectors from the SVD. Using increasing rank1 will increase the accuracy of the estimation. But note that the computational complexity is $O(P \text{ choose}(\text{rank}, 2))$ , where $P$ is the number of features in the dataset
rank2	rank of eclairs() decomposition returned
varianceFraction	fraction of variance to retain after truncating trailing eigen values

**Details**

Consider a data matrix  $X_{N \times P}$  of  $P$  features and  $N$  samples where  $N \ll P$ . Let the columns of  $X$  be scaled so that  $C_{P \times P} = XX^T$ .  $C$  is often too big to compute directly since it is  $O(P^2)$  and  $O(P^3)$  to invert. But we can compute the SVD of  $X$  in  $O(PN^2)$ . The goal is to compute the SVD of the matrix  $C^2$ , given only the SVD of  $C$  in less than  $O(P^2)$  time. Here we compute this SVD

of  $C^2$  in  $O(PN^4)$  time, which is tractible for small  $N$ . Moreover, if we use an SVD  $X = UDV^T$  with of rank  $R$ , we can approximate the SVD of  $C^2$  in  $O(PR^4)$  using only  $D$  and  $V$

### Value

compute the eclairs of  $C^2$

### Examples

```
# Compute correlations directly and using eclairs decomp

n <- 600 # number of samples
p <- 100 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- Rfast::rmvnorm(n, rep(0, p), sigma = Sigma, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# correlation computed directly
C <- cor(Y)

# correlation from eclairs decomposition
ecl <- eclairs(Y, compute = "cor")
C.eclairs <- getCor(ecl, lambda = 0)

all.equal(C, C.eclairs)

# Correlation of Y^2
#-----

# exact quadratic way
C <- 2 * cor(Y)^2

# faster low rank
ecl2 <- eclairs_sq(ecl)
C.eclairs <- 2 * getCor(ecl2, lambda = 0)

all.equal(C.eclairs, C)
```

---

fastcca-class

*Class fastcca*

---

### Description

Class fastcca

**Details**

Object storing:

**n.comp:** number of canonical components

**cors:** canonical correlations

**x.coefs:** canonical coefficients for X

**x.vars:** canonical variates for X

**y.coefs:** canonical coefficients for Y

**y.vars:** canonical variates for Y

**lambdas:** shrinkage parameters from eclairs

---

getCov

*Get full covariance/correlation matrix from [eclairs](#)*

---

**Description**

Get full covariance/correlation matrix from [eclairs](#) decomposition

**Usage**

```
getCov(ecl, lambda, ...)
```

```
getCor(ecl, lambda, ...)
```

```
## S4 method for signature 'eclairs'
getCov(ecl, lambda, ...)
```

```
## S4 method for signature 'eclairs'
getCor(ecl, lambda, ...)
```

**Arguments**

ecl            eclairs decomposition

lambda        shrinkage parameter for the convex combination.

...            other arguments

**Details**

The full matrix is computationally expensive to compute and uses a lot of memory for large  $p$ . So it is better to use [decorrelate](#) or [mult\\_eclairs](#) to perform projections in  $O(np)$  time.

**Value**

$p \times p$  covariance/correlation matrix

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# eclairs decomposition
ecl <- eclairs(Y)

# extract covariance implied by eclairs decomposition
getCov(ecl)[1:3, 1:3]

```

---

getShrinkageParams      *Estimate shrinkage parameter by empirical Bayes*

---

**Description**

Estimate shrinkage parameter by empirical Bayes

**Usage**

```
getShrinkageParams(ecl, k = ecl$k, minimum = 1e-04, lambda = NULL)
```

**Arguments**

ecl	eclairs() decomposition
k	number of singular vectors to use
minimum	minimum value of lambda
lambda	(default: NULL) If NULL, estimate lambda from data. Else evaluate logML using specified lambda value.

**Details**

Estimate shrinkage parameter for covariance matrix estimation using empirical Bayes method (Hannart and Naveau, 2014; Leday and Richardson 2019). The shrinkage estimate of the covariance matrix is  $(1 - \lambda)\hat{\Sigma} + \lambda\nu I$ , where  $\hat{\Sigma}$  is the sample covariance matrix, given a value of  $\lambda$ . A large value of  $\lambda$  indicates more weight on the prior.

**Value**

value  $\lambda$  and  $\nu$  indicating the shrinkage between sample and prior covariance matrices.

**References**

Hannart, A., & Naveau, P. (2014). Estimating high dimensional covariance matrices: A new look at the Gaussian conjugate framework. *Journal of Multivariate Analysis*, 131, 149-162.

Leday, G. G., & Richardson, S. (2019). Fast Bayesian inference in large Gaussian graphical models. *Biometrics*, 75(4), 1288-1298.

**Examples**

```
library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# eclairs decomposition: covariance
ecl <- eclairs(Y, compute = "correlation")

# For full SVD
getShrinkageParams(ecl)

# For truncated SVD at k = 20
getShrinkageParams(ecl, k = 20)
```

---

getWhiteningMatrix      *Get whitening matrix*

---

**Description**

Get whitening matrix implied by [eclairs](#) decomposition

**Usage**

```
getWhiteningMatrix(ecl, lambda)
```

**Arguments**

ecl estimate of covariance/correlation matrix from `eclairs` storing  $U$ ,  $d_1^2$ ,  $\lambda$  and  $\nu$   
 lambda specify lambda and override value from ecl

**Value**

whitening matrix

**Examples**

```
library(Rfast)

n <- 2000
p <- 3

Y <- matrnorm(n, p, seed = 1) * 10

# decorrelate with implicit whitening matrix
# give same result as explicit whitening matrix
ecl <- eclairs(Y, compute = "covariance")

# get explicit whitening matrix
W <- getWhiteningMatrix(ecl)

# apply explicit whitening matrix
Z1 <- tcrossprod(Y, W)

# use implicit whitening matrix
Z2 <- decorrelate(Y, ecl)

range(Z1 - Z2)
```

---

kappa,eclairs-method *Compute condition number*

---

**Description**

Compute condition number of matrix from `eclairs` decomposition

**Usage**

```
## S4 method for signature 'eclairs'
kappa(z, lambda = NULL)
```

**Arguments**

z `eclairs()` decomposition  
 lambda specify lambda to override value from z

**Value**

condition number of the correlation matrix. If  $z$  is a covariance matrix, kappa is only computed for the correlation component

**Examples**

```
library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# eclairs decomposition
ecl <- eclairs(Y, compute = "correlation")

# compute condition number
kappa(ecl)
```

---

lm\_each\_eclairs

*Fit linear model on each feature after decorrelating*

---

**Description**

Fit linear model on each feature after applying decorrelation projection to response and predictors.

**Usage**

```
lm_each_eclairs(
  formula,
  data,
  X,
  ecl,
  subset,
  weights,
  na.action,
  method = "qr",
  model = TRUE,
  x = FALSE,
  y = FALSE,
  qr = TRUE,
```

```

singular.ok = TRUE,
contrasts = NULL,
offset,
...
)

```

### Arguments

formula	an object of class 'formula' (or one that can be coerced to that class): a symbolic description of the model to be fitted.
data	a matrix or data.frame containing the variables in the model
X	matrix or data.frame where each column stores a predictor to be evaluated by the regression model one at a time. The $i^{th}$ model includes $X[, i]$ as a predictor.
ecl	estimate of covariance/correlation matrix from <a href="#">eclairs</a> storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
subset	same as for <a href="#">lm</a>
weights	same as for <a href="#">lm</a>
na.action	same as for <a href="#">lm</a>
method	same as for <a href="#">lm</a>
model	same as for <a href="#">lm</a>
x	same as for <a href="#">lm</a>
y	same as for <a href="#">lm</a>
qr	same as for <a href="#">lm</a>
singular.ok	same as for <a href="#">lm</a>
contrasts	same as for <a href="#">lm</a>
offset	same as for <a href="#">lm</a>
...	other arguments passed to <code>lm()</code>

### Value

data.frame with columns beta, se, tsat, pvalue storing results for regression model fit for each feature

### Examples

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)

```

```

# eclairs decomposition
ecl <- eclairs(Y)

# simulate covariates
data <- data.frame(matrnorm(p, 2, seed = 1))
colnames(data) <- paste0("v", 1:2)

# simulate response
y <- rnorm(p)

# Simulate 1000 features to test
X <- matrnorm(p, 1000, seed = 1)
colnames(X) <- paste0("set_", seq(ncol(X)))

# Use linear model to test each feature stored as columns in X
res <- lm_each_eclairs(y ~ v1 + v2, data, X, ecl)

head(res)

# Analysis after non-linear transform
#-----

# Apply function to transforme data
f <- function(x) log(x^2 + 0.001)

# evaluate covariance of transformed data
ecl_transform <- cov_transform(ecl, f, 100)

# Use linear model to test each feature stored as columns in X
# in data transformed by f()
res2 <- lm_each_eclairs(f(y) ~ v1 + v2, data, X, ecl_transform)

head(res)

```

---

lm\_eclairs

*Fit linear model after decorrelating*


---

## Description

Fit linear model after applying decorrelation projection to response and predictors.

## Usage

```

lm_eclairs(
  formula,
  data,
  ecl,
  subset,
  weights,

```

```

na.action,
method = "qr",
model = TRUE,
x = FALSE,
y = FALSE,
qr = TRUE,
singular.ok = TRUE,
contrasts = NULL,
offset,
...
)

```

### Arguments

formula	an object of class 'formula' (or one that can be coerced to that class): a symbolic description of the model to be fitted.
data	a matrix or data.frame containing the variables in the model
ecl	estimate of covariance/correlation matrix from <a href="#">eclairs</a> storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
subset	same as for <a href="#">lm</a>
weights	same as for <a href="#">lm</a>
na.action	same as for <a href="#">lm</a>
method	same as for <a href="#">lm</a>
model	same as for <a href="#">lm</a>
x	same as for <a href="#">lm</a>
y	same as for <a href="#">lm</a>
qr	same as for <a href="#">lm</a>
singular.ok	same as for <a href="#">lm</a>
contrasts	same as for <a href="#">lm</a>
offset	same as for <a href="#">lm</a>
...	same as for <a href="#">lm</a>

### Details

This function fit a linear regression to the transformed response, and transformed design matrix. Note that the design matrix, not just the data.frame of variables is transformed so that 1) factors are transformed and 2) the intercept term is transformed.

### Value

Object of class `lm` returned by function [lm](#)

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)

# eclairs decomposition
ecl <- eclairs(Y)

# simulate covariates
data <- data.frame(matrnorm(p, 2, seed = 1))
colnames(data) <- paste0("v", 1:2)

# simulate response
y <- rnorm(p)

# fit linear model on transformed data
lm_eclairs(y ~ v1 + v2, data, ecl)

```

---

logDet

*Evaluate the log determinant*


---

**Description**

Evaluate the log determinant of the matrix

**Usage**

```
logDet(ecl, alpha = 1)
```

**Arguments**

ecl	estimate of covariance/correlation matrix from eclairs() storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
alpha	exponent to be applied to eigen-values

**Value**

log determinant

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# eclairs decomposition
ecl <- eclairs(Y)

logDet(ecl)

```

---

mahalanobisDistance    *Mahalanobis Distance*

---

**Description**

Mahalanobis Distance using eclairs() decomposition

**Usage**

```
mahalanobisDistance(ecl, X, lambda, center = FALSE)
```

**Arguments**

ecl	estimate of covariance/correlation matrix from eclairs storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
X	data matrix
lambda	specify lambda and override value from 'ecl'
center	logical: should columns be centered internally

**Details**

Evaluate quadratic form  $(X - \mu)^T \Sigma^{-1} (X - \mu)$  where covariance is estimated from finite sample

**Value**

array of distances

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)

# eclairs decomposition
ecl <- eclairs(Y)

# Mahalanobis distance after mean centering
Y_center <- scale(Y, scale = FALSE)
mu <- colMeans(Y)

# Standard R method
a <- mahalanobis(Y, mu, cov = cov(Y))

# distance using eclairs decomposition, no shrinkage
b <- mahalanobisDistance(ecl, Y_center, lambda = 0)
range(a - b)

# with shrinkage
d <- mahalanobisDistance(ecl, Y_center)

# centering internally
e <- mahalanobisDistance(ecl, Y, center = TRUE)
range(d - e)
#

```

---

mult\_eclairs

*Multiply by eclairs matrix*


---

**Description**

Multiply by [eclairs](#) matrix using special structure to achieve linear instead of cubic time complexity.

**Usage**

```
mult_eclairs(X, U1, dSq1, lambda, nu, alpha, sigma, transpose = FALSE)
```

**Arguments**

X	matrix to be transformed so <i>*columns*</i> are independent
U1	orthonormal matrix with k columns representing the low rank component

dSq1	eigen values so that $U_1 \text{diag}(d_1^2) U_1^T$ is the low rank component
lambda	shrinkage parameter for the convex combination.
nu	diagonal value of target matrix in shrinkage
alpha	exponent to be evaluated
sigma	standard deviation of each feature
transpose	logical, (default FALSE) indicating if X should be transposed first

### Details

Let  $\Sigma = U_1 \text{diag}(d_1^2) U_1^T * (1 - \lambda) + \text{diag}(\nu \lambda, p)$ , where  $\lambda$  shrinkage parameter for the convex combination between a low rank matrix and the diagonal matrix with values  $\nu$ .

Evaluate  $X \Sigma^\alpha$  using special structure of the [eclairs](#) decomposition in  $O(k^2 p)$  when there are  $k$  components in the decomposition.

### Value

a matrix product

---

optimal_SVHT_coef	<i>Optimal Hard Threshold for Singular Values</i>
-------------------	---

---

### Description

A function for the calculation of the coefficient determining optimal location of hard threshold for matrix denoising by singular values hard thresholding when noise level is known or unknown. Recreation of MATLAB code by Matan Gavish and David Donoho.

### Usage

```
optimal_SVHT_coef(beta, sigma_known = FALSE)
```

### Arguments

beta	A single value or a vector that represents aspect ratio $m/n$ of the matrix to be denoised. $0 < \text{beta} \leq 1$ .
sigma_known	A logical value. TRUE if noise level known, FALSE if unknown.

### Value

Optimal location of hard threshold, up the median data singular value (sigma unknown) or up to  $\text{sigma} * \sqrt{n}$  (sigma known); a vector of the same dimension as beta, where  $\text{coef}[i]$  is the coefficient corresponding to  $\text{beta}[i]$ .

### References

Gavish, M., & Donoho, D. L. (2014). The optimal hard threshold for singular values is  $4/\sqrt{3}$ . IEEE Transactions on Information Theory, 60(8), 5040-5053.

---

plot,eclairs-method     *Plot eclairs object*

---

**Description**

Plot eclairs object

**Usage**

```
## S4 method for signature 'eclairs'
plot(x, y, ...)
```

**Arguments**

x	eclairs object
y	extra argument, not used
...	additional arguments

**Value**

plot

---

quadForm     *Evaluate quadratic form*

---

**Description**

Evaluate quadratic form

**Usage**

```
quadForm(ecl, A, alpha = -1/2)
```

**Arguments**

ecl	estimate of covariance/correlation matrix from eclairs storing $U$ , $d_1^2$ , $\lambda$ and $\nu$
A	matrix
alpha	default = -1/2. Exponent of eigen-values

**Details**

Evaluate quadratic form  $A^T \Sigma^{2\alpha} A$

**Value**

scalar value

**Examples**

```

library(Rfast)
n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)

# eclairs decomposition
ecl <- eclairs(Y)

# return scalar
quadForm(ecl, Y[1, ])

# return matrix
quadForm(ecl, Y[1:2, ])

```

---

reform\_decomp

*Recompute eclairs after dropping features*


---

**Description**

Recompute eclairs after dropping features

**Usage**

```
reform_decomp(ecl, k = ecl$k, drop = NULL)
```

**Arguments**

ecl	covariance/correlation matrix as an <a href="#">eclairs</a> object
k	the rank of the low rank component
drop	array of variable names to drop.

**Details**

Reform the dataset from the eclairs decomposition, drop features, then recompute the eclairs decomposition. If the original SVD/eigen was truncated, then the reconstruction of the original data will be approximate. Note that the target shrinkage matrix is the same as in ecl, so  $\nu$  is not recomputed from the retained features.

**Value**

[eclairs](#) decomposition for a subset of features

**Examples**

```

library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)
rownames(Y) <- paste0("sample_", seq(n))
colnames(Y) <- paste0("gene_", seq(p))

# Correlation
#-----

# eclairs decomposition
Sigma.eclairs <- eclairs(Y, compute = "correlation")

# features to drop
drop <- paste0("gene_", 1:100)

# Compute SVD on subset of eclairs decomposition
ecl1 <- reform_decomp(Sigma.eclairs, drop = drop)

ecl1

```

---

rmvnorm\_eclairs

*Draw from multivariate normal and t distributions*


---

**Description**

Draw from multivariate normal and t distributions using eclairs decomposition

**Usage**

```
rmvnorm_eclairs(n, mu, ecl, v = Inf, seed = NULL)
```

**Arguments**

n	sample size
mu	mean vector
ecl	covariance matrix as an <a href="#">eclairs</a> object
v	degrees of freedom, defaults to Inf. If finite, uses a multivariate t distribution
seed	If you want the same to be generated again use a seed for the generator, an integer number

**Details**

Draw from multivariate normal and t distributions using eclairs decomposition. If the (implied) covariance matrix is  $p \times p$ , the standard approach is  $O(p^3)$ . Taking advantage of the previously computed eclairs decomposition of rank  $k$ , this can be done in  $O(pk^2)$ .

**Value**

matrix where rows are samples from multivariate normal or t distribution where columns have covariance specified by ecl

**Examples**

```
library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma, seed = 1)

# perform eclairs decomposition
ecl <- eclairs(Y)

# draw from multivariate normal
n <- 10000
mu <- rep(0, ncol(Y))

# using eclairs decomposition
X.draw1 <- rmvnorm_eclairs(n, mu, ecl)

# using full covariance matrix implied by eclairs model
X.draw2 <- rmvnorm(n, mu, getCov(ecl))

# assess difference between covariances from two methods
range(cov(X.draw1) - cov(X.draw2))

# compare covariance to the covariance matrix used to simulated the data
range(cov(X.draw1) - getCov(ecl))
```

---

sv\_threshold

*Singular value thresholding*


---

**Description**

Singular value thresholding evaluates the optimal number of singular values to retain

**Usage**

```
sv_threshold(n, p, d)
```

**Arguments**

n	number of samples
p	number of features
d	singular values

**Value**

Number of singular values to retain

**References**

Gavish, M., & Donoho, D. L. (2014). The optimal hard threshold for singular values is  $4/\sqrt{3}$ . *IEEE Transactions on Information Theory*, 60(8), 5040-5053.

**Examples**

```
# simulate data

n <- 500
p <- 5000
Y <- Rfast::matrnorm(n, p, seed = 1)

# SVD
dcmp <- svd(Y)

# how many components to retain
sv_threshold(n, p, dcmp$d)

# in this case the data has no structure, so no components are retained
```

---

whiten

*Decorrelation projection + eclairs*

---

**Description**

Efficient decorrelation projection using eclairs decomposition

**Usage**

```
whiten(X, k = ncol(X), lambda = NULL)
```

**Arguments**

X                    matrix to be transformed so \*columns\* are independent  
k                    the rank of the low rank component  
lambda              specify lambda and override value estimated by eclairs()

**Value**

data rotated and scaled according to the regularized sample covariance of the input data

**Examples**

```
library(Rfast)

n <- 800 # number of samples
p <- 200 # number of features

# create correlation matrix
Sigma <- autocorr.mat(p, .9)

# draw data from correlation matrix Sigma
Y <- rmvnorm(n, rep(0, p), sigma = Sigma * 5.1, seed = 1)

# eclairs decomposition
ecl <- eclairs(Y)

# whitened Y
Y.transform <- decorrelate(Y, ecl)

# Combine eclairs and decorrelate into one step
Y.transform2 <- whiten(Y)
```

# Index

autocorr.mat, 2  
averageCorr, 3  
averageCorrSq (averageCorr), 3  
cov\_transform, 5  
decorrelate, 7, 15  
dmult, 8  
eclairs, 5–7, 9, 10, 12, 15, 17, 18, 20, 22, 25,  
26, 28, 29  
eclairs-class, 11  
eclairs\_corMat, 12  
eclairs\_sq, 13  
effVariance (averageCorr), 3  
fastcca-class, 14  
getCor (getCov), 15  
getCor, eclairs-method (getCov), 15  
getCov, 15  
getCov, eclairs-method (getCov), 15  
getShrinkageParams, 16  
getWhiteningMatrix, 17  
kappa, eclairs-method, 18  
lm, 20, 22  
lm\_each\_eclairs, 19  
lm\_eclairs, 21  
logDet, 23  
mahalanobisDistance, 24  
mult\_eclairs, 15, 25  
optimal\_SVHT\_coef, 26  
plot, eclairs-method, 27  
quadForm, 27  
reform\_decomp, 28  
rmvnorm\_eclairs, 29  
sumInverseCorr (averageCorr), 3  
sv\_threshold, 30  
tr (averageCorr), 3  
whiten, 31