

Package ‘fbardl’

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Type Package

Title Fourier Bootstrap ARDL Cointegration Test

Version 1.0.2

Description Implements the Fourier Bootstrap Autoregressive Distributed Lag (FBARDL) bounds testing approach for cointegration analysis. Combines the Pesaran, Shin & Smith (2001) <[doi:10.1002/jae.616](https://doi.org/10.1002/jae.616)> ARDL bounds testing framework with Fourier terms to capture structural breaks following Yilanci, Bozoklu & Gorus (2020) <[doi:10.1080/00036846.2019.1686454](https://doi.org/10.1080/00036846.2019.1686454)>, and bootstrap critical values based on McNown, Sam & Goh (2018) <[doi:10.1080/00036846.2017.1366643](https://doi.org/10.1080/00036846.2017.1366643)> and Bertelli, Vacca & Zoia (2022) <[doi:10.1016/j.econmod.2022.105987](https://doi.org/10.1016/j.econmod.2022.105987)>. Features include automatic lag selection via AIC/BIC, optimal Fourier frequency selection by minimum SSR, long-run and short-run coefficient estimation, diagnostic tests, and dynamic multiplier analysis.

License GPL-3

Encoding UTF-8

Depends R (>= 3.5.0)

Imports stats

Suggests testthat (>= 3.0.0), knitr, rmarkdown

LazyData true

RoxygenNote 7.3.3

URL <https://github.com/muhammedalkhalaf/fbardl>

BugReports <https://github.com/muhammedalkhalaf/fbardl/issues>

NeedsCompilation no

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Description

Performs the Fourier Bootstrap ARDL (FBARDL) bounds testing approach for cointegration analysis. This function combines the Pesaran, Shin & Smith (2001) ARDL bounds testing framework with Fourier terms to capture structural breaks, and provides bootstrap critical values for robust inference.

Usage

```
fbardl(
  formula,
  data,
  type = c("fardl", "fbardl_mcnown", "fbardl_bvz"),
  maxlag = 4,
  maxk = 5,
  ic = c("aic", "bic"),
  case = 3,
  reps = 999,
  fourier = TRUE,
  level = 0.95,
  horizon = 20
)
```

Arguments

formula	A formula of the form $y \sim x_1 + x_2 + \dots$ specifying the dependent and independent variables.
data	A data frame containing the time series variables.
type	Character string specifying the test type: <ul style="list-style-type: none"> "fardl": Fourier ARDL with PSS bounds test (default) "fbardl_mcnown": Bootstrap ARDL (McNown, Sam & Goh, 2018) "fbardl_bvz": Bootstrap ARDL (Bertelli, Vacca & Zoia, 2022)
maxlag	Integer. Maximum lag order for grid search (default: 4).
maxk	Numeric. Maximum Fourier frequency (default: 5).
ic	Character string. Information criterion for lag selection: "aic" (default) or "bic".

case	Integer. PSS case specification (2, 3, 4, or 5). Default is 3 (unrestricted intercept, no trend).
reps	Integer. Number of bootstrap replications (default: 999).
fourier	Logical. Whether to include Fourier terms (default: TRUE).
level	Numeric. Confidence level for intervals (default: 0.95).
horizon	Integer. Horizon for dynamic multipliers (default: 20).

Details

The FBARDL approach extends the standard ARDL bounds testing procedure by:

1. Incorporating Fourier terms to capture smooth structural breaks
2. Using bootstrap methods to generate finite-sample critical values
3. Implementing the McNown et al. (2018) procedure to detect degenerate cases

The procedure involves three main steps:

1. Selection of optimal Fourier frequency k^* by minimum SSR
2. Selection of lag orders (p, q) by AIC or BIC
3. Cointegration testing with bootstrap or PSS critical values

Three test statistics are computed:

- **F.overall**: Joint test on all lagged level variables
- **t.dependent**: t-test on lagged dependent variable
- **F.independent**: Joint test on lagged independent variables

Value

An object of class "fbardl" containing:

coefficients Named vector of estimated coefficients

std.errors Standard errors of coefficients

t.values t-statistics

p.values p-values

long.run Long-run coefficient estimates with standard errors

short.run Short-run coefficient estimates

ecm.coef Error correction coefficient (speed of adjustment)

best.p Selected lag order for dependent variable

best.q Selected lag orders for independent variables

best.kstar Selected Fourier frequency

F.overall F-statistic for overall cointegration test

t.dependent t-statistic on lagged dependent variable

F.independent F-statistic on lagged independent variables

cointegration Cointegration test results with critical values

diagnostics Diagnostic test results

model.fit Model fit statistics (R2, AIC, BIC, etc.)

residuals Model residuals

fitted.values Fitted values

nobs Number of observations

call The matched call

References

Pesaran, M. H., Shin, Y., & Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics*, 16(3), 289-326. doi:10.1002/jae.616

McNown, R., Sam, C. Y., & Goh, S. K. (2018). Bootstrapping the autoregressive distributed lag test for cointegration. *Applied Economics*, 50(13), 1509-1521. doi:10.1080/00036846.2017.1366643

Yilanci, V., Bozoklu, S., & Gorus, M. S. (2020). Are BRICS countries pollution havens? Evidence from a bootstrap ARDL bounds testing approach with a Fourier function. *Sustainable Cities and Society*, 55, 102035. doi:10.1016/j.scs.2020.102035

Kripfganz, S., & Schneider, D. C. (2020). Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models. *Oxford Bulletin of Economics and Statistics*, 82(6), 1456-1481. doi:10.1111/obes.12377

Examples

```
# Load example data
data(fbardl_data)

# Basic Fourier ARDL test
result <- fbardl(y ~ x1 + x2, data = fbardl_data, type = "fardl")
summary(result)

# Bootstrap ARDL (McNown approach)
result_boot <- fbardl(y ~ x1 + x2, data = fbardl_data,
                    type = "fbardl_mcnown", reps = 499)
summary(result_boot)

# Without Fourier terms
result_nofourier <- fbardl(y ~ x1 + x2, data = fbardl_data,
                          fourier = FALSE)
```

`fbardl_data`*Example Data for Fourier Bootstrap ARDL Analysis*

Description

A simulated time series dataset suitable for demonstrating the Fourier Bootstrap ARDL cointegration testing procedure. The data contains a dependent variable y and two independent variables x_1 and x_2 with a cointegrating relationship and structural breaks.

Usage`fbardl_data`**Format**

A data frame with 150 observations and 3 variables:

y Dependent variable (simulated I(1) series)

x1 First independent variable (simulated I(1) series)

x2 Second independent variable (simulated I(1) series)

Details

The data is generated from a data-generating process (DGP) that includes:

- A long-run cointegrating relationship: $y_t = 2 + 0.8x_{1t} - 0.5x_{2t} + u_t$
- Short-run dynamics with AR(1) errors
- A structural break modeled by Fourier terms
- Error correction mechanism with adjustment speed of -0.3

This dataset is designed to produce clear cointegration test results when analyzed with the [fbardl](#) function.

Source

Simulated data for package demonstration.

See Also

[fbardl](#)

Examples

```
data(fbardl_data)
head(fbardl_data)
summary(fbardl_data)

# Plot the series
ts.plot(ts(fbardl_data), col = 1:3, lty = 1:3)
legend("topleft", colnames(fbardl_data), col = 1:3, lty = 1:3)
```

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* **datasets**

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