

# Package ‘deepgp’

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**Type** Package

**Title** Bayesian Deep Gaussian Processes using MCMC

**Version** 1.2.1

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**Depends** R (>= 3.6)

**Description** Performs Bayesian posterior inference for deep Gaussian processes following Sauer, Gramacy, and Higdon (2023, <[doi:10.48550/arXiv.2012.08015](https://doi.org/10.48550/arXiv.2012.08015)>). See Sauer (2023, <<http://hdl.handle.net/10919/114845>>) for comprehensive methodological details and <<https://bitbucket.org/gramacylab/deepgp-ex/>> for a variety of coding examples. Models are trained through MCMC including elliptical slice sampling of latent Gaussian layers and Metropolis-Hastings sampling of kernel hyperparameters. Gradient-enhancement and gradient predictions are offered following Booth (2025, <[doi:10.48550/arXiv.2512.18066](https://doi.org/10.48550/arXiv.2512.18066)>). Vecchia approximation for faster computation is implemented following Sauer, Cooper, and Gramacy (2023, <[doi:10.48550/arXiv.2204.02904](https://doi.org/10.48550/arXiv.2204.02904)>). Optional monotonic warpings are implemented following Barnett et al. (2025, <[doi:10.48550/arXiv.2408.01540](https://doi.org/10.48550/arXiv.2408.01540)>). Downstream tasks include sequential design through active learning Cohn/integrated mean squared error (ALC/IMSE; Sauer, Gramacy, and Higdon, 2023), optimization through expected improvement (EI; Gramacy, Sauer, and Wycoff, 2022, <[doi:10.48550/arXiv.2112.07457](https://doi.org/10.48550/arXiv.2112.07457)>), and contour location through entropy (Booth, Renganathan, and Gramacy, 2025, <[doi:10.48550/arXiv.2308.04420](https://doi.org/10.48550/arXiv.2308.04420)>). Models extend up to three layers deep; a one layer model is equivalent to typical Gaussian process regression. Incorporates OpenMP and SNOW parallelization and utilizes C/C++ under the hood.

**License** LGPL

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**LinkingTo** Rcpp, RcppArmadillo,

**Suggests** interp, knitr, rmarkdown

**VignetteBuilder** knitr

**RoxygenNote** 7.3.3**Author** Annie S. Booth [aut, cre]**Maintainer** Annie S. Booth <annie\_booth@vt.edu>**Repository** CRAN**Date/Publication** 2026-02-09 14:50:02 UTC**Contents**

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deepgp-package	<i>Package deepgp</i>
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**Description**

Performs Bayesian posterior inference for deep Gaussian processes following Sauer, Gramacy, and Higdon (2023). See Sauer (2023) for comprehensive methodological details and <https://bitbucket.org/gramacylab/deepgp-ex/> for a variety of coding examples. Models are trained through MCMC including elliptical slice sampling of latent Gaussian layers and Metropolis-Hastings sampling of kernel hyperparameters. Gradient-enhancement and gradient predictions are offered following Booth (2025). Vecchia approximation for faster computation is implemented following Sauer, Cooper, and Gramacy (2023). Optional monotonic warpings are implemented following Barnett et al. (2025). Downstream tasks include sequential design through active learning Cohn/integrated mean squared error (ALC/IMSE; Sauer, Gramacy, and Higdon, 2023), optimization through expected improvement (EI; Gramacy, Sauer, and Wycoff, 2022), and contour location through entropy (Booth, Renganathan, and Gramacy, 2025). Models extend up to three layers deep; a one layer model is equivalent to typical Gaussian process regression. Incorporates OpenMP and SNOW parallelization and utilizes C/C++ under the hood.

### Important Functions

- `fit_one_layer`: conducts MCMC sampling of hyperparameters for a one layer GP
- `fit_two_layer`: conducts MCMC sampling of hyperparameters and hidden layer for a two layer deep GP
- `fit_three_layer`: conducts MCMC sampling of hyperparameters and hidden layers for a three layer deep GP
- `continue`: collects additional MCMC samples
- `trim`: cuts off burn-in and optionally thins samples
- `predict`: calculates posterior mean and variance over a set of input locations (optionally calculates EI or entropy)
- `plot`: produces trace plots, hidden layer plots, and posterior predictive plots
- `ALC`: calculates active learning Cohn over set of input locations using reference grid
- `IMSE`: calculates integrated mean-squared error over set of input locations

### Author(s)

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### References

- Sauer, A. (2023). Deep Gaussian process surrogates for computer experiments. \*Ph.D. Dissertation, Department of Statistics, Virginia Polytechnic Institute and State University.\* <http://hdl.handle.net/10919/114845>
- Booth, A. S. (2025). Deep Gaussian processes with gradients. arXiv:2512.18066
- Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. \*Technometrics, 65,\* 4-18. arXiv:2012.08015
- Sauer, A., Cooper, A., & Gramacy, R. B. (2023). Vecchia-approximated deep Gaussian processes for computer experiments. \*Journal of Computational and Graphical Statistics, 32\*(3), 824-837. arXiv:2204.02904
- Gramacy, R. B., Sauer, A. & Wycoff, N. (2022). Triangulation candidates for Bayesian optimization. \*Advances in Neural Information Processing Systems (NeurIPS), 35,\* 35933-35945. arXiv:2112.07457
- Booth, A., Renganathan, S. A. & Gramacy, R. B. (2025). Contour location for reliability in air-foil simulation experiments using deep Gaussian processes. \*Annals of Applied Statistics, 19\*(1), 191-211. arXiv:2308.04420
- Barnett, S., Beesley, L. J., Booth, A. S., Gramacy, R. B., & Osthus D. (2025). Monotonic warpings for additive and deep Gaussian processes. \*Statistics and Computing, 35\*(3), 65. arXiv:2408.01540

**Examples**

```
# See vignette, ?fit_one_layer, ?fit_two_layer, ?fit_three_layer,
# ?ALC, or ?IMSE for examples
# Many more examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/
```

ALC

*Active Learning Cohn for Sequential Design***Description**

Acts on a gp, dgp2, or dgp3 object. Current version requires squared exponential covariance (cov = "exp2"). Calculates ALC over the input locations `x_new` using specified reference grid. If no reference grid is specified, `x_new` is used as the reference. Optionally utilizes SNOW parallelization. User should select the point with the highest ALC to add to the design.

**Usage**

```
ALC(object, x_new, ref, cores)

## S3 method for class 'gp'
ALC(object, x_new = NULL, ref = NULL, cores = 1)

## S3 method for class 'dgp2'
ALC(object, x_new = NULL, ref = NULL, cores = 1)

## S3 method for class 'dgp3'
ALC(object, x_new = NULL, ref = NULL, cores = 1)
```

**Arguments**

<code>object</code>	object of class gp, dgp2, or dgp3
<code>x_new</code>	matrix of possible input locations, if object has been run through predict the previously stored <code>x_new</code> is used
<code>ref</code>	optional reference grid for ALC approximation, if <code>ref = NULL</code> then <code>x_new</code> is used
<code>cores</code>	number of cores to utilize for SNOW parallelization

**Details**

Not yet implemented for Vecchia-approximated fits or Matern kernels.

All iterations in the object are used in the calculation, so samples should be burned-in. Thinning the samples using `trim` will speed up computation. This function may be used in two ways:

- Option 1: called on an object with only MCMC iterations, in which case `x_new` must be specified

- Option 2: called on an object that has been predicted over, in which case the `x_new` from `predict` is used

In Option 2, it is recommended to set `store_latent = TRUE` for `dgp2` and `dgp3` objects so latent mappings do not have to be re-calculated. Through `predict`, the user may specify a mean mapping (`mean_map = TRUE`) or a full sample from the MVN distribution over `w_new` (`mean_map = FALSE`). When the object has not yet been predicted over (Option 1), the mean mapping is used.

SNOW parallelization reduces computation time but requires more memory storage. C code derived from the "laGP" package (Robert B Gramacy and Furong Sun).

### Value

list with elements:

- `value`: vector of ALC values, indices correspond to `x_new`
- `time`: computation time in seconds

### References

Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. *\*Technometrics*, 65,\* 4-18. arXiv:2012.08015

Seo, S, M Wallat, T Graepel, and K Obermayer. 2000. Gaussian Process Regression: Active Data Selection and Test Point Rejection. In *Mustererkennung 2000*, 2734. New York, NY: SpringerVerlag.

Gramacy, RB and F Sun. (2016). laGP: Large-Scale Spatial Modeling via Local Approximate Gaussian Processes in R. *Journal of Statistical Software* 72 (1), 1-46. doi:10.18637/jss.v072.i01

### Examples

```
# Additional examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/
```

```
# -----
# Example 1: toy step function, runs in less than 5 seconds
# -----
```

```
f <- function(x) {
  if (x <= 0.4) return(-1)
  if (x >= 0.6) return(1)
  if (x > 0.4 & x < 0.6) return(10*(x-0.5))
}

x <- seq(0.05, 0.95, length = 7)
y <- sapply(x, f)
x_new <- seq(0, 1, length = 100)

# Fit model and calculate ALC
fit <- fit_two_layer(x, y, nmc = 100, cov = "exp2")
fit <- trim(fit, 50)
```

```

fit <- predict(fit, x_new, cores = 1, store_latent = TRUE)
alc <- ALC(fit)

# -----
# Example 2: damped sine wave
# -----

f <- function(x) {
  exp(-10*x) * (cos(10*pi*x - 1) + sin(10*pi*x - 1)) * 5 - 0.2
}

# Training data
x <- seq(0, 1, length = 30)
y <- f(x) + rnorm(30, 0, 0.05)

# Testing data
xx <- seq(0, 1, length = 100)
yy <- f(xx)

plot(xx, yy, type = "l")
points(x, y, col = 2)

# Conduct MCMC (can replace fit_two_layer with fit_one_layer/fit_three_layer)
# nugget estimated
fit <- fit_two_layer(x, y, D = 1, nmcmc = 2000, cov = "exp2")
plot(fit)
fit <- trim(fit, 1000, 2)

# Option 1 - calculate ALC from MCMC iterations
alc <- ALC(fit, xx)

# Option 2 - calculate ALC after predictions
fit <- predict(fit, xx, cores = 1, store_latent = TRUE)
alc <- ALC(fit)

# Visualize fit
plot(fit)
par(new = TRUE) # overlay ALC
plot(xx, alc$value, type = 'l', lty = 2, axes = FALSE, xlab = '', ylab = '')

# Select next design point
x_new <- xx[which.max(alc$value)]

```

**Description**

Acts on a `gp`, `gpvec`, `dgp2`, `dgp2vec`, `dgp3`, or `dgp3vec` object. Continues MCMC sampling of hyperparameters and hidden layers using settings from the original object. Appends new samples to existing samples. When `vecchia = TRUE`, this function provides the option to update Vecchia ordering/conditioning sets based on latent layer warpings through the specification of `re_approx = TRUE`.

**Usage**

```
continue(object, new_mcmc, verb, re_approx, ...)

## S3 method for class 'gp'
continue(object, new_mcmc = 1000, verb = TRUE, ...)

## S3 method for class 'dgp2'
continue(object, new_mcmc = 1000, verb = TRUE, ...)

## S3 method for class 'dgp3'
continue(object, new_mcmc = 1000, verb = TRUE, ...)

## S3 method for class 'gpvec'
continue(
  object,
  new_mcmc = 1000,
  verb = TRUE,
  re_approx = FALSE,
  ord = NULL,
  ...
)

## S3 method for class 'dgp2vec'
continue(
  object,
  new_mcmc = 1000,
  verb = TRUE,
  re_approx = FALSE,
  ord = NULL,
  ...
)

## S3 method for class 'dgp3vec'
continue(
  object,
  new_mcmc = 1000,
  verb = TRUE,
  re_approx = FALSE,
  ord = NULL,
  ...
)
```

)

**Arguments**

object	object from <code>fit_one_layer</code> , <code>fit_two_layer</code> , or <code>fit_three_layer</code>
new_mcmc	number of new MCMC iterations to conduct and append
verb	logical indicating whether to print iteration progress
re_approx	logical indicating whether to re-randomize the ordering and update Vecchia nearest-neighbor conditioning sets (only for fits with <code>vecchia = TRUE</code> )
...	N/A
ord	optional ordering to be used in Vecchia re-approximation (only for fits with <code>vecchia = TRUE</code> when <code>re_approx = TRUE</code> )

**Details**

See `fit_one_layer`, `fit_two_layer`, or `fit_three_layer` for details on MCMC. The resulting object will have `nmc` equal to the previous `nmc` plus `new_mcmc`. It is recommended to start an MCMC fit then investigate trace plots to assess burn-in. The primary use of this function is to gather more MCMC iterations in order to obtain burned-in samples.

Specifying `re_approx = TRUE` updates random orderings and nearest-neighbor conditioning sets (only for `vecchia = TRUE` fits). In one-layer, there is no latent warping but the Vecchia approximation is still re-randomized and nearest-neighbors are adjusted accordingly. In two- and three-layers, the latest samples of hidden layers are used to update nearest-neighbors. If you update the Vecchia approximation, you should later remove previous samples (updating the approximation effectively starts a new chain). When `re_approx = FALSE` the previous orderings and conditioning sets are used (maintaining the continuity of the previous chain).

**Value**

object of the same class with the new iterations appended

**Examples**

```
# See ?fit_two_layer for an example
```

---

crps

*Calculates CRPS*


---

**Description**

Calculates continuous ranked probability score (lower CRPS indicate better fits, better uncertainty quantification).

**Usage**

```
crps(y, mu, s2)
```

**Arguments**

y	response vector
mu	predicted mean
s2	predicted point-wise variances

**References**

Gneiting, T, and AE Raftery. 2007. Strictly Proper Scoring Rules, Prediction, and Estimation. *Journal of the American Statistical Association* 102 (477), 359-378.

**Examples**

```
# Additional examples including real-world computer experiments are available at:  
# https://bitbucket.org/gramacylab/deepgp-ex/
```

---

fit_one_layer	<i>MCMC sampling for one layer GP</i>
---------------	---------------------------------------

---

**Description**

Conducts MCMC sampling of hyperparameters for a one layer GP. Length scale parameter theta governs the strength of the correlation and nugget parameter g governs noise. In Matern covariance, v governs smoothness.

**Usage**

```
fit_one_layer(  
  x,  
  y,  
  dydx = NULL,  
  nmcmc = 10000,  
  sep = FALSE,  
  verb = TRUE,  
  theta_0 = 0.01,  
  g_0 = 0.001,  
  true_g = NULL,  
  v = 2.5,  
  settings = NULL,  
  cov = c("matern", "exp2"),  
  vecchia = FALSE,  
  m = NULL,  
  ord = NULL,  
  cores = NULL  
)
```

**Arguments**

x	vector or matrix of input locations
y	vector of response values
dydx	optional matrix of observed gradients, rows correspond to x locations, columns contain partial derivatives with respect to that input dimension (dim(dy) must match dim(x))
nmc	number of MCMC iterations
sep	logical indicating whether to use separable (sep = TRUE) or isotropic (sep = FALSE) lengthscales
verb	logical indicating whether to print iteration progress
theta_0	initial value for theta
g_0	initial value for g (only used if true_g = NULL)
true_g	if true nugget is known it may be specified here (set to a small value to make fit deterministic). Note - values that are too small may cause numerical issues in matrix inversions.
v	Matern smoothness parameter (only used if cov = "matern")
settings	hyperparameters for proposals and priors (see details)
cov	covariance kernel, either Matern ("matern") or squared exponential ("exp2")
vecchia	logical indicating whether to use Vecchia approximation
m	size of Vecchia conditioning sets, defaults to the lower of 25 or the maximum available (only used if vecchia = TRUE)
ord	optional ordering for Vecchia approximation, must correspond to rows of x, defaults to random
cores	number of cores to use for OpenMP parallelization (vecchia = TRUE only). Defaults to min(4, maxcores - 1) where maxcores is the number of detectable available cores.

**Details**

Utilizes Metropolis Hastings sampling of the length scale and nugget parameters with proposals and priors controlled by settings. When true\_g is set to a specific value, the nugget is not estimated. When vecchia = TRUE, all calculations leverage the Vecchia approximation with specified conditioning set size m.

NOTE on OpenMP: The Vecchia implementation relies on OpenMP parallelization for efficient computation. This function will produce a warning message if the package was installed without OpenMP (this is the default for CRAN packages installed on Apple machines). To set up OpenMP parallelization, download the package source code and install using the gcc/g++ compiler.

Proposals for g and theta follow a uniform sliding window scheme, e.g.,

```
g_star <- runif(1, l * g_t / u, u * g_t / l),
```

with defaults l = 1 and u = 2 provided in settings. To adjust these, set settings = list(l = new\_l, u = new\_u).

Priors on  $g$  and  $\theta$  follow Gamma distributions with shape parameters ( $\alpha$ ) and rate parameters ( $\beta$ ) controlled within the `settings` list object. Default priors differ for noisy/deterministic settings. All default values are visible in the internal `deepgp:::check_settings` function. These priors are designed for  $x$  scaled to  $[0, 1]$  and  $y$  scaled to have mean 0 and variance 1. These may be adjusted using the `settings` input.

The output object of class `gp` is designed for use with `continue`, `trim`, `plot`, and `predict`.

## Value

a list of the S3 class `gp` or `gpvec` with elements:

- `x`: copy of input matrix
- `y`: copy of response vector
- `nmcnc`: number of MCMC iterations
- `settings`: copy of proposal/prior settings
- `v`: copy of Matern smoothness parameter (`v = 999` indicates `cov = "exp2"`)
- `dydx`: copy of `dydx` (if not `NULL`)
- `grad_indx`: stacked partial derivative indices (only if `dydx` is provided)
- `g`: vector of MCMC samples for  $g$
- `theta`: vector of MCMC samples for  $\theta$
- `tau2`: vector of MLE estimates for  $\tau^2$  (scale parameter)
- `x_approx`: Vecchia approximation object (`vecchia = TRUE` only)
- `ll`: vector of MVN log likelihood for each Gibbs iteration
- `time`: computation time in seconds

## References

Sauer, A. (2023). Deep Gaussian process surrogates for computer experiments. \*Ph.D. Dissertation, Department of Statistics, Virginia Polytechnic Institute and State University.\*

Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. \*Technometrics, 65,\* 4-18. arXiv:2012.08015

Booth, A. S. (2025). Deep Gaussian processes with gradients. arXiv:2512.18066

Sauer, A., Cooper, A., & Gramacy, R. B. (2023). Vecchia-approximated deep Gaussian processes for computer experiments. \*Journal of Computational and Graphical Statistics, 32\*(3), 824-837. arXiv:2204.02904

## Examples

```
# Additional examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/

# Booth function (inspired by the Higdon function)
f <- function(x) {
```

```

    i <- which(x <= 0.58)
    x[i] <- sin(pi * x[i] * 6) + cos(pi * x[i] * 12)
    x[-i] <- 5 * x[-i] - 4.9
    return(x)
}

# Training data
x <- seq(0, 1, length = 25)
y <- f(x)

# Testing data
xx <- seq(0, 1, length = 100)
yy <- f(xx)

plot(xx, yy, type = "l")
points(x, y, col = 2)

# Example 1: nugget fixed, calculating EI
fit <- fit_one_layer(x, y, nmcmc = 2000, true_g = 1e-6)
plot(fit)
fit <- trim(fit, 1000, 2)
fit <- predict(fit, xx, cores = 1, EI = TRUE)
plot(fit)
par(new = TRUE) # overlay EI
plot(xx[order(xx)], fit$EI[order(xx)], type = 'l', lty = 2,
      axes = FALSE, xlab = '', ylab = '')

# Example 2: convert fit to Vecchia object before predicting
# (this is faster if the training data set is large)
fit <- to_vec(fit)
fit <- predict(fit, xx, cores = 1)
plot(fit)

# Example 3: using Vecchia for training and testing
fit <- fit_one_layer(x, y, nmcmc = 2000, true_g = 1e-6, vecchia = TRUE, m = 10)
plot(fit)
fit <- trim(fit, 1000, 2)
fit <- predict(fit, xx, cores = 1)
plot(fit)

```

---

fit\_three\_layer

*MCMC sampling for three layer deep GP*


---

### Description

Conducts MCMC sampling of hyperparameters, hidden layer z, and hidden layer w for a three layer deep GP. Separate length scale parameters  $\theta_z$ ,  $\theta_w$ , and  $\theta_y$  govern the correlation

strength of the inner layer, middle layer, and outer layer respectively. Nugget parameter  $g$  governs noise on the outer layer. In Matern covariance,  $\nu$  governs smoothness.

Currently, there are no `pmx`, `monowarp`, or `dydx` options.

### Usage

```
fit_three_layer(
  x,
  y,
  nmcmc = 10000,
  D = ifelse(is.matrix(x), ncol(x), 1),
  verb = TRUE,
  w_0 = NULL,
  z_0 = NULL,
  theta_y_0 = 0.01,
  theta_w_0 = 0.1,
  theta_z_0 = 0.1,
  g_0 = 0.001,
  true_g = NULL,
  v = 2.5,
  settings = NULL,
  cov = c("matern", "exp2"),
  vecchia = FALSE,
  m = NULL,
  ord = NULL,
  cores = NULL
)
```

### Arguments

<code>x</code>	vector or matrix of input locations
<code>y</code>	vector of response values
<code>nmcmc</code>	number of MCMC iterations
<code>D</code>	integer designating dimension of hidden layers, defaults to dimension of <code>x</code>
<code>verb</code>	logical indicating whether to print iteration progress
<code>w_0</code>	initial value for hidden layer <code>w</code> (rows must correspond to rows of <code>x</code> , requires <code>ncol(w_0) = D</code> . Defaults to the identity mapping. If <code>nrow(w_0) &lt; nrow(x)</code> , missing initial values are filled-in with the GP posterior mean.
<code>z_0</code>	initial value for hidden layer <code>z</code> (rows must correspond to rows of <code>x</code> , requires <code>ncol(z_0) = D</code> . Defaults to the identity mapping. If <code>nrow(z_0) &lt; nrow(x)</code> , missing initial values are filled-in with the GP posterior mean.
<code>theta_y_0</code>	initial value for <code>theta_y</code> (length scale of outer layer)
<code>theta_w_0</code>	initial value for <code>theta_w</code> (length scale of middle layer), may be single value or vector of length <code>D</code>
<code>theta_z_0</code>	initial value for <code>theta_z</code> (length scale of inner layer), may be single value or vector of length <code>D</code>

<code>g_0</code>	initial value for <code>g</code>
<code>true_g</code>	if true nugget is known it may be specified here (set to a small value to make fit deterministic). Note - values that are too small may cause numerical issues in matrix inversions.
<code>v</code>	Matern smoothness parameter (only used if <code>cov = "matern"</code> )
<code>settings</code>	hyperparameters for proposals and priors (see details)
<code>cov</code>	covariance kernel, either Matern (" <code>matern</code> ") or squared exponential (" <code>exp2</code> ")
<code>vecchia</code>	logical indicating whether to use Vecchia approximation
<code>m</code>	size of Vecchia conditioning sets, defaults to the lower of 25 or the maximum available (only used if <code>vecchia = TRUE</code> )
<code>ord</code>	optional ordering for Vecchia approximation, must correspond to rows of <code>x</code> , defaults to random, is applied to <code>x</code> , <code>w</code> , and <code>z</code>
<code>cores</code>	number of cores to use for OpenMP parallelization ( <code>vecchia = TRUE</code> only). Defaults to $\min(4, \text{maxcores} - 1)$ where <code>maxcores</code> is the number of detectable available cores.

## Details

Maps inputs `x` through hidden layer `z` then hidden layer `w` to outputs `y`. Conducts sampling of the hidden layers using elliptical slice sampling. Utilizes Metropolis Hastings sampling of the length scale and nugget parameters with proposals and priors controlled by `settings`. When `true_g` is set to a specific value, the nugget is not estimated. When `vecchia = TRUE`, all calculations leverage the Vecchia approximation with specified conditioning set size `m`.

NOTE on OpenMP: The Vecchia implementation relies on OpenMP parallelization for efficient computation. This function will produce a warning message if the package was installed without OpenMP (this is the default for CRAN packages installed on Apple machines). To set up OpenMP parallelization, download the package source code and install using the `gcc/g++` compiler.

Proposals for `g`, `theta_y`, `theta_w`, and `theta_z` follow a uniform sliding window scheme, e.g.,

```
g_star <- runif(1, 1 * g_t / u, u * g_t / 1),
```

with defaults `l = 1` and `u = 2` provided in `settings`. To adjust these, set `settings = list(l = new_l, u = new_u)`. Priors on `g`, `theta_y`, `theta_w`, and `theta_z` follow Gamma distributions with shape parameters (`alpha`) and rate parameters (`beta`) controlled within the `settings` list object. Default priors differ for noisy/deterministic settings. All default values are visible in the internal `deepgp:::check_settings` function. These priors are designed for `x` scaled to  $[0, 1]$  and `y` scaled to have mean 0 and variance 1. These may be adjusted using the `settings` input.

The scale on the latent layers (`tau2_z` and `tau2_w`) may also be specified in `settings`. Defaults to 1.

When `w_0 = NULL` and/or `z_0 = NULL`, the hidden layers are initialized at `x` (i.e., the identity mapping). If `w_0` and/or `z_0` is of dimension  $nrow(x) - 1$  by `D`, the final row is filled-in using the GP posterior mean. This is helpful in sequential design when adding a new input location and starting the MCMC at the place where the previous MCMC left off.

The output object of class `dgp3` or `dgp3vec` is designed for use with `continue`, `trim`, and `predict`.

**Value**

a list of the S3 class `dgp3` or `dgp3vec` with elements:

- `x`: copy of input matrix
- `y`: copy of response vector
- `nmcmc`: number of MCMC iterations
- `settings`: copy of proposal/prior settings
- `v`: copy of Matern smoothness parameter (`v = 999` indicates `cov = "exp2"`)
- `g`: vector of MCMC samples for `g`
- `tau2_y`: vector of MLE estimates for `tau2` on outer layer
- `theta_y`: vector of MCMC samples for `theta_y` (length scale of outer layer)
- `theta_w`: matrix of MCMC samples for `theta_w` (length scale of middle layer)
- `theta_z`: matrix of MCMC samples for `theta_z` (length scale of inner layer)
- `w`: list of MCMC samples for middle hidden layer `w`
- `z`: list of MCMC samples for inner hidden layer `z`
- `w_approx`: Vecchia approximation object for outer layer (`vecchia = TRUE` only)
- `z_approx`: Vecchia approximation object for middle layer (`vecchia = TRUE` only)
- `x_approx`: Vecchia approximation object for inner layer (`vecchia = TRUE` only)
- `ll`: vector of MVN log likelihood of the outer layer for reach Gibbs iteration
- `time`: computation time in seconds

**References**

- Sauer, A. (2023). Deep Gaussian process surrogates for computer experiments. \*Ph.D. Dissertation, Department of Statistics, Virginia Polytechnic Institute and State University.\* <http://hdl.handle.net/10919/114845>
- Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. \*Technometrics, 65,\* 4-18. arXiv:2012.08015
- Sauer, A., Cooper, A., & Gramacy, R. B. (2023). Vecchia-approximated deep Gaussian processes for computer experiments. \*Journal of Computational and Graphical Statistics, 32\*(3), 824-837. arXiv:2204.02904

**Examples**

```
# Additional examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/

# G function in 2 dimensions (https://www.sfu.ca/~ssurjano/gfunc.html)
f <- function(xx, a = (c(1:length(xx)) - 1) / 2) {
  newf <- abs(4 * xx - 2) + a
}
```

```

    new2 <- 1 + a
    prod <- prod(new1 / new2)
    return((prod - 1) / 0.86)
}

# Training data
d <- 2
n <- 30
x <- matrix(runif(n * d), ncol = d)
y <- apply(x, 1, f)

# Testing data
n_test <- 500
xx <- matrix(runif(n_test * d), ncol = d)
yy <- apply(xx, 1, f)

i <- interp::interp(xx[, 1], xx[, 2], yy)
image(i, col = heat.colors(128))
contour(i, add = TRUE)
contour(i, level = -0.5, col = 4, add = TRUE) # potential failure limit
points(x)

# Example 1: nugget fixed, calculating entropy
fit <- fit_three_layer(x, y, nmcmc = 2000, true_g = 1e-6)
plot(fit)
fit <- trim(fit, 1000, 2)
fit <- predict(fit, xx, entropy_limit = -0.5, cores = 1)
plot(fit)
i <- interp::interp(xx[, 1], xx[, 2], fit$entropy)
image(i, col = heat.colors(128), main = "Entropy")

# Example 2: using Vecchia
fit <- fit_three_layer(x, y, nmcmc = 2000, true_g = 1e-6, vecchia = TRUE, m = 10)
plot(fit)
fit <- trim(fit, 1000, 2)
fit <- predict(fit, xx, cores = 1)
plot(fit)

```

### Description

Conducts MCMC sampling of hyperparameters and hidden layer  $w$  for a two layer deep GP. Separate length scale parameters  $\theta_w$  and  $\theta_y$  govern the correlation strength of the hidden layer and outer layer respectively. Nugget parameter  $g$  governs noise on the outer layer. In Matern covariance,  $\nu$  governs smoothness.

**Usage**

```

fit_two_layer(
  x,
  y,
  dydx = NULL,
  nmcmc = 10000,
  D = ifelse(is.matrix(x), ncol(x), 1),
  monowarp = FALSE,
  pmx = FALSE,
  verb = TRUE,
  w_0 = NULL,
  theta_y_0 = 0.01,
  theta_w_0 = 0.1,
  g_0 = 0.001,
  true_g = NULL,
  v = 2.5,
  settings = NULL,
  cov = c("matern", "exp2"),
  vecchia = FALSE,
  m = NULL,
  ord = NULL,
  cores = NULL
)

```

**Arguments**

x	vector or matrix of input locations
y	vector of response values
dydx	optional matrix of observed gradients, rows correspond to x locations, columns contain partial derivatives with respect to that input dimension ( $\dim(dy)$ must match $\dim(x)$ )
nmcmc	number of MCMC iterations
D	integer designating dimension of hidden layer, defaults to dimension of x
monowarp	logical or numeric. If FALSE, warpings are not forced to be monotonic. If TRUE, each input dimension is individually monotonically warped with a default grid size of 50. If numeric, triggers monotonic warpings with the provided grid size.
pmx	"prior mean x", logical indicating whether w should have prior mean of x (TRUE, requires $D = ncol(x)$ ) or prior mean zero (FALSE). $pmx = TRUE$ is recommended for higher dimensions.
verb	logical indicating whether to print iteration progress
w_0	initial value for hidden layer w (rows must correspond to rows of x, requires $ncol(w_0) = D$ . Defaults to the identity mapping. If $nrow(w_0) < nrow(x)$ , missing initial values are filled-in with the GP posterior mean.
theta_y_0	initial value for theta_y (length scale of outer layer)
theta_w_0	initial value for theta_w (length scale of inner layer), may be single value or vector of length D

<code>g_0</code>	initial value for <code>g</code> (only used if <code>true_g = NULL</code> )
<code>true_g</code>	if true nugget is known it may be specified here (set to a small value to make fit deterministic). Note - values that are too small may cause numerical issues in matrix inversions.
<code>v</code>	Matern smoothness parameter (only used if <code>cov = "matern"</code> )
<code>settings</code>	hyperparameters for proposals and priors (see details)
<code>cov</code>	covariance kernel, either Matern (" <code>matern</code> ") or squared exponential (" <code>exp2</code> ")
<code>vecchia</code>	logical indicating whether to use Vecchia approximation
<code>m</code>	size of Vecchia conditioning sets, defaults to the lower of 25 or the maximum available (only used if <code>vecchia = TRUE</code> )
<code>ord</code>	optional ordering for Vecchia approximation, must correspond to rows of <code>x</code> , defaults to random, is applied to both <code>x</code> and <code>w</code>
<code>cores</code>	number of cores to use for OpenMP parallelization ( <code>vecchia = TRUE</code> only). Defaults to $\min(4, \text{maxcores} - 1)$ where <code>maxcores</code> is the number of detectable available cores.

## Details

Maps inputs `x` through hidden layer `w` to outputs `y`. Conducts sampling of the hidden layer using elliptical slice sampling. Utilizes Metropolis Hastings sampling of the length scale and nugget parameters with proposals and priors controlled by `settings`. When `true_g` is set to a specific value, the nugget is not estimated. When `vecchia = TRUE`, all calculations leverage the Vecchia approximation with specified conditioning set size `m`.

When `monowarp = TRUE`, each input dimension is warped separately and monotonically. This requires  $D = \text{ncol}(x)$  with `x` scaled to the unit cube. New in version 1.2.0 - monotonic warpings estimate separate scale parameters (`tau2_w`) on each latent node and use an isotropic lengthscale on the outer layer. As a default, monotonic warpings use the reference grid: `seq(0, 1, length = 50)`. The grid size may be controlled by passing a numeric integer to `monowarp` (i.e., `monowarp = 100` uses the grid `seq(0, 1, length = 100)`).

When `pmx = TRUE`, the prior on the latent layer is set at `x` (rather than the default of zero). This requires  $D = \text{ncol}(x)$ .

NOTE on OpenMP: The Vecchia implementation relies on OpenMP parallelization for efficient computation. This function will produce a warning message if the package was installed without OpenMP (this is the default for CRAN packages installed on Apple machines). To set up OpenMP parallelization, download the package source code and install using the `gcc/g++` compiler.

Proposals for `g`, `theta_y`, and `theta_w` follow a uniform sliding window scheme, e.g.,

```
g_star <- runif(1, 1 * g_t / u, u * g_t / 1),
```

with defaults `l = 1` and `u = 2` provided in `settings`. To adjust these, set `settings = list(l = new_l, u = new_u)`. Priors on `g`, `theta_y`, and `theta_w` follow Gamma distributions with shape parameters (`alpha`) and rate parameters (`beta`) controlled within the `settings` list object. Default priors differ for noisy/deterministic settings and depend on whether `monowarp = TRUE`. All default values are visible in the internal `deepgp:::check_settings` function. These priors are designed for `x` scaled to  $[0, 1]$  and `y` scaled to have mean 0 and variance 1. These may be adjusted using the `settings` input.

The scale on the latent layer ( $\tau_{2_w}$ ) may also be specified in `settings`. Defaults to 1.

When  $w_{\theta} = \text{NULL}$ , the hidden layer is initialized at  $x$  (i.e., the identity mapping). If  $w_{\theta}$  is of dimension  $nrow(x) - 1$  by  $D$ , the final row is filled-in using the GP posterior mean. This is helpful in sequential design when adding a new input location and starting the MCMC at the place where the previous MCMC left off.

The output object of class `dgp2` or `dgp2vec` is designed for use with `continue`, `trim`, and `predict`.

## Value

a list of the S3 class `dgp2` or `dgp2vec` with elements:

- `x`: copy of input matrix
- `y`: copy of response vector
- `nmcmc`: number of MCMC iterations
- `settings`: copy of proposal/prior settings
- `v`: copy of Matern smoothness parameter ( $v = 999$  indicates `cov = "exp2"`)
- `x_grid`: grid used for monotonic warpings (`monowarp = TRUE` only)
- `dydx`: copy of `dydx` (if not `NULL`)
- `grad_indx`: stacked partial derivative indices (only if `dydx` is provided)
- `g`: vector of MCMC samples for `g`
- `tau2_y`: vector of MLE estimates for  $\tau_2$  on the outer layer
- `theta_y`: vector of MCMC samples for  $\theta_y$  (length scale of outer layer)
- `tau2_w`: matrix of MLE estimates for  $\tau_2$  on inner layer (only returned if `monowarp = TRUE`, otherwise this is fixed in `settings`)
- `theta_w`: matrix of MCMC samples for  $\theta_w$  (length scale of inner layer)
- `w`: list of MCMC samples for hidden layer `w`
- `w_grid`: `w` values at `x_grid` locations (`monowarp = TRUE` only)
- `w_approx`: Vecchia approximation object for outer layer (`vecchia = TRUE` only)
- `x_approx`: Vecchia approximation object for inner layer (`vecchia = TRUE` only)
- `ll`: vector of MVN log likelihood of the outer layer for each Gibbs iteration
- `time`: computation time in seconds

## References

Sauer, A. (2023). Deep Gaussian process surrogates for computer experiments. \*Ph.D. Dissertation, Department of Statistics, Virginia Polytechnic Institute and State University.\* <http://hdl.handle.net/10919/114845>

Booth, A. S. (2025). Deep Gaussian processes with gradients. arXiv:2512.18066

Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. \*Technometrics, 65,\* 4-18. arXiv:2012.08015

Sauer, A., Cooper, A., & Gramacy, R. B. (2023). Vecchia-approximated deep Gaussian processes for computer experiments. *Journal of Computational and Graphical Statistics*, 32\*(3), 824-837. arXiv:2204.02904

Barnett, S., Beesley, L. J., Booth, A. S., Gramacy, R. B., & Osthus D. (2025). Monotonic warpings for additive and deep Gaussian processes. *Statistics and Computing*, 35\*(3), 65. arXiv:2408.01540

## Examples

```
# Additional examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/
```

```
# Booth function (inspired by the Higdon function)
f <- function(x) {
  i <- which(x <= 0.58)
  x[i] <- sin(pi * x[i] * 6) + cos(pi * x[i] * 12)
  x[-i] <- 5 * x[-i] - 4.9
  return(x)
}
```

```
# Training data
x <- seq(0, 1, length = 25)
y <- f(x)
```

```
# Testing data
xx <- seq(0, 1, length = 100)
yy <- f(xx)
```

```
plot(xx, yy, type = "l")
points(x, y, col = 2)
```

```
# Example 1: nugget fixed, using continue
fit <- fit_two_layer(x, y, nmcmc = 1000, true_g = 1e-6)
plot(fit)
fit <- continue(fit, 1000)
plot(fit, hidden = TRUE) # trace plots and ESS samples
fit <- trim(fit, 1000, 2)
fit <- predict(fit, xx, cores = 1)
plot(fit)
```

```
# Example 2: using Vecchia, re-approximated after burn-in
fit <- fit_two_layer(x, y, nmcmc = 1000, true_g = 1e-6, vecchia = TRUE, m = 10)
fit <- continue(fit, 1000, re_approx = TRUE)
plot(fit, hidden = TRUE) # trace plots and ESS samples
fit <- trim(fit, 1000, 2)
fit <- predict(fit, xx, cores = 1)
plot(fit)
```

```
# Example 3: using monotonic warpings
fit <- fit_two_layer(x, y, nmcmc = 2000, true_g = 1e-6, monowarp = TRUE)
plot(fit, hidden = TRUE) # trace plots and ESS samples
fit <- trim(fit, 1000, 2)
```

```
fit <- predict(fit, xx, cores = 1)
plot(fit)
```

---

IMSE

*Integrated Mean-Squared (prediction) Error for Sequential Design*

---

### Description

Acts on a gp, dgp2, or dgp3 object. Current version requires squared exponential covariance (cov = "exp2"). Calculates IMSE over the input locations `x_new`. Optionally utilizes SNOW parallelization. User should select the point with the lowest IMSE to add to the design.

### Usage

```
IMSE(object, x_new, cores)

## S3 method for class 'gp'
IMSE(object, x_new = NULL, cores = 1)

## S3 method for class 'dgp2'
IMSE(object, x_new = NULL, cores = 1)

## S3 method for class 'dgp3'
IMSE(object, x_new = NULL, cores = 1)
```

### Arguments

<code>object</code>	object of class gp, dgp2, or dgp3
<code>x_new</code>	matrix of possible input locations, if object has been run through predict the previously stored <code>x_new</code> is used
<code>cores</code>	number of cores to utilize for SNOW parallelization

### Details

Not yet implemented for Vecchia-approximated fits or Matern kernels.

All iterations in the object are used in the calculation, so samples should be burned-in. Thinning the samples using `trim` will speed up computation. This function may be used in two ways:

- Option 1: called on an object with only MCMC iterations, in which case `x_new` must be specified
- Option 2: called on an object that has been predicted over, in which case the `x_new` from `predict` is used

In Option 2, it is recommended to set `store_latent = TRUE` for `dgp2` and `dgp3` objects so latent mappings do not have to be re-calculated. Through `predict`, the user may specify a mean mapping (`mean_map = TRUE`) or a full sample from the MVN distribution over `w_new` (`mean_map = FALSE`). When the object has not yet been predicted over (Option 1), the mean mapping is used.

SNOW parallelization reduces computation time but requires more memory storage.

## Value

list with elements:

- `value`: vector of IMSE values, indices correspond to `x_new`
- `time`: computation time in seconds

## References

Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. *Technometrics*, 65,\* 4-18. arXiv:2012.08015

Binois, M, J Huang, RB Gramacy, and M Ludkovski. 2019. "Replication or Exploration? Sequential Design for Stochastic Simulation Experiments." *Technometrics* 61, 7-23. Taylor & Francis. doi:10.1080/00401706.2018.1469433

## Examples

```
# Additional examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/
```

```
# -----
# Example 1: toy step function, runs in less than 5 seconds
# -----
```

```
f <- function(x) {
  if (x <= 0.4) return(-1)
  if (x >= 0.6) return(1)
  if (x > 0.4 & x < 0.6) return(10*(x-0.5))
}

x <- seq(0.05, 0.95, length = 7)
y <- sapply(x, f)
x_new <- seq(0, 1, length = 100)

# Fit model and calculate IMSE
fit <- fit_one_layer(x, y, nmcmc = 100, cov = "exp2")
fit <- trim(fit, 50)
fit <- predict(fit, x_new, cores = 1, store_latent = TRUE)
imse <- IMSE(fit)

# -----
# Example 2: Higdon function
# -----
```

```

f <- function(x) {
  i <- which(x <= 0.48)
  x[i] <- 2 * sin(pi * x[i] * 4) + 0.4 * cos(pi * x[i] * 16)
  x[-i] <- 2 * x[-i] - 1
  return(x)
}

# Training data
x <- seq(0, 1, length = 30)
y <- f(x) + rnorm(30, 0, 0.05)

# Testing data
xx <- seq(0, 1, length = 100)
yy <- f(xx)

plot(xx, yy, type = "l")
points(x, y, col = 2)

# Conduct MCMC (can replace fit_three_layer with fit_one_layer/fit_two_layer)
# nugget estimated
fit <- fit_three_layer(x, y, D = 1, nmcmc = 2000, cov = "exp2")
plot(fit)
fit <- trim(fit, 1000, 2)

# Option 1 - calculate IMSE from only MCMC iterations
imse <- IMSE(fit, xx)

# Option 2 - calculate IMSE after predictions
fit <- predict(fit, xx, cores = 1, store_latent = TRUE)
imse <- IMSE(fit)

# Visualize fit
plot(fit)
par(new = TRUE) # overlay IMSE
plot(xx, imse$value, col = 2, type = 'l', lty = 2, axes = FALSE,
      xlab = '', ylab = '')

# Select next design point
x_new <- xx[which.min(imse$value)]

```

---

plot

*Plots object from deepgp package*


---

### Description

Acts on a gp, gpvec, dgp2, dgp2vec, dgp3, or dgp3vec object. Generates trace plots for outer log likelihood, length scale, and nugget hyperparameters. Generates plots of hidden layers for

low dimensions or monotonic warpings. Generates plots of the posterior mean and estimated 90% prediction intervals for one-dimensional inputs; generates heat maps of the posterior mean and point-wise variance for two-dimensional inputs.

### Usage

```
## S3 method for class 'gp'
plot(x, trace = NULL, predict = NULL, ...)

## S3 method for class 'gpvec'
plot(x, trace = NULL, predict = NULL, ...)

## S3 method for class 'dgp2'
plot(x, trace = NULL, hidden = NULL, predict = NULL, ...)

## S3 method for class 'dgp2vec'
plot(x, trace = NULL, hidden = NULL, predict = NULL, ...)

## S3 method for class 'dgp3'
plot(x, trace = NULL, hidden = NULL, predict = NULL, ...)

## S3 method for class 'dgp3vec'
plot(x, trace = NULL, hidden = NULL, predict = NULL, ...)
```

### Arguments

x	object of class gp, gpvec, dgp2, dgp2vec, dgp3, or dgp3vec
trace	logical indicating whether to generate trace plots (default is TRUE if the object has not been through predict)
predict	logical indicating whether to generate posterior predictive plot (default is TRUE if the object has been through predict)
...	N/A
hidden	logical indicating whether to generate plots of hidden layers (two or three layer only, default is FALSE)

### Details

Trace plots are useful in assessing burn-in. If there are too many hyperparameters to plot them all, then it is most useful to visualize the log likelihood (e.g., `plot(fit$l1, type = "l")`).

In one dimension, hidden layer plots show 100 evenly distributed samples. In two dimensions (two layer only), hidden layer plots show 3 samples. Hidden layer plots are colored on a gradient - red lines represent earlier iterations and yellow lines represent later iterations - to help assess burn-in of the hidden layers.

### Examples

```
# See ?fit_one_layer, ?fit_two_layer, or ?fit_three_layer
# for examples
```

---

post_sample	<i>Generates joint posterior samples from a trained GP/DGP</i>
-------------	--

---

**Description**

Acts on a gp, gpvec, dgp2, dgp2vec, dgp3, or dgp3vec object. Generates joint samples from the posterior distribution at the provided locations.

**Usage**

```
post_sample(object, x_new, nper = 1, ...)  
  
## S3 method for class 'gp'  
post_sample(object, x_new, nper = 1, grad = FALSE, cores = 1, ...)  
  
## S3 method for class 'gpvec'  
post_sample(  
  object,  
  x_new,  
  nper = 1,  
  m = NULL,  
  ord_new = NULL,  
  grad = FALSE,  
  cores = 1,  
  ...  
)  
  
## S3 method for class 'dgp2'  
post_sample(  
  object,  
  x_new,  
  nper = 1,  
  grad = FALSE,  
  mean_map = TRUE,  
  cores = 1,  
  ...  
)  
  
## S3 method for class 'dgp2vec'  
post_sample(  
  object,  
  x_new,  
  nper = 1,  
  m = NULL,  
  ord_new = NULL,  
  grad = FALSE,  
  mean_map = TRUE,
```

```

    cores = 1,
    ...
)

## S3 method for class 'dgp3'
post_sample(object, x_new, nper = 1, mean_map = TRUE, cores = 1, ...)

## S3 method for class 'dgp3vec'
post_sample(
  object,
  x_new,
  nper = 1,
  m = NULL,
  ord_new = NULL,
  mean_map = TRUE,
  cores = 1,
  ...
)

```

### Arguments

object	object from <code>fit_one_layer</code> , <code>fit_two_layer</code> , or <code>fit_three_layer</code> with burn-in already removed
x_new	vector or matrix of predictive input locations
nper	the number of samples to generate from each MCMC iteration. The total number of samples will equal <code>nper*object\$nmcmc</code>
...	N/A
grad	logical indicating whether to additionally calculate/return samples of the gradient (one and two layer models only)
cores	number of cores to use for SNOW parallelization
m	size of Vecchia conditioning sets (only for fits with <code>vecchia = TRUE</code> ), defaults to the twice the <code>m</code> used for MCMC
ord_new	optional ordering for Vecchia approximation with <code>lite = FALSE</code> , must correspond to rows of <code>x_new</code> , defaults to random, is applied to all layers in deeper models
mean_map	logical indicating whether to map hidden layers using conditional mean ( <code>mean_map = TRUE</code> ) or using a random sample from the full MVN distribution (two or three layer models only)

### Details

By default, one sample is generated per each MCMC iteration. This may be increased with the `nper` argument.

SNOW parallelization reduces computation time but requires more memory storage.

**Value**

If `grad = FALSE`, returns matrix of samples. Rows correspond to `x_new` locations. If `grad = TRUE`, returns a list with `y` and `dydx` containing the respective samples.

**References**

Sauer, A. (2023). Deep Gaussian process surrogates for computer experiments. \*Ph.D. Dissertation, Department of Statistics, Virginia Polytechnic Institute and State University.\*

**Examples**

```
# Simple step function
f <- function(x) {
  return(pnorm((x - 0.5) / 0.065))
}

# Training data
x <- seq(0, 1, length = 5)
y <- f(x)

# Testing data
xx <- seq(0, 1, length = 100)
yy <- f(xx)

plot(xx, yy, type = "l")
points(x, y, col = 2)

# Conduct MCMC
fit <- fit_two_layer(x, y, nmc = 2000, true_g = 1e-6, cov = "exp2")
plot(fit, hidden = TRUE)
fit <- trim(fit, 1000, 2)

# Generate posterior samples, including gradients
samples <- post_sample(fit, xx, grad = TRUE, cores = 1)

# Plot samples
par(mfrow = c(1, 2))
matplot(xx, t(samples$y), type = "l")
points(x, y, pch = 20)
matplot(xx, t(samples$dy), type = "l")
```

**Description**

Acts on a `gp`, `gpvec`, `dgp2`, `dgp2vec`, `dgp3`, or `dgp3vec` object. Calculates posterior mean and variance/covariance over specified input locations. Optionally calculates expected improvement (EI) or entropy over candidate inputs. Optionally utilizes SNOW parallelization.

**Usage**

```
## S3 method for class 'gp'
predict(
  object,
  x_new,
  lite = TRUE,
  grad = FALSE,
  return_all = FALSE,
  EI = FALSE,
  entropy_limit = NULL,
  cores = 1,
  ...
)

## S3 method for class 'dgp2'
predict(
  object,
  x_new,
  lite = TRUE,
  grad = FALSE,
  store_latent = FALSE,
  mean_map = TRUE,
  return_all = FALSE,
  EI = FALSE,
  entropy_limit = NULL,
  cores = 1,
  ...
)

## S3 method for class 'dgp3'
predict(
  object,
  x_new,
  lite = TRUE,
  store_latent = FALSE,
  mean_map = TRUE,
  return_all = FALSE,
  EI = FALSE,
  entropy_limit = NULL,
  cores = 1,
  ...
)
```

```
## S3 method for class 'gpvec'
predict(
  object,
  x_new,
  m = NULL,
  ord_new = NULL,
  lite = TRUE,
  grad = FALSE,
  return_all = FALSE,
  EI = FALSE,
  entropy_limit = NULL,
  cores = 1,
  ...
)

## S3 method for class 'dgp2vec'
predict(
  object,
  x_new,
  m = NULL,
  ord_new = NULL,
  lite = TRUE,
  grad = FALSE,
  store_latent = FALSE,
  mean_map = TRUE,
  return_all = FALSE,
  EI = FALSE,
  entropy_limit = NULL,
  cores = 1,
  ...
)

## S3 method for class 'dgp3vec'
predict(
  object,
  x_new,
  m = NULL,
  ord_new = NULL,
  lite = TRUE,
  store_latent = FALSE,
  mean_map = TRUE,
  return_all = FALSE,
  EI = FALSE,
  entropy_limit = NULL,
  cores = 1,
  ...
)
```

**Arguments**

<code>object</code>	object from <code>fit_one_layer</code> , <code>fit_two_layer</code> , or <code>fit_three_layer</code> with burn-in already removed
<code>x_new</code>	vector or matrix of predictive input locations
<code>lite</code>	logical indicating whether to calculate only point-wise variances ( <code>lite = TRUE</code> ) or full covariance ( <code>lite = FALSE</code> )
<code>grad</code>	logical indicating whether to additionally calculate/return predictions of the gradient (one and two layer models only)
<code>return_all</code>	logical indicating whether to return mean and point-wise variance prediction for ALL samples (only available for <code>lite = TRUE</code> )
<code>EI</code>	logical indicating whether to calculate expected improvement (for minimizing the response)
<code>entropy_limit</code>	optional limit state for entropy calculations (separating passes and failures), default value of <code>NULL</code> bypasses entropy calculations
<code>cores</code>	number of cores to utilize for SNOW parallelization
<code>...</code>	N/A
<code>store_latent</code>	logical indicating whether to store and return mapped values of latent layers (two or three layer models only)
<code>mean_map</code>	logical indicating whether to map hidden layers using conditional mean ( <code>mean_map = TRUE</code> ) or using a random sample from the full MVN distribution (two or three layer models only)
<code>m</code>	size of Vecchia conditioning sets, defaults to the lower of twice the <code>m</code> used for MCMC or the maximum available (only for fits with <code>vecchia = TRUE</code> ),
<code>ord_new</code>	optional ordering for Vecchia approximation with <code>lite = FALSE</code> , must correspond to rows of <code>x_new</code> , defaults to random, is applied to all layers in deeper models

**Details**

All iterations in the object are used for prediction, so samples should be burned-in. Thinning the samples using `trim` will speed up computation. Posterior moments are calculated using conditional expectation and variance. As a default, only point-wise variance is calculated. Full covariance may be calculated using `lite = FALSE`.

Expected improvement is calculated with the goal of minimizing the response. See Chapter 7 of Gramacy (2020) for details. Entropy is calculated based on two classes separated by the specified limit. See Sauer (2023, Chapter 3) for details.

SNOW parallelization reduces computation time but requires more memory storage.

**Value**

object of the same class with the following additional elements:

- `x_new`: copy of predictive input locations
- `mean`: predicted posterior mean, indices correspond to `x_new` locations

- `s2`: predicted point-wise variances, indices correspond to `x_new` locations (only returned when `lite = TRUE`)
- `mean_all`: predicted posterior mean for each sample (rows correspond to iterations), only returned when `return_all = TRUE`
- `s2_all`: predicted point-wise variances for each sample (rows correspond to iterations), only returned when `return_all = TRUE`
- `Sigma`: predicted posterior covariance, indices correspond to `x_new` locations (only returned when `lite = FALSE`)
- `grad_mean`: predicted posterior mean of the gradient (rows correspond to `x_new`, columns correspond to dimension, only returned when `grad = TRUE`)
- `grad_s2`: predicted point-wise variances of the gradient (rows correspond to `x_new`, columns correspond to dimension, only returned when `grad = TRUE`)
- `EI`: vector of expected improvement values, indices correspond to `x_new` locations (only returned when `EI = TRUE`)
- `entropy`: vector of entropy values, indices correspond to `x_new` locations (only returned when `entropy_limit` is numeric)
- `w_new`: array of hidden layer mappings, with dimensions corresponding to iteration, then `x_new` location, then dimension (only returned when `store_latent = TRUE`)
- `z_new`: array of hidden layer mappings, with dimensions corresponding to iteration, then `x_new` location, then dimension (only returned when `store_latent = TRUE`)

Computation time is added to the computation time of the existing object.

## References

- Sauer, A. (2023). Deep Gaussian process surrogates for computer experiments. \*Ph.D. Dissertation, Department of Statistics, Virginia Polytechnic Institute and State University.\* <http://hdl.handle.net/10919/114845>
- Booth, A. S. (2025). Deep Gaussian processes with gradients. arXiv:2512.18066
- Sauer, A., Gramacy, R.B., & Higdon, D. (2023). Active learning for deep Gaussian process surrogates. \*Technometrics, 65,\* 4-18. arXiv:2012.08015
- Sauer, A., Cooper, A., & Gramacy, R. B. (2023). Vecchia-approximated deep Gaussian processes for computer experiments. \*Journal of Computational and Graphical Statistics, 32\*(3), 824-837. arXiv:2204.02904
- Gramacy, R. B., Sauer, A. & Wycoff, N. (2022). Triangulation candidates for Bayesian optimization. \*Advances in Neural Information Processing Systems (NeurIPS), 35,\* 35933-35945. arXiv:2112.07457
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- Barnett, S., Beesley, L. J., Booth, A. S., Gramacy, R. B., & Osthus D. (2025). Monotonic warpings for additive and deep Gaussian processes. \*Statistics and Computing, 35\*(3), 65. arXiv:2408.01540

**Examples**

```
# See ?fit_one_layer, ?fit_two_layer, or ?fit_three_layer
# for examples
```

---

rmse	<i>Calculates RMSE</i>
------	------------------------

---

**Description**

Calculates root mean square error (lower RMSE indicate better fits).

**Usage**

```
rmse(y, mu)
```

**Arguments**

y	response vector
mu	predicted mean

**Examples**

```
# Additional examples including real-world computer experiments are available at:
# https://bitbucket.org/gramacylab/deepgp-ex/
```

---

score	<i>Calculates score</i>
-------	-------------------------

---

**Description**

Calculates score, proportional to the multivariate normal log likelihood. Higher scores indicate better fits. Only applicable to noisy data. Requires full covariance matrix (e.g. predict with `lite = FALSE`).

**Usage**

```
score(y, mu, sigma)
```

**Arguments**

y	response vector
mu	predicted mean
sigma	predicted covariance

**References**

Gneiting, T, and AE Raftery. 2007. Strictly Proper Scoring Rules, Prediction, and Estimation. *Journal of the American Statistical Association* 102 (477), 359-378.

**Examples**

```
# Additional examples including real-world computer experiments are available at:  
# https://bitbucket.org/gramacylab/deepgp-ex/
```

---

sq_dist	<i>Calculates squared pairwise distances</i>
---------	--

---

**Description**

Calculates squared pairwise euclidean distances using C.

**Usage**

```
sq_dist(X1, X2 = NULL)
```

**Arguments**

X1	matrix of input locations
X2	matrix of second input locations (if NULL, distance is calculated between X1 and itself)

**Details**

C code derived from the "laGP" package (Robert B Gramacy and Furong Sun).

**Value**

symmetric matrix of squared euclidean distances

**References**

Gramacy, RB and F Sun. (2016). laGP: Large-Scale Spatial Modeling via Local Approximate Gaussian Processes in R. *Journal of Statistical Software* 72 (1), 1-46. doi:10.18637/jss.v072.i01

**Examples**

```
x <- seq(0, 1, length = 10)  
d2 <- sq_dist(x)
```

---

to_vec	<i>Converts non-Vecchia object to its Vecchia version</i>
--------	---

---

**Description**

Converts an object of class "gp", "dgp2", or "dgp3" to its Vecchia equivalent ("gpvec", "dgp2vec", or "dgp3vec").

**Usage**

```
to_vec(object, m = NULL, ord = NULL, cores = NULL)
```

**Arguments**

object	object from <code>fit_one_layer</code> , <code>fit_two_layer</code> , or <code>fit_three_layer</code> with <code>vecchia = FALSE</code>
m	size of Vecchia conditioning sets, defaults to the lower of 25 or maximum available
ord	optional ordering for Vecchia approximation, defaults to random
cores	number of cores to use for OpenMP parallelization. Defaults to $\min(4, \text{maxcores} - 1)$ where <code>maxcores</code> is the number of detectable available cores.

**Details**

Creates and appends Vecchia-approximation objects to the provided fit. Defaults to random ordering and nearest neighbors conditioning. Useful for speeding up predictions when the testing size is large but the training size is small.

**Value**

The same object, with `x_approx`, `w_approx`, and `z_approx` appended (depending on the number of layers).

**Examples**

```
# See ?fit_one_layer for an example
```

---

trim *Trim/Thin MCMC iterations*

---

### Description

Acts on a `gp`, `gpvec`, `dgp2`, `dgp2vec`, `dgp3vec`, or `dgp3` object. Removes the specified number of MCMC iterations (starting at the first iteration). After these samples are removed, the remaining samples are optionally thinned.

### Usage

```
trim(object, burn, thin)

## S3 method for class 'gp'
trim(object, burn, thin = 1)

## S3 method for class 'gpvec'
trim(object, burn, thin = 1)

## S3 method for class 'dgp2'
trim(object, burn, thin = 1)

## S3 method for class 'dgp2vec'
trim(object, burn, thin = 1)

## S3 method for class 'dgp3'
trim(object, burn, thin = 1)

## S3 method for class 'dgp3vec'
trim(object, burn, thin = 1)
```

### Arguments

<code>object</code>	object from <code>fit_one_layer</code> , <code>fit_two_layer</code> , or <code>fit_three_layer</code>
<code>burn</code>	integer specifying number of iterations to cut off as burn-in
<code>thin</code>	integer specifying amount of thinning ( <code>thin = 1</code> keeps all iterations, <code>thin = 2</code> keeps every other iteration, <code>thin = 10</code> keeps every tenth iteration, etc.)

### Details

The resulting object will have `nmc` equal to the previous `nmc` minus `burn` divided by `thin`. It is recommended to start an MCMC fit then investigate trace plots to assess burn-in. Once burn-in has been achieved, use this function to remove the starting iterations. Thinning reduces the size of the resulting object while accounting for the high correlation between consecutive iterations.

### Value

object of the same class with the selected iterations removed

**Examples**

```
# See ?fit_one_layer, ?fit_two_layer, or ?fit_three_layer  
# for examples
```

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