

Package ‘frontiles’

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Type Package

Title Partial Frontier Efficiency Analysis

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Description It calculates the alpha-quantile proposed by Daouia and Simar (2007) <[doi:10.1016/j.jeconom.2006.07.002](https://doi.org/10.1016/j.jeconom.2006.07.002)> and order-m efficiency score in multi-dimension proposed by Daouia and Gijbels (2011) <[doi:10.1016/j.jeconom.2010.12.002](https://doi.org/10.1016/j.jeconom.2010.12.002)> and computes several summaries and representation of the associated frontiers in 2d and 3d.

License GPL (>= 2)

LazyLoad yes

Collate alphafrontier.2d.r alphafrontier.3d.r alphascore.r
orderscore.r orderscore.boot.r ordermfrontier.2d.r ROCscore.r

NeedsCompilation yes

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frontiles-package	<i>alpha-quantile efficiency score and 2d/3d representation of frontier</i>
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Description

It calculates the alpha-quantile efficiency score and order-m score in multi-dimension and gives representation of alpha-quantile efficiency frontier. This work was supported by the agence nationale de la recherche through the EPI project (ANR-08-BLAN-0106-01)

Details

Package:	frontiles
Type:	Package
Version:	1.2
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License:	GPL Vesion 2 or later
LazyLoad:	yes

Author(s)

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References

Daouia, A. and L. Simar (2007), Nonparametric efficiency analysis: A multivariate conditional quantile approach, *Journal of Econometrics* 140, 375-400.

Examples

```
data(spain)
xobs=as.matrix(spain[,3:4])
yobs=as.matrix(spain[,1])
score.ref.1<-alphascore(xobs,yobs)
```

alphafrontier.2d	<i>Representation of alpha-quantile efficiency frontier for 1 output and 1 input</i>
------------------	--

Description

Representation of the alpha-quantile efficiency frontier (output, input or hyperbolic direction) for a set of reference points (xobs, yobs) in 2D (1 output and 1 input).

Usage

```
alphafrontier.2d(xobs, yobs, type="output", alpha=0.95, add=FALSE,
confidence=FALSE, shade=FALSE,...)
```

Arguments

xobs	a matrix of size $n_1 \times 1$, input of sample points
yobs	a matrix of size $n_1 \times 1$, output of sample points
type	a direction to choose among "output", "input" and "hyper"
alpha	a scalar between 0 and 1
add	a boolean with TRUE for keeping the active device
confidence	a boolean for representing a confidence interval
shade	a boolean for shading the confidence interval
...	usual options for plotting the frontier, lty, col, etc.

Details

Actually, there is no confidence interval when type="hyper". If type="input" and confidence=TRUE, the y-axis is permuted with the x-axis

Value

No return value, used for plotting the alpha-quantile efficiency frontier for 1 output and 1 input

Author(s)

Abdelaati Daouia and Thibault Laurent

References

Daouia, A. and L. Simar (2007), Nonparametric efficiency analysis: A multivariate conditional quantile approach, *Journal of Econometrics* 140, 375-400.

See Also

[ordermfrontier.2d](#), [alphascore](#)

Examples

```
# 1st example
data(spain)
plot(y ~ x2, data = spain)
alphafrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "output",
  alpha = 0.95, col = 'red', lty = 2, add = TRUE)
alphafrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "input",
  alpha = 0.95, col = 'royalblue', lty = 3, add = TRUE)
alphafrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "hyper",
  alpha = 0.95, col = 'green', lty = 4, add = TRUE)
legend("topleft", title = "alpha-quantile frontier; alpha=0.95",
  legend = c("output direction", "input direction", "hyper direction"),
  lty = 2:4, col = c("red", "royalblue", "green"))

# 2nd example
plot(y ~ x2, data = spain)
alphafrontier.2d(as.matrix(spain$x2), as.matrix(spain$y),
  type = "output", alpha = 1, add = TRUE)
alphafrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "output",
  alpha = 0.95, col = 'blue', lty = 2, add = TRUE)
ordermfrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "output",
  m = 30, col = 'green', lty = 3, add = TRUE)
legend("topleft", title = "output direction", legend = c("FDH", "alpha=0.95", "m=30"),
  lty = 1:3, col = c("black", "royalblue", "green"))

# 3rd example
alphafrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "output",
  confidence = TRUE, shade = TRUE, alpha = 0.98)
title("Alpha-quantile frontier with alpha=0.98 and its confidence interval")
```

alphafrontier.3d	<i>3d representation of alpha-quantile frontier in the case of 2 input and 1 output.</i>
------------------	--

Description

3d representation of alpha-quantile frontier for a set of reference points (xobs,yobs) in the case of two input and one output. No representation yet for hyperbolic direction.

Usage

```
alphafrontier.3d(xobs, yobs, type="output",alpha=0.95, digits=4,
  box.leg=TRUE, palette=heat_hcl, rgl=FALSE, n.class=NULL, ...)
```

Arguments

xobs	a matrix of size $n_1 \times 2$, input of reference points
yobs	a matrix of size $n_1 \times 1$, output of reference points

type	a character, "output" or "input" direction
alpha	a scalar
digits	a precision parameter to compute the alpha-frontier
box.leg	representation of a legend-box on the plot with the values of frontier
palette	function to use for colors in case where option col has not been called. See package colorspace for more informations.
rgl	a boolean, for output direction, representation of the graphic in 3d if TRUE
n.class	a numeric, for output direction, the number of class
...	usual parameters of function plot. Use the parameter asp to modify the scale of window...

Details

In "input" direction: You choose a value of output in the legend box, the efficiency-frontier of the input is represented with the corresponding color on the 2d graphic. In "output" direction : For calculate the alpha-quantile efficiency output frontier everywhere, we have constructed a grid of size $n \times n$ by drawing vertical and horizontal lines which intersect each reference observations. Then, we have calculated for each cell C_k $k = 1, \dots, n^2$ the prediction of the alpha-quantile output efficiency frontier which correspond to the α^{th} elements of the suite $\{y_j\}_{j=1, \dots, n_k}$ where reference observations j , $j = 1, \dots, n_k$ verify

$$x_j^1 \leq \inf_{(x^1, x^2) \in C_k} x^1$$

and $x^2 \leq \inf_{(x^1, x^2) \in C_k} x^2$.

We propose to use an algorithm which filled up cells with colors depending on the values taken by the alpha-quantile output efficiency frontier. The algorithm attach the row and vary the column as the folling figure can show it.

Value

no values

Note

The algorithm used is certainly not optimized. For a data set of 61 observations, the function necessits 15.17s on an Optiplex GX745 2 duo 2.13GHz under Windows Vista and probably bugs beyond a certain number of observation

Author(s)

Abdelaati Daouia and Thibault Laurent

See Also

[alphascore](#)

Examples

```

data(spain)
xyn <- cbind(spain[, 3:4],spain[, 1])
xtab <- as.matrix(xyn[, c(1,2)])
ytab <- matrix(xyn[, 3])

# representation in 2-d

op <- par(no.readonly = TRUE) # the whole list of settable par's.
alphafrontier.3d(xtab, ytab, type = "output", alpha = 0.6, xlab = "input 1",
  ylab = "input 2", main = "blabla")
points(xtab, pch = 16)
par(op)

```

alphascore

Calculates alpha-quantile efficiency score

Description

Calculates alpha-quantile efficiency score (output, input and hyperbolic direction) for a set of evaluation points (xeval, yeval) depending on reference points (xobs, yobs).

Usage

```
alphascore(xobs, yobs, xeval=xobs, yeval=yobs, alpha=0.95)
```

Arguments

xobs	a matrix of size $n_1 \times p$, input of sample points
yobs	a matrix of size $n_1 \times q$, output of sample points
xeval	a matrix of size $n_2 \times p$, input of assessment points
yeval	a matrix of size $n_2 \times q$, output of assessment points
alpha	a scalar

Details

A score between 0 and 1 means that DMU is inefficient. If DMU greater than 1, DMU is super-efficient.

Value

a data.frame object with the alpha-quantile efficiency score in:

output	output direction
input	input direction
hyper	hyperbolic direction

Author(s)

Abdelaati Daouia and Thibault Laurent

References

Daouia, A. and L. Simar (2007), Nonparametric efficiency analysis: A multivariate conditional quantile approach, *Journal of Econometrics* 140, 375-400.

See Also

[alphafrontier.2d](#), [ordermscore](#)

Examples

```
# 1st example
data(spain)
res.alqf <- alphascore(xobs = as.matrix(spain[, c(2, 3, 4)]),
                     yobs = as.matrix(spain[, 1]), alpha = 0.8)

# 2nd example
data(burposte)
bur.samp <- burposte[which(burposte$input < 50000), ]
ind.samp <- sample(nrow(bur.samp), 500)
xeval <- as.matrix(bur.samp[ind.samp[1:100], 2])
yeval <- as.matrix(bur.samp[ind.samp[1:100], 3])
xobs <- as.matrix(bur.samp[ind.samp[101:500], 2])
yobs <- as.matrix(bur.samp[ind.samp[101:500], 3])

alphafrontier.2d(xobs, yobs, alpha = 0.95)
points(xeval, yeval, pch = 16, col = 'red')
text(xeval, yeval, as.character(1:100), adj = 2, cex = 0.8)
score.new.0.95 <- alphascore(xobs, yobs, xeval, yeval, alpha = 0.95)
```

burposte

Burposte data

Description

Burposte data

Usage

```
data(burposte)
```

Format

A data frame with 9521 observations on the following 3 variables.

ident a numeric vector

xinput a numeric vector

yprod a numeric vector

References

Cazals C., Florens J.-P., Simar L. (2002), Nonparametric frontier estimation: a robust approach, *Journal of Econometrics* 106, 1-25.

Examples

```
data(burposte)
```

ordermfrontier.2d	<i>Representation of m-order efficiency frontier for 1 output and 1 input</i>
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Description

Representation of the m-order efficiency score (output, input or hyperbolic direction) for a set of reference points (xobs, yobs) in 2D (1 output and 1 input).

Usage

```
ordermfrontier.2d(xobs, yobs, type = "output", m = 30, add = FALSE,
  confidence = FALSE, shade = FALSE,...)
```

Arguments

xobs	a matrix of size $n_1 \times 1$, input of sample points
yobs	a matrix of size $n_1 \times 1$, output of sample points
type	a direction to choose among "output", "input" and "hyper"
m	an integer
add	a boolean with TRUE for keeping the active device
confidence	a boolean for representing a confidence interval
shade	a boolean for shading the confidence interval
...	usual options for plotting the frontier, lty, col, etc.

Details

Actually, there is no confidence interval when type="hyper". If type="input" and confidence=TRUE, the y-axis is permuted with the x-axis

Value

No return value, used for plotting the m-order efficiency frontier for 1 output and 1 input

Author(s)

Abdelaati Daouia and Thibault Laurent

References

Daouia, A. and L. Simar (2007), Nonparametric efficiency analysis: A multivariate conditional quantile approach, *Journal of Econometrics* 140, 375-400.

Examples

```
# 1st example
data(spain)
plot(y ~ x2, data = spain)
ordermfrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "output",
  m = 30, col = 'red', lty = 2, add = TRUE)
ordermfrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "input", m = 30,
  col = 'royalblue', lty = 3, add = TRUE)
ordermfrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "hyper", m = 30,
  col = 'green', lty = 4, add = TRUE)
legend("topleft", legend = c("output direction", "input direction", "hyper direction"),
  lty = 2:4, col = c("red", "royalblue", "green"))

# 2nd example
ordermfrontier.2d(as.matrix(spain$x2), as.matrix(spain$y), type = "output",
  confidence = TRUE, shade = TRUE, m = 30)
```

ordermscore

Calculates order-m efficiency score with asymptotic formula

Description

Calculates order-m efficiency score (output, input and hyperbolic direction) for a set of assessment points (xeval, yeval) depending on sample points (xobs, yobs), using the formulas of Daouia and Gijbels (2011).

Usage

```
ordermscore(xobs, yobs, xeval=xobs, yeval=yobs, m=30)
```

Arguments

xobs	a matrix of size $n_1 \times p$, input of sample points
yobs	a matrix of size $n_1 \times q$, output of sample points
xeval	a matrix of size $n_2 \times p$, input of assessment points
yeval	a matrix of size $n_2 \times q$, output of assessment points
m	an integer

Details

A score between 0 and 1 means that DMU is inefficient. If DMU greater than 1, DMU is super-efficient. The asymptotic formula of the order-m score are given in Daouia and Gijbels (2011).

Value

a data.frame object with the order-m efficiency score in:

output	output direction
input	input direction
hyper	hyperbolic direction

Author(s)

Abdelaati Daouia and Thibault Laurent

References

Daouia and Gijbels (2011), Robustness and inference in nonparametric partial-frontier modeling, *Journal of Econometrics*.

See Also

[alphascore](#), [ordermfrontier.2d](#)

Examples

```
# 1st example
data(spain)
score.orderm<-ordermscore(xobs = as.matrix(spain[, c(2, 3, 4)]),
                           yobs = as.matrix(spain[, 1]))

# 2nd example
data(burposte)
ind.samp <- sample(nrow(burposte), 500)
xobs <- as.matrix(burposte[ind.samp[1:100], 2])
yobs <- as.matrix(burposte[ind.samp[1:100], 3])
xeval <- as.matrix(burposte[ind.samp[101:500],2])
yeval <- as.matrix(burposte[ind.samp[101:500], 3])
score.orderm.2 <- ordermscore(xobs, yobs, xeval, yeval)
```

ordermscore.boot	<i>Calculates order-m efficiency score with bootstrap algorithm</i>
------------------	---

Description

Calculates order-m efficiency score (output, input and hyperbolic direction) for a set of assessment points (xeval, yeval) depending on sample points (xobs, yobs), using the initial algorithm of Cazals et al. (2002).

Usage

```
ordermscore.boot(xobs, yobs, xeval=xobs, yeval=yobs, m=30, B=200, m.move=FALSE)
```

Arguments

xobs	a matrix of size $n_1 \times p$, input of sample points
yobs	a matrix of size $n_1 \times q$, output of sample points
xeval	a matrix of size $n_2 \times p$, input of assessment points
yeval	a matrix of size $n_2 \times q$, output of assessment points
m	an integer, the number of selected firms
B	an integer, the number of replication
m.move	a boolean, to choose different values of m

Details

This function computes the algorithm initially proposed by Cazals et al. (2002). If m.move=TRUE, different values of m are given as suggested by Daouia et al (2009).

Value

a data.frame object with the average mean order-m efficiency score and standard deviation associated:

output	output direction
output	output direction
input	input direction
input	input direction
hyper	hyperbolic direction
hyper	hyperbolic direction

Author(s)

Abdelaati Daouia and Thibault Laurent

References

Cazals et al. (2002), Nonparametric frontier estimation: a robust approach, *Journal of Econometrics*.

Daouia et al. (2009), Regularization of Nonparametric Frontier Estimators, *TSE working paper*.

See Also

[ordermscore](#), [alphascore](#)

Examples

```
# 1st example
data(spain)
score.orderm.b <- ordermscore.boot(xobs = as.matrix(spain[, c(2, 3, 4)]),
                                   yobs = as.matrix(spain[, 1]))

system.time(
  ordermscore.boot(xobs = as.matrix(spain[, c(2, 3, 4)]),
                  yobs = as.matrix(spain[, 1]))
)
system.time(
  ordermscore(xobs = as.matrix(spain[, c(2, 3, 4)]),
              yobs = as.matrix(spain[, 1]))
)

# 2nd example

data(burposte)
ind.samp <- sample(nrow(burposte), 500)
xobs <- as.matrix(burposte[ind.samp[1:100], 2])
yobs <- as.matrix(burposte[ind.samp[1:100], 3])
xeval <- as.matrix(burposte[ind.samp[101:500], 2])
yeval <- as.matrix(burposte[ind.samp[101:500], 3])
score.orderm.2.b <- ordermscore.boot(xobs, yobs, xeval, yeval)
```

ROCscore

The ROC curve to help choosing alpha and m parameters

Description

Computes the percentage of firms super-efficient according to the parameter alpha for alpha-quantile score and m for m-order score in a given direction.

Usage

```
ROCscore(xobs, yobs, type="output")
```

Arguments

xobs	a matrix of size $n_1 \times p$, input of sample points
yobs	a matrix of size $n_1 \times q$, output of sample points
type	a direction to choose among "output", "input" and "hyper"

Details

A firm is super-efficient if its score is greater than 1. By consulting this graph, we may choose the values of alpha and m which correspond to the desired degree of robustness, i.e. the percentage of high performers of the population we want to exclude in our more realistic benchmarking comparison (see p.78 of Daraio and Simar, 2010).

Value

a data.frame object with:

alpha	different values of alpha
f(alpha)	the percentage of firms super-efficient
m	different values of m
f(m)	the percentage of firms super-efficient

Author(s)

Abdelaati Daouia and Thibault Laurent

References

Daraio and Simar (2007), *Advanced Robust and Nonparametric Methods in Efficiency Analysis*, Springer.

Examples

```
# 1st example
data(spain)
res.roc <- ROCscore(xobs = as.matrix(spain[,c(2,3,4)]),
                   yobs = as.matrix(spain[,1]),
                   type = "output")
```

spain

Spain data

Description

Spain data

Usage

```
data(spain)
```

Format

A data frame with 61 observations on the following 4 variables.

y a numeric vector of output

x1 a numeric vector of input

x2 a numeric vector of input

x3 a numeric vector of input

Examples

```
data(spain)
```

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