Package 'dtw'

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Type Package

Title Dynamic Time Warping Algorithms

Description A comprehensive implementation of dynamic time warping (DTW) algorithms in R. DTW computes the optimal (least cumulative distance) alignment between points of two time series. Common DTW variants covered include local (slope) and global (window) constraints, subsequence matches, arbitrary distance definitions, normalizations, minimum variance matching, and so on. Provides cumulative distances, alignments, specialized plot styles, etc., as described in Giorgino (2009) <doi:10.18637/jss.v031.i07>.

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Imports graphics, grDevices, stats, utils

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Author Toni Giorgino [aut, cre]

Maintainer Toni Giorgino <toni.giorgino@gmail.com>

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Contents

`dtw-package` *Comprehensive implementation of Dynamic Time Warping (DTW) algorithms in R.*

Description

The DTW algorithm computes the stretch of the time axis which optimally maps one given timeseries (query) onto whole or part of another (reference). It yields the remaining cumulative distance after the alignment and the point-by-point correspondence (warping function). DTW is widely used e.g. for classification and clustering tasks in econometrics, chemometrics and general timeseries mining.

Details

Please see documentation for function `dtw()`, which is the main entry point to the package.

The R implementation in dtw provides:

- arbitrary windowing functions (global constraints), eg. the Sakoe-Chiba band; see `dtwWindowingFunctions()`
- arbitrary transition types (also known as step patterns, slope constraints, local constraints, or DP-recursion rules). This includes dozens of well-known types; see `stepPattern()`:
  - all step patterns classified by Rabiner-Juang, Sakoe-Chiba, and Rabiner-Myers;
  - symmetric and asymmetric;
  - Rabiner's smoothed variants;
  - arbitrary, user-defined slope constraints
- partial matches: open-begin, open-end, substring matches
- proper, pattern-dependent, normalization (exact average distance per step)
- the Minimum Variance Matching (MVM) algorithm (Latecki et al.)

Multivariate timeseries can be aligned with arbitrary local distance definitions, leveraging the `proxy::dist()` function of package **proxy**. DTW itself becomes a distance function with the dist semantics.

In addition to computing alignments, the package provides:

- methods for plotting alignments and warping functions in several classic styles (see plot gallery);

- graphical representation of step patterns;
- functions for applying a warping function, either direct or inverse; and more.

If you use this software, please cite it according to `citation("dtw")`. The package home page is at https://dynamictimewarping.github.io.

Author(s)

Toni Giorgino

References

- Toni Giorgino. *Computing and Visualizing Dynamic Time Warping Alignments in R: The dtw Package.* Journal of Statistical Software, 31(7), 1-24. doi:10.18637/jss.v031.i07
- Tormene, P.; Giorgino, T.; Quaglini, S. & Stefanelli, M. *Matching incomplete time series with dynamic time warping: an algorithm and an application to post-stroke rehabilitation.* Artif Intell Med, 2009, 45, 11-34 doi:10.1016/j.artmed.2008.11.007
- Rabiner, L. R., & Juang, B.-H. (1993). Chapter 4 in *Fundamentals of speech recognition.* Englewood Cliffs, NJ: Prentice Hall.

See Also

`dtw()` for the main entry point to the package; `dtwWindowingFunctions()` for global constraints; `stepPattern()` for local constraints; `proxy::dist()`, `analogue::distance()`, `vegan::vegdist()` to build local cost matrices for multivariate timeseries and custom distance functions.

Examples

```
library(dtw);
## demo(dtw);
```

---

aami *ANSI/AAMI EC13 Test Waveforms, 3a and 3b*

Description

ANSI/AAMI EC13 Test Waveforms 3a and 3b, as obtained from the PhysioBank database.

Format

Time-series objects (class `ts`).

Details

The following text is reproduced (abridged) from PhysioBank, page https://www.physionet.org/content/aami-ec13/1.0.0/. Other recordings belong to the dataset and can be obtained from the same page.

The files in this set can be used for testing a variety of devices that monitor the electrocardiogram. The recordings include both synthetic and real waveforms. For details on these test waveforms and how to use them, please refer to section 5.1.2.1, paragraphs (e) and (g) in the reference below. Each recording contains one ECG signal sampled at 720 Hz with 12-bit resolution.

Note

Timestamps in the datasets have been re-created at the indicated frequency of 720 Hz, whereas the original timestamps in ms (at least in text format) only had three decimal digits' precision, and were therefore affected by substantial jittering.

Source

https://www.physionet.org/content/aami-ec13/1.0.0/

References

- Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PCh, Mark RG, Mietus JE, Moody GB, Peng CK, Stanley HE. *PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals.* Circulation 101(23):e215-e220; 2000 (June 13).
- Cardiac monitors, heart rate meters, and alarms; American National Standard (ANSI/AAMI EC13:2002). Arlington, VA: Association for the Advancement of Medical Instrumentation, 2002.

Examples

```
data(aami3a);
data(aami3b);

## Plot both as a multivariate TS object
##  only extract the first 10 seconds

plot( main="ECG (mV)",
 window(
  cbind(aami3a,aami3b)   ,end=10)
)
```

countPaths *Count the number of warping paths consistent with the constraints.*

Description

Count how many possible warping paths exist in the alignment problem passed as an argument. The object passed as an argument is used to look up the problem parameters such as the used step pattern, windowing, open ends, and so on. The actual alignment is ignored.

Usage

```
countPaths(d, debug = FALSE)
```

Arguments

d	an object of class dtw
debug	return an intermediate result

Details

Note that the number of paths grows exponentially with problems size. The result may be approximate when windowing functions are used.

If debug=TRUE, a matrix used for the computation is returned instead of the final result.

Value

The number of paths.

Author(s)

Toni Giorgino

Examples

```
ds<-dtw(1:7+2,1:8,keep=TRUE,step=asymmetric);
countPaths(ds)
## Result: 126
```

dtw *Dynamic Time Warp*

Description

Compute Dynamic Time Warp and find optimal alignment between two time series.

Usage

```
dtw(
  x,
  y = NULL,
  dist.method = "Euclidean",
  step.pattern = symmetric2,
  window.type = "none",
  keep.internals = FALSE,
  distance.only = FALSE,
  open.end = FALSE,
  open.begin = FALSE,
  ...
)

is.dtw(d)

## S3 method for class 'dtw'
print(x, ...)
```

Arguments

x	query vector *or* local cost matrix
y	reference vector, or NULL if x given as a local cost matrix
dist.method	pointwise (local) distance function to use. See proxy::dist() in package **proxy**
step.pattern	a stepPattern object describing the local warping steps allowed with their cost (see stepPattern())
window.type	windowing function. Character: "none", "itakura", "sakoechiba", "slantedband", or a function (see details).
keep.internals	preserve the cumulative cost matrix, inputs, and other internal structures
distance.only	only compute distance (no backtrack, faster)
open.begin, open.end	perform open-ended alignments
...	additional arguments, passed to window.type
d	an arbitrary R object

Details

The function performs Dynamic Time Warp (DTW) and computes the optimal alignment between two time series `x` and `y`, given as numeric vectors. The "optimal" alignment minimizes the sum of distances between aligned elements. Lengths of `x` and `y` may differ.

The local distance between elements of `x` (query) and `y` (reference) can be computed in one of the following ways:

1. if `dist.method` is a string, `x` and `y` are passed to the `proxy::dist()` function in package **proxy** with the method given;
2. if `dist.method` is a function of two arguments, it invoked repeatedly on all pairs `x[i],y[j]` to build the local cost matrix;
3. multivariate time series and arbitrary distance metrics can be handled by supplying a precomputed local cost matrix. Element `[i,j]` of the local cost matrix is understood as the distance between element `x[i]` and `y[j]`. The distance matrix has therefore `n=length(x)` rows and `m=length(y)` columns (see note below).

Several common variants of the DTW recursion are supported via the `step.pattern` argument, which defaults to `symmetric2`. Step patterns are commonly used to *locally* constrain the slope of the alignment function. See `stepPattern()` for details.

Windowing enforces a *global* constraint on the envelope of the warping path. It is selected by passing a string or function to the `window.type` argument. Commonly used windows are (abbreviations allowed):

- `"none"` No windowing (default)
- `"sakoechiba"` A band around main diagonal
- `"slantedband"` A band around slanted diagonal
- `"itakura"` So-called Itakura parallelogram

`window.type` can also be an user-defined windowing function. See `dtwWindowingFunctions()` for all available windowing functions, details on user-defined windowing, and a discussion of the (mis)naming of the "Itakura" parallelogram as a global constraint. Some windowing functions may require parameters, such as the `window.size` argument.

Open-ended alignment, i.e. semi-unconstrained alignment, can be selected via the `open.end` switch. Open-end DTW computes the alignment which best matches all of the query with a *leading part* of the reference. This is proposed e.g. by Mori (2006), Sakoe (1979) and others. Similarly, open-begin is enabled via `open.begin`; it makes sense when `open.end` is also enabled (subsequence finding). Subsequence alignments are similar e.g. to UE2-1 algorithm by Rabiner (1978) and others. Please find a review in Tormene et al. (2009).

If the warping function is not required, computation can be sped up enabling the `distance.only=TRUE` switch, which skips the backtracking step. The output object will then lack the `index{1,2,1s,2s}` and `stepsTaken` fields.

`is.dtw` tests whether the argument is of class `dtw`.

Value

An object of class `dtw` with the following items:

- `distance` the minimum global distance computed, *not* normalized.
- `normalizedDistance` distance computed, *normalized* for path length, if normalization is known for chosen step pattern.
- `N,M` query and reference length
- `call` the function call that created the object
- `index1` matched elements: indices in `x`
- `index2` corresponding mapped indices in `y`
- `stepPattern` the `stepPattern` object used for the computation
- `jmin` last element of reference matched, if `open.end=TRUE`
- `directionMatrix` if `keep.internals=TRUE`, the directions of steps that would be taken at each alignment pair (integers indexing production rules in the chosen step pattern)
- `stepsTaken` the list of steps taken from the beginning to the end of the alignment (integers indexing chosen step pattern)
- `index1s, index2s` same as `index1/2`, excluding intermediate steps for multi-step patterns like `asymmetricP05()`
- `costMatrix` if `keep.internals=TRUE`, the cumulative cost matrix
- `query, reference` if `keep.internals=TRUE` and passed as the `x` and `y` arguments, the query and reference timeseries.

Note

Cost matrices (both input and output) have query elements arranged row-wise (first index), and reference elements column-wise (second index). They print according to the usual convention, with indexes increasing down- and rightwards. Many DTW papers and tutorials show matrices according to plot-like conventions, i.e. reference index growing upwards. This may be confusing.

Author(s)

Toni Giorgino

References

1. Toni Giorgino. *Computing and Visualizing Dynamic Time Warping Alignments in R: The dtw Package.* Journal of Statistical Software, 31(7), 1-24. doi:10.18637/jss.v031.i07
2. Tormene, P.; Giorgino, T.; Quaglini, S. & Stefanelli, M. *Matching incomplete time series with dynamic time warping: an algorithm and an application to post-stroke rehabilitation.* Artif Intell Med, 2009, 45, 11-34. doi:10.1016/j.artmed.2008.11.007
3. Sakoe, H.; Chiba, S., *Dynamic programming algorithm optimization for spoken word recognition,* Acoustics, Speech, and Signal Processing, IEEE Transactions on , vol.26, no.1, pp. 43-49, Feb 1978. doi:10.1109/TASSP.1978.1163055
4. Mori, A.; Uchida, S.; Kurazume, R.; Taniguchi, R.; Hasegawa, T. & Sakoe, H. *Early Recognition and Prediction of Gestures* Proc. 18th International Conference on Pattern Recognition ICPR 2006, 2006, 3, 560-563 doi:10.1109/ICPR.2006.467

5. Sakoe, H. *Two-level DP-matching–A dynamic programming-based pattern matching algorithm for connected word recognition* Acoustics, Speech, and Signal Processing, IEEE Transactions on, 1979, 27, 588-595 doi:10.1109/TASSP.1979.1163310
6. Rabiner L, Rosenberg A, Levinson S (1978). *Considerations in dynamic time warping algorithms for discrete word recognition.* IEEE Trans. Acoust., Speech, Signal Process., 26(6), 575-582. doi:10.1109/TASSP.1978.1163164
7. Muller M. *Dynamic Time Warping* in *Information Retrieval for Music and Motion*. Springer Berlin Heidelberg; 2007. p. 69-84. doi:10.1007/9783540740483_4

See Also

`dtwDist()`, for iterating dtw over a set of timeseries; `dtwWindowingFunctions()`, for windowing and global constraints; `stepPattern()`, step patterns and local constraints; `plot.dtw()`, plot methods for DTW objects. To generate a local cost matrix, the functions `proxy::dist()`, `analogue::distance()`, `vegan::vegdist()`, or `outer()` may come handy.

Examples

```
## A noisy sine wave as query
idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;

## A cosine is for reference; sin and cos are offset by 25 samples
reference<-cos(idx)
plot(reference); lines(query,col="blue");

## Find the best match
alignment<-dtw(query,reference);


## Display the mapping, AKA warping function - may be multiple-valued
## Equivalent to: plot(alignment,type="alignment")
plot(alignment$index1,alignment$index2,main="Warping function");

## Confirm: 25 samples off-diagonal alignment
lines(1:100-25,col="red")




#########
##
## Partial alignments are allowed.
##

alignmentOBE <-
  dtw(query[44:88],reference,
      keep.internals=TRUE,step.pattern=asymmetric,
      open.end=TRUE,open.begin=TRUE);
plot(alignmentOBE,type="two",off=1);
```

```
#########
##
## Subsetting allows warping and unwarping of
## timeseries according to the warping curve.
## See first example below.
##

## Most useful: plot the warped query along with reference
plot(reference)
lines(query[alignment$index1]~alignment$index2,col="blue")

## Plot the (unwarped) query and the inverse-warped reference
plot(query,type="l",col="blue")
points(reference[alignment$index2]~alignment$index1)



#########
##
## Contour plots of the cumulative cost matrix
##    similar to: plot(alignment,type="density") or
##                dtwPlotDensity(alignment)
## See more plots in ?plot.dtw
##

## keep.internals = TRUE so we can look into the cost matrix

alignment<-dtw(query,reference,keep.internals=TRUE);

contour(alignment$costMatrix,col=terrain.colors(100),x=1:100,y=1:100,
xlab="Query (noisy sine)",ylab="Reference (cosine)");

lines(alignment$index1,alignment$index2,col="red",lwd=2);




#########
##
## An hand-checkable example
##

ldist<-matrix(1,nrow=6,ncol=6);  # Matrix of ones
ldist[2,]<-0; ldist[,5]<-0;      # Mark a clear path of zeroes
ldist[2,5]<-.01;  # Forcely cut the corner

ds<-dtw(ldist);  # DTW with user-supplied local
                                 #   cost matrix
da<-dtw(ldist,step.pattern=asymmetric);  # Also compute the asymmetric
plot(ds$index1,ds$index2,pch=3); # Symmetric: alignment follows
                                 #   the low-distance marked path
```

```
points(da$index1,da$index2,col="red");  # Asymmetric: visiting
                                        #   1 is required twice

ds$distance;
da$distance;



#########
##
## A multivariate alignment example
##

## Reference: one lap around the unit circle
t.ref <- seq(0, 2*pi, length.out = 100)
reference <- cbind(cos(t.ref), sin(t.ref))

## Query: same path, traversed with a nonlinear time axis
u <- seq(0, 1, length.out = 70)
t.query <- 2*pi*(u^1.5)
query <- cbind(cos(t.query), sin(t.query))

## Explicitly choose the local distance for the multivariate samples
alignment.mv <- dtw(query, reference,
                    dist.method = "Euclidean",
                    keep.internals = TRUE)

## Equivalent precomputed local cost matrix:
## local.cost <- proxy::dist(query, reference, method = "Euclidean")
## alignment.mv <- dtw(local.cost, keep.internals = TRUE)

plot(reference, type = "l", asp = 1, col = "black",
     xlab = "x", ylab = "y",
     main = "Multivariate alignment: same path, different timing")
lines(query, col = "blue")

plot(alignment.mv$index1, alignment.mv$index2, type = "l",
     xlab = "query index", ylab = "reference index",
     main = "Warping function for multivariate alignment")
```

---

dtwDist *Compute a dissimilarity matrix*

Description

Compute the dissimilarity matrix between a set of single-variate timeseries.

Usage

```
dtwDist(mx, my = mx, ...)
```

Arguments

mx	numeric matrix, containing timeseries as rows
my	numeric matrix, containing timeseries as rows (for cross-distance)
...	arguments passed to the dtw() call

Details

dtwDist computes a dissimilarity matrix, akin to proxy::dist(), based on the Dynamic Time Warping definition of a distance between single-variate timeseries.

The dtwDist command is a synonym for the proxy::dist() function of package **proxy**; the DTW distance is registered as method="DTW" (see examples below).

The timeseries are stored as rows in the matrix argument m. In other words, if m is an N * T matrix, dtwDist will build N*N ordered pairs of timeseries, perform the corresponding N*N dtw alignments, and return all of the results in a matrix. Each of the timeseries is T elements long.

dtwDist returns a square matrix, whereas the dist object is lower-triangular. This makes sense because in general the DTW "distance" is not symmetric (see e.g. asymmetric step patterns). To make a square matrix with the proxy::dist() function semantics, use the two-arguments call as dist(m,m). This will return a square crossdist object.

Value

A square matrix whose element [i,j] holds the Dynamic Time Warp distance between row i (query) and j (reference) of mx and my, i.e. dtw(mx[i,],my[j,])$distance.

Note

To convert a square cross-distance matrix (crossdist object) to a symmetric proxy::dist() object, use a suitable conversion strategy (see examples).

Author(s)

Toni Giorgino

Examples

```
## Symmetric step pattern => symmetric dissimilarity matrix;
## no problem coercing it to a dist object:

m <- matrix(0,ncol=3,nrow=4)
m <- row(m)
dist(m,method="DTW");

# Old-fashioned call style would be:
```

```
#   dtwDist(m)
#   as.dist(dtwDist(m))



## Find the optimal warping _and_ scale factor at the same time.
## (There may be a better, analytic way)

# Prepare a query and a reference

query<-sin(seq(0,4*pi,len=100))
reference<-cos(seq(0,4*pi,len=100))

# Make a set of several references, scaled from 0 to 3 in .1 increments.
# Put them in a matrix, in rows

scaleSet <- seq(0.1,3,by=.1)
referenceSet<-outer(1/scaleSet,reference)

# The query has to be made into a 1-row matrix.
# Perform all of the alignments at once, and normalize the result.

dist(t(query),referenceSet,meth="DTW")->distanceSet

# The optimal scale for the reference is 1.0
plot(scaleSet,scaleSet*distanceSet,
  xlab="Reference scale factor (denominator)",
  ylab="DTW distance",type="o",
  main="Sine vs scaled cosine alignment, 0 to 4 pi")




## Asymmetric step pattern: we can either disregard part of the pairs
## (as.dist), or average with the transpose

mm <- matrix(runif(12),ncol=3)
dm <- dist(mm,mm,method="DTW",step=asymmetric); # a crossdist object

# Old-fashioned call style would be:
#   dm <- dtwDist(mm,step=asymmetric)
#   as.dist(dm)


## Symmetrize by averaging:
(dm+t(dm))/2


## check definition
stopifnot(dm[2,1]==dtw(mm[2,],mm[1,],step=asymmetric)$distance)
```

dtwPlot *Plotting of dynamic time warp results*

Description

Methods for plotting dynamic time warp alignment objects returned by dtw().

Usage

```
## S3 method for class 'dtw'
plot(x, type = "alignment", ...)

dtwPlotAlignment(
  d,
  xlab = "Query index",
  ylab = "Reference index",
  plot.type = "l",
  ...
)
```

Arguments

x, d	dtw object, usually result of call to dtw()
type	general style for the plot, see below
...	additional arguments, passed to plotting functions
xlab	label for the query axis
ylab	label for the reference axis
plot.type	type of line to be drawn, used as the type argument in the underlying plot call

Details

dtwPlot displays alignment contained in dtw objects.

Various plotting styles are available, passing strings to the type argument (may be abbreviated):

- alignment plots the warping curve in d;
- twoway plots a point-by-point comparison, with matching lines; see dtwPlotTwoWay();
- threeway vis-a-vis inspection of the timeseries and their warping curve; see dtwPlotThreeWay();
- density displays the cumulative cost landscape with the warping path overimposed; see dtwPlotDensity()

Additional parameters are passed to the plotting functions: use with care.

Warning

These functions are incompatible with mechanisms for arranging plots on a device: `par(mfrow)`, `layout` and `split.screen`.

Author(s)

Toni Giorgino

See Also

`dtwPlotTwoWay()`, `dtwPlotThreeWay()` and `dtwPlotDensity()` for details on the respective plotting styles.

Other plot: `dtwPlotDensity()`, `dtwPlotThreeWay()`, `dtwPlotTwoWay()`

Examples

```
## Same example as in dtw

idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx)

alignment<-dtw(query,reference,keep=TRUE);

# A sample of the plot styles. See individual plotting functions for details

plot(alignment, type="alignment",
  main="DTW sine/cosine: simple alignment plot")

plot(alignment, type="twoway",
  main="DTW sine/cosine: dtwPlotTwoWay")

plot(alignment, type="threeway",
  main="DTW sine/cosine: dtwPlotThreeWay")

plot(alignment, type="density",
  main="DTW sine/cosine: dtwPlotDensity")
```

---

dtwPlotDensity *Display the cumulative cost density with the warping path overimposed*

Description

The plot is based on the cumulative cost matrix. It displays the optimal alignment as a "ridge" in the global cost landscape.

Usage

```
dtwPlotDensity(
  d,
  normalize = FALSE,
  xlab = "Query index",
  ylab = "Reference index",
  ...
)
```

Arguments

d	an alignment result, object of class dtw
normalize	show per-step average cost instead of cumulative cost
xlab	label for the query axis
ylab	label for the reference axis
...	additional parameters forwarded to plotting functions

Details

The alignment must have been constructed with the keep.internals=TRUE parameter set.

If normalize is TRUE, the *average* cost per step is plotted instead of the cumulative one. Step averaging depends on the stepPattern() used.

See Also

Other plot: dtwPlot(), dtwPlotThreeWay(), dtwPlotTwoWay()

Examples

```
## A study of the "Itakura" parallelogram
##
## A widely held misconception is that the "Itakura parallelogram" (as
## described in the original article) is a global constraint.  Instead,
## it arises from local slope restrictions. Anyway, an "itakuraWindow",
## is provided in this package. A comparison between the two follows.

## The local constraint: three sides of the parallelogram are seen
idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx)

ita <- dtw(query,reference,keep=TRUE,step=typeIIIc)
dtwPlotDensity(ita, main="Slope-limited asymmetric step (Itakura)")

## Symmetric step with global parallelogram-shaped constraint. Note how
## long (>2 steps) horizontal stretches are allowed within the window.

dtw(query,reference,keep=TRUE,window=itakuraWindow)->ita;
dtwPlotDensity(ita,
```

```
          main="Symmetric step with Itakura parallelogram window")
```

`dtwPlotThreeWay` *Plotting of dynamic time warp results: annotated warping function*

Description

Display the query and reference time series and their warping curve, arranged for visual inspection.

Usage

```
dtwPlotThreeWay(
  d,
  xts = NULL,
  yts = NULL,
  type.align = "l",
  type.ts = "l",
  match.indices = NULL,
  margin = 4,
  inner.margin = 0.2,
  title.margin = 1.5,
  xlab = "Query index",
  ylab = "Reference index",
  main = "Timeseries alignment",
  ...
)
```

Arguments

`d`	an alignment result, object of class `dtw`
`xts`	query vector
`yts`	reference vector
`type.align`	line style for warping curve plot
`type.ts`	line style for timeseries plot
`match.indices`	indices for which to draw a visual guide
`margin`	outer figure margin
`inner.margin`	inner figure margin
`title.margin`	space on the top of figure
`xlab`	label for the query axis
`ylab`	label for the reference axis
`main`	main title
`...`	additional arguments, used for the warping curve

Details

The query time series is plotted in the bottom panel, with indices growing rightwards and values upwards. Reference is in the left panel, indices growing upwards and values leftwards. The warping curve panel matches indices, and therefore element (1,1) will be at the lower left, (N,M) at the upper right.

Argument `match.indices` is used to draw a visual guide to matches; if a vector is given, guides are drawn for the corresponding indices in the warping curve (match lines). If integer, it is used as the number of guides to be plotted. The corresponding style is customized via the `match.col` and `match.lty` arguments.

If `xts` and `yts` are not supplied, they will be recovered from `d`, as long as it was created with the two-argument call of `dtw()` with `keep.internals=TRUE`. Only single-variate time series can be plotted.

Warning

The function is incompatible with mechanisms for arranging plots on a device: `par(mfrow)`, `layout` and `split.screen`. Appearance of the match lines and timeseries currently can not be customized.

Author(s)

Toni Giorgino

See Also

Other plot: `dtwPlot()`, `dtwPlotDensity()`, `dtwPlotTwoWay()`

Examples

```
## A noisy sine wave as query
## A cosine is for reference; sin and cos are offset by 25 samples

idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx)
dtw(query,reference,keep=TRUE)->alignment;


## Beware of the reference's y axis, may be confusing
## Equivalent to plot(alignment,type="three");
dtwPlotThreeWay(alignment);


## Highlight matches of chosen QUERY indices. We will do some index
## arithmetics to recover the corresponding indices along the warping
## curve

hq <- (0:8)/8
hq <- round(hq*100)      #  indices in query for  pi/4 .. 7/4 pi
```

```
hw <- (alignment$index1 %in% hq)   # where are they on the w. curve?
hi <- (1:length(alignment$index1))[hw];   # get the indices of TRUE elems

dtwPlotThreeWay(alignment,match.indices=hi);
```

`dtwPlotTwoWay` *Plotting of dynamic time warp results: pointwise comparison*

Description

Display the query and reference time series and their alignment, arranged for visual inspection.

Usage

```
dtwPlotTwoWay(
  d,
  xts = NULL,
  yts = NULL,
  offset = 0,
  ts.type = "l",
  pch = 21,
  match.indices = NULL,
  match.col = "gray70",
  match.lty = 3,
  xlab = "Index",
  ylab = "Query value",
  ...
)
```

Arguments

`d`	an alignment result, object of class `dtw`
`xts`	query vector
`yts`	reference vector
`offset`	displacement between the timeseries, summed to reference
`ts.type`, `pch`	graphical parameters for timeseries plotting, passed to `matplot`
`match.indices`	indices for which to draw a visual guide
`match.col`, `match.lty`	color and line type of the match guide lines
`xlab`, `ylab`	axis labels
`...`	additional arguments, passed to `matplot`

Details

The two vectors are displayed via the matplot() functions; their appearance can be customized via the type and pch arguments (constants or vectors of two elements). If offset is set, the reference is shifted vertically by the given amount; this will be reflected by the *right-hand* axis.

Argument match.indices is used to draw a visual guide to matches; if a vector is given, guides are drawn for the corresponding indices in the warping curve (match lines). If integer, it is used as the number of guides to be plotted. The corresponding style is customized via the match.col and match.lty arguments.

If xts and yts are not supplied, they will be recovered from d, as long as it was created with the two-argument call of dtw() with keep.internals=TRUE. Only single-variate time series can be plotted this way.

Warning

The function is incompatible with mechanisms for arranging plots on a device: par(mfrow), layout and split.screen.

Note

When offset is set values on the left axis only apply to the query.

Author(s)

Toni Giorgino

See Also

dtwPlot() for other dtw plotting functions, matplot() for graphical parameters.

Other plot: dtwPlot(), dtwPlotDensity(), dtwPlotThreeWay()

Examples

```
## A noisy sine wave as query
## A cosine is for reference; sin and cos are offset by 25 samples

idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx)
dtw(query,reference,step=asymmetricP1,keep=TRUE)->alignment;


## Equivalent to plot(alignment,type="two");
dtwPlotTwoWay(alignment);


## Highlight matches of chosen QUERY indices. We will do some index
## arithmetics to recover the corresponding indices along the warping
## curve
```

```
hq <- (0:8)/8
hq <- round(hq*100)      #  indices in query for  pi/4 .. 7/4 pi

hw <- (alignment$index1 %in% hq)   # where are they on the w. curve?
hi <- (1:length(alignment$index1))[hw];   # get the indices of TRUE elems


## Beware of the reference's y axis, may be confusing
plot(alignment,offset=-2,type="two", lwd=3, match.col="grey50",
     match.indices=hi,main="Match lines shown every pi/4 on query");

legend("topright",c("Query","Reference (rt. axis)"), pch=21, col=1:6)
```

---

dtwWindowingFunctions *Global constraints and windowing functions for DTW*

Description

Various global constraints (windows) which can be applied to the window.type argument of dtw(), including the Sakoe-Chiba band, the Itakura parallelogram, and custom functions.

Usage

```
sakoeChibaWindow(iw, jw, window.size, ...)

slantedBandWindow(iw, jw, query.size, reference.size, window.size, ...)

itakuraWindow(iw, jw, query.size, reference.size, ...)

dtwWindow.plot(fun, query.size = 200, reference.size = 220, ...)

noWindow(iw, jw, ...)
```

Arguments

iw	index in the query (row) – automatically set
jw	index in the reference (column) – automatically set
window.size	window size, used by some windowing functions – must be set
...	additional arguments passed to windowing functions
query.size	size of the query time series – automatically set
reference.size	size of the reference time series – automatically set
fun	a windowing function

Details

Windowing functions can be passed to the `window.type` argument in `dtw()` to put a global constraint to the warping paths allowed. They take two integer arguments (plus optional parameters) and must return a boolean value `TRUE` if the coordinates fall within the allowed region for warping paths, `FALSE` otherwise.

User-defined functions can read variables `reference.size`, `query.size` and `window.size`; these are pre-set upon invocation. Some functions require additional parameters which must be set (e.g. `window.size`). User-defined functions are free to implement any window shape, as long as at least one path is allowed between the initial and final alignment points, i.e., they are compatible with the DTW constraints.

The `sakoeChibaWindow` function implements the Sakoe-Chiba band, i.e. `window.size` elements around the `main` diagonal. If the window size is too small, i.e. if `reference.size-query.size > window.size`, warping becomes impossible.

An `itakuraWindow` global constraint is still provided with this package. See example below for a demonstration of the difference between a local the two.

The `slantedBandWindow` (package-specific) is a band centered around the (jagged) line segment which joins element `[1,1]` to element `[query.size,reference.size]`, and will be `window.size` columns wide. In other words, the "diagonal" goes from one corner to the other of the possibly rectangular cost matrix, therefore having a slope of `M/N`, not 1.

`dtwWindow.plot` visualizes a windowing function. By default it plots a 200 x 220 rectangular region, which can be changed via `reference.size` and `query.size` arguments.

Value

Windowing functions return `TRUE` if the coordinates passed as arguments fall within the chosen warping window, `FALSE` otherwise. User-defined functions should do the same.

Note

Although `dtwWindow.plot` resembles object-oriented notation, there is not a such a dtwWindow class currently.

A widely held misconception is that the "Itakura parallelogram" (as described in reference 2) is a *global* constraint, i.e. a window. To the author's knowledge, it instead arises from the local slope restrictions imposed to the warping path, such as the one implemented by the `typeIIIc()` step pattern.

Author(s)

Toni Giorgino

References

1. Sakoe, H.; Chiba, S., *Dynamic programming algorithm optimization for spoken word recognition,* Acoustics, Speech, and Signal Processing, IEEE Transactions on , vol.26, no.1, pp. 43-49, Feb 1978 doi:10.1109/TASSP.1978.1163055

2. Itakura, F., *Minimum prediction residual principle applied to speech recognition,* Acoustics, Speech, and Signal Processing, IEEE Transactions on , vol.23, no.1, pp. 67-72, Feb 1975. doi:10.1109/TASSP.1975.1162641

Examples

```
## Display some windowing functions
dtwWindow.plot(itakuraWindow, main="So-called Itakura parallelogram window")
dtwWindow.plot(slantedBandWindow, window.size=2,
  reference=13, query=17, main="The slantedBandWindow at window.size=2")


## Asymmetric step with Sakoe-Chiba band

idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx);

asyband<-dtw(query,reference,keep=TRUE,
             step=asymmetric,
             window.type=sakoeChibaWindow,
             window.size=30                 );

dtwPlot(asyband,type="density",main="Sine/cosine: asymmetric step, S-C window")
```

`mvm` *Minimum Variance Matching algorithm*

Description

Step patterns to compute the Minimum Variance Matching (MVM) correspondence between time series

Usage

```
mvmStepPattern(elasticity = 20)
```

Arguments

`elasticity`	integer: maximum consecutive reference elements skippable

Details

The Minimum Variance Matching algorithm (1) finds the non-contiguous parts of reference which best match the query, allowing for arbitrarily long "stretches" of reference to be excluded from the match. All elements of the query have to be matched. First and last elements of the query are anchored at the boundaries of the reference.

The `mvmStepPattern` function creates a `stepPattern` object which implements this behavior, to be used with the usual `dtw()` call (see example). MVM is computed as a special case of DTW, with a very large, asymmetric-like step pattern.

The `elasticity` argument limits the maximum run length of reference which can be skipped at once. If no limit is desired, set `elasticity` to an integer at least as large as the reference (computation time grows linearly).

Value

A step pattern object.

Author(s)

Toni Giorgino

References

Latecki, L. J.; Megalooikonomou, V.; Wang, Q. & Yu, D. *An elastic partial shape matching technique* Pattern Recognition, 2007, 40, 3069-3080. doi:10.1016/j.patcog.2007.03.004

See Also

Other objects in `stepPattern()`.

Examples

```
## The hand-checkable example given in Fig. 5, ref. [1] above
diffmx <- matrix( byrow=TRUE, nrow=5, c(
  0,  1,  8,  2,  2,  4,  8,
  1,  0,  7,  1,  1,  3,  7,
 -7, -6,  1, -5, -5, -3,  1,
 -5, -4,  3, -3, -3, -1,  3,
 -7, -6,  1, -5, -5, -3,  1 ) ) ;

## Cost matrix
costmx <- diffmx^2;

## Compute the alignment
al <- dtw(costmx,step.pattern=mvmStepPattern(10))

## Elements 4,5 are skipped
print(al$index2)

plot(al,main="Minimum Variance Matching alignment")
```

stepPattern *Step patterns for DTW*

Description

A stepPattern object lists the transitions allowed while searching for the minimum-distance path. DTW variants are implemented by passing one of the objects described in this page to the stepPattern argument of the dtw() call.

Usage

```
## S3 method for class 'stepPattern'
t(x)

## S3 method for class 'stepPattern'
plot(x, ...)

## S3 method for class 'stepPattern'
print(x, ...)

rabinerJuangStepPattern(type, slope.weighting = "d", smoothed = FALSE)
```

Arguments

x	a step pattern object
...	additional arguments to print().
type	path specification, integer 1..7 (see (Rabiner1993), table 4.5)
slope.weighting	slope weighting rule: character "a" to "d" (see (Rabiner1993), sec. 4.7.2.5)
smoothed	logical, whether to use smoothing (see (Rabiner1993), fig. 4.44)

Details

A step pattern characterizes the matching model and slope constraint specific of a DTW variant. They also known as local- or slope-constraints, transition types, production or recursion rules (GiorginoJSS).

Pre-defined step patterns

```
## Well-known step patterns
symmetric1
symmetric2
asymmetric
```

```
## Step patterns classified according to Rabiner-Juang (Rabiner1993)
rabinerJuangStepPattern(type,slope.weighting="d",smoothed=FALSE)

## Slope-constrained step patterns from Sakoe-Chiba (Sakoe1978)
symmetricP0;  asymmetricP0
symmetricP05; asymmetricP05
symmetricP1;  asymmetricP1
symmetricP2;  asymmetricP2

## Step patterns classified according to Rabiner-Myers (Myers1980)
typeIa;   typeIb;   typeIc;   typeId;
typeIas;  typeIbs;  typeIcs;  typeIds;  # smoothed
typeIIa;  typeIIb;  typeIIc;  typeIId;
typeIIIc; typeIVc;

## Miscellaneous
mori2006;
rigid;
```

A variety of classification schemes have been proposed for step patterns, including Sakoe-Chiba (Sakoe1978); Rabiner-Juang (Rabiner1993); and Rabiner-Myers (Myers1980). The `dtw` package implements all of the transition types found in those papers, with the exception of Itakura's and Velichko-Zagoruyko's steps, which require subtly different algorithms (this may be rectified in the future). Itakura recursion is almost, but not quite, equivalent to `typeIIIc`.

For convenience, we shall review pre-defined step patterns grouped by classification. Note that the same pattern may be listed under different names. Refer to paper (GiorginoJSS) for full details.

1. Well-known step patterns

Common DTW implementations are based on one of the following transition types.

`symmetric2` is the normalizable, symmetric, with no local slope constraints. Since one diagonal step costs as much as the two equivalent steps along the sides, it can be normalized dividing by `N+M` (query+reference lengths). It is widely used and the default.

`asymmetric` is asymmetric, slope constrained between 0 and 2. Matches each element of the query time series exactly once, so the warping path `index2~index1` is guaranteed to be single-valued. Normalized by `N` (length of query).

`symmetric1` (or White-Neely) is quasi-symmetric, no local constraint, non-normalizable. It is biased in favor of oblique steps.

2. The Rabiner-Juang set

A comprehensive table of step patterns is proposed in Rabiner-Juang's book (Rabiner1993), tab. 4.5. All of them can be constructed through the `rabinerJuangStepPattern(type,slope.weighting,smoothed)` function.

The classification foresees seven families, labelled with Roman numerals I-VII; here, they are selected through the integer argument `type`. Each family has four slope weighting sub-types, named in sec. 4.7.2.5 as "Type (a)" to "Type (d)"; they are selected passing a character argument `slope.weighting`, as in the table below. Furthermore, each subtype can be either plain or smoothed

(figure 4.44); smoothing is enabled setting the logical argument `smoothed`. (Not all combinations of arguments make sense.)

```
Subtype | Rule       | Norm | Unbiased
--------|------------|------|---------
   a    | min step   |  --  |   NO
   b    | max step   |  --  |   NO
   c    | Di step    |   N  |  YES
   d    | Di+Dj step | N+M  |  YES
```

3. The Sakoe-Chiba set

Sakoe-Chiba (Sakoe1978) discuss a family of slope-constrained patterns; they are implemented as shown in page 47, table I. Here, they are called `symmetricP<x>` and `asymmetricP<x>`, where `<x>` corresponds to Sakoe's integer slope parameter *P*. Values available are accordingly: `0` (no constraint), `1`, `05` (one half) and `2`. See (Sakoe1978) for details.

4. The Rabiner-Myers set

The `type<XX><y>` step patterns follow the older Rabiner-Myers' classification proposed in (Myers1980) and (MRR1980). Note that this is a subset of the Rabiner-Juang set (Rabiner1993), and the latter should be preferred in order to avoid confusion. `<XX>` is a Roman numeral specifying the shape of the transitions; `<y>` is a letter in the range `a-d` specifying the weighting used per step, as above; `typeIIx` patterns also have a version ending in `s`, meaning the smoothing is used (which does not permit skipping points). The `typeId, typeIId` and `typeIIds` are unbiased and symmetric.

5. Others

The `rigid` pattern enforces a fixed unitary slope. It only makes sense in combination with `open.begin=TRUE`, `open.end=TRUE` to find gapless subsequences. It may be seen as the `P->inf` limiting case in Sakoe's classification.

`mori2006` is Mori's asymmetric step-constrained pattern (Mori2006). It is normalized by the matched reference length.

`mvmStepPattern()` implements Latecki's Minimum Variance Matching algorithm, and it is described in its own page.

Methods

`print.stepPattern` prints an user-readable description of the recurrence equation defined by the given pattern.

`plot.stepPattern` graphically displays the step patterns productions which can lead to element (0,0). Weights are shown along the step leading to the corresponding element.

`t.stepPattern` transposes the productions and normalization hint so that roles of query and reference become reversed.

Note

Constructing `stepPattern` objects is tricky and thus undocumented. For a commented example please see source code for `symmetricP1`.

Author(s)

Toni Giorgino

References

- (GiorginoJSS) Toni Giorgino. *Computing and Visualizing Dynamic Time Warping Alignments in R: The dtw Package.* Journal of Statistical Software, 31(7), 1-24. doi:10.18637/jss.v031.i07
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- (MRR1980) Myers, C.; Rabiner, L. & Rosenberg, A. *Performance tradeoffs in dynamic time warping algorithms for isolated word recognition*, IEEE Trans. Acoust., Speech, Signal Process., 1980, 28, 623-635. doi:10.1109/TASSP.1980.1163491
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- (Rabiner1993) Rabiner, L. R., & Juang, B.-H. (1993). *Fundamentals of speech recognition.* Englewood Cliffs, NJ: Prentice Hall.
- (Sakoe1978) Sakoe, H.; Chiba, S., *Dynamic programming algorithm optimization for spoken word recognition,* Acoustics, Speech, and Signal Processing, IEEE Transactions on , vol.26, no.1, pp. 43-49, Feb 1978 doi:10.1109/TASSP.1978.1163055

See Also

`mvmStepPattern()`, implementing Latecki's Minimal Variance Matching algorithm.

Examples

```
#########
##
## The usual (normalizable) symmetric step pattern
## Step pattern recursion, defined as:
## g[i,j] = min(
##      g[i,j-1] + d[i,j] ,
##      g[i-1,j-1] + 2 * d[i,j] ,
##      g[i-1,j] + d[i,j] ,
##   )

print(symmetric2)   # or just "symmetric2"



#########
```

```
##
## The well-known plotting style for step patterns

plot(symmetricP2,main="Sakoe's Symmetric P=2 recursion")



#########
##
## Same example seen in ?dtw , now with asymmetric step pattern

idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx);

## Do the computation
asy<-dtw(query,reference,keep=TRUE,step=asymmetric);

dtwPlot(asy,type="density",main="Sine and cosine, asymmetric step")


#########
##
##  Hand-checkable example given in [Myers1980] p 61
##

`tm` <-
structure(c(1, 3, 4, 4, 5, 2, 2, 3, 3, 4, 3, 1, 1, 1, 3, 4, 2,
3, 3, 2, 5, 3, 4, 4, 1), .Dim = c(5L, 5L))
```

---

warp *Apply a warping to a given timeseries*

Description

Returns the indexing required to apply the optimal warping curve to a given timeseries (warps either into a query or into a reference).

Usage

```
warp(d, index.reference = FALSE)
```

Arguments

d	dtw object specifying the warping curve to apply
index.reference	TRUE to warp a reference, FALSE to warp a query

Details

The warping is returned as a set of indices, which can be used to subscript the timeseries to be warped (or rows in a matrix, if one wants to warp a multivariate time series). In other words, `warp` converts the warping curve, or its inverse, into a function in the explicit form.

Multiple indices that would be mapped to a single point are averaged, with a warning. Gaps in the index sequence are filled by linear interpolation.

Value

A list of indices to subscript the timeseries.

Author(s)

Toni Giorgino

See Also

Examples in `dtw()` show how to *graphically* apply the warping via parametric plots.

Examples

```
idx<-seq(0,6.28,len=100);
query<-sin(idx)+runif(100)/10;
reference<-cos(idx)

alignment<-dtw(query,reference);


wq<-warp(alignment,index.reference=FALSE);
wt<-warp(alignment,index.reference=TRUE);

old.par <- par(no.readonly = TRUE);
par(mfrow=c(2,1));

plot(reference,main="Warping query");
  lines(query[wq],col="blue");

plot(query,type="l",col="blue",
  main="Warping reference");
  points(reference[wt]);

par(old.par);


##############
##
## Asymmetric step makes it "natural" to warp
## the reference, because every query index has
## exactly one image (q->t is a function)
##
```

```
alignment<-dtw(query,reference,step=asymmetric)
wt<-warp(alignment,index.reference=TRUE);

plot(query,type="l",col="blue",
  main="Warping reference, asymmetric step");
  points(reference[wt]);
```

---

warpArea *Compute Warping Path Area*

Description

Compute the area between the warping function and the diagonal (no-warping) path, in unit steps.

Usage

```
warpArea(d)
```

Arguments

d	an object of class dtw

Details

Above- and below- diagonal unit areas all count *plus* one (they do not cancel with each other). The "diagonal" goes from one corner to the other of the possibly rectangular cost matrix, therefore having a slope of M/N, not 1, as in slantedBandWindow().

The computation is approximate: points having multiple correspondences are averaged, and points without a match are interpolated. Therefore, the area can be fractionary.

Value

The area, not normalized by path length or else.

Note

There could be alternative definitions to the area, including considering the envelope of the path.

Author(s)

Toni Giorgino

Examples

```
ds<-dtw(1:4,1:8);

plot(ds);lines(seq(1,8,len=4),col="red");

warpArea(ds)

## Result: 6
##  index 2 is 2 while diag is 3.3  (+1.3)
##        3    3               5.7  (+2.7)
##        4   4:8 (avg to 6)    8   (+2  )
##                                 --------
##                                     6
```

Index