

# Package ‘geess’

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**Type** Package

**Title** Modified Generalized Estimating Equations for Small-Sample Data

**Version** 1.0.1

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**Description** Analyze small-sample clustered or longitudinal data using modified generalized estimating equations with bias-adjusted covariance estimator. The package provides any combination of three modified generalized estimating equations and 11 bias-adjusted covariance estimators.

**Depends** R (>= 4.5.0)

**Imports** MASS (>= 7.3-45)

**License** GPL (>= 2)

**Encoding** UTF-8

**RoxygenNote** 7.3.2

**Suggests** testthat (>= 3.0.0)

**Config/testthat/edition** 3

**NeedsCompilation** no

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**Repository** CRAN

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**Description**

geess is an extension of geessbin package (Ishii et al., 2024). geess analyzes small-sample clustered or longitudinal data using modified generalized estimating equations (GEE) with bias-adjusted covariance estimator. This function provides any combination of three GEE methods (conventional and two modified GEE methods) and 12 covariance estimators (unadjusted and 11 bias-adjusted estimators).

**Usage**

```
geess(
  formula,
  family = gaussian,
  data = parent.frame(),
  id = NULL,
  corstr = "independence",
  repeated = NULL,
  beta.method = "GEE",
  SE.method = "SA",
  b = NULL,
  maxitr = 50,
  tol = 1e-05,
  scale.fix = FALSE,
  conf.level = 0.95
)
```

**Arguments**

formula	Object of class formula: symbolic description of model to be fitted (see documentation of <code>lm</code> and <code>formula</code> for details).
family	Description of the error distribution and link function to be used in the model. The gaussian family accepts the identity link function. The binomial family accepts the links <code>logit</code> and <code>probit</code> . The poisson family accepts the log link function.
data	Data frame.
id	Vector that identifies the subjects or clusters (NULL by default).
corstr	Working correlation structure. The following are permitted: "independence", "exchangeable", "ar1", and "unstructured" ("independence" by default).
repeated	Vector that identifies repeatedly measured variable within each subject or cluster. If <code>repeated = NULL</code> , as is the case in function <code>gee</code> , data are assumed to be sorted so that observations on a cluster are contiguous rows for all entities in the formula.

<code>beta.method</code>	Method for estimating regression parameters (see Details section). The following are permitted: "GEE", "PGEE", and "BCGEE" ("GEE" by default).
<code>SE.method</code>	Method for estimating standard errors (see Details section). The following are permitted: "SA", "MK", "KC", "MD", "FG", "PA", "GS", "MB", "WL", "WB", "FW", and "FZ" ("SA" by default).
<code>b</code>	Numeric vector specifying initial values of regression coefficients. If <code>b = NULL</code> (default value), the initial values are calculated using the ordinary or Firth logistic regression assuming that all the observations are independent.
<code>maxitr</code>	Maximum number of iterations (50 by default).
<code>tol</code>	Tolerance used in fitting algorithm (1e-5 by default).
<code>scale.fix</code>	Logical variable; if TRUE, the scale parameter is fixed at 1 (FALSE by default).
<code>conf.level</code>	Numeric value of confidence level for confidence intervals (0.95 by default).

### Details

Details of `beta.method` are as follows:

- "GEE" is the conventional GEE method (Liang and Zeger, 1986)
- "BCGEE" is the bias-corrected GEE method (Paul and Zhang, 2014; Lunardon and Scharfstein, 2017)
- "PGEE" is the bias reduction of the GEE method obtained by adding a Firth-type penalty term to the estimating equation (Mondol and Rahman, 2019)

Details of `SE.method` are as follows:

- "SA" is the unadjusted sandwich variance estimator (Liang and Zeger, 1986)
- "MK" is the MacKinnon and White estimator (MacKinnon and White, 1985)
- "KC" is the Kauermann and Carroll estimator (Kauermann and Carroll, 2001)
- "MD" is the Mancl and DeRouen estimator (Mancl and DeRouen, 2001)
- "FG" is the Fay and Graubard estimator (Fay and Graubard, 2001)
- "PA" is the Pan estimator (Pan, 2001)
- "GS" is the Gosho et al. estimator (Gosho et al., 2014)
- "MB" is the Morel et al. estimator (Morel et al., 2003)
- "WL" is the Wang and Long estimator (Wang and Long, 2011)
- "WB" is the Westgate and Burchett estimator (Westgate and Burchett, 2016)
- "FW" is the Ford and Wastgate estimator (Ford and Wastgate, 2017)
- "FZ" is the Fan et al. estimator (Fan et al., 2013)

Descriptions and performances of some of the above methods in the case of binary outcomes can be found in Gosho et al. (2023).

### Value

The object of class "geess" representing the results of modified generalized estimating equations with bias-adjusted covariance estimators. Generic function `summary` provides details of the results.

## References

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## Examples

```
library(geess)
library(MASS)

# analysis of longitudinal count data using BCg GEE method with Morel et al. covariance estimator
res <- geess(formula = y ~ trt + period + lbase + lage,
             family = poisson, data = epil, id = subject,
             repeated = period, corstr = "unstructured",
             beta.method = "BCGEE", SE.method = "MB")

print(res)

# hypothesis tests for regression coefficients
summary(res)
```

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sqrtmat

*Square root of nonsymmetric matrix*

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## Description

sqrtmat is used to calculate the square root of  $E_i - H_{ii}$ , which is an adjustment factor in Kauermann and Carroll-type method.

## Usage

```
sqrtmat(M)
```

**Arguments**

M                      Square matrix whose square root is to be computed.

**Value**

The square root of M

**References**

Kauermann, G. and Carroll, R. J. (2001). A note on the efficiency of sandwich covariance matrix estimation. *Journal of the American Statistical Association*, 96, 1387–1396, [doi:10.1198/016214501753382309](https://doi.org/10.1198/016214501753382309).

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