

Package ‘hda’

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Title Heteroscedastic Discriminant Analysis

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Suggests mvtnorm, klaR, MASS

Imports e1071

Description Functions to perform dimensionality reduction for classification if the covariance matrices of the classes are unequal.

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hda	<i>Heteroscedastic discriminant analysis</i>
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Description

Computes a linear transformation loadings matrix for discrimination of classes with unequal covariance matrices.

Usage

```

hda(x, ...)
## Default S3 method:
hda(x, grouping, newdim = 1:(ncol(x)-1), crule = FALSE,
     reg.lamb = NULL, reg.gamm = NULL, initial.loadings = NULL,
     sig.levs = c(0.05,0.05), noutit = 7, ninit = 10, verbose = TRUE, ...)
## S3 method for class 'formula'
hda(formula, data = NULL, ...)

```

Arguments

<code>x</code>	A matrix or data frame containing the explanatory variables. The method is restricted to numerical data.
<code>grouping</code>	A factor specifying the class for each observation.
<code>formula</code>	A formula of the form <code>grouping ~ x1 + x2 + ...</code> . That is, the response is the grouping factor and the right hand side specifies the (non-factor) discriminators.
<code>data</code>	Data frame from which variables specified in formula are to be taken.
<code>newdim</code>	Dimension of the discriminative subspace. The class distributions are assumed to be equal in the remaining dimensions. Alternatively, a vector of integers can be specified which is then computed until for the first time both tests on equal means as well as homoscedasticity do not reject. This option is to be applied with care and the resulting dimension should be checked manually.
<code>crule</code>	Logical specifying whether a naiveBayes classification rule should be computed. Requires package <code>e1071</code> .
<code>reg.lamb</code>	Parameter in $[0,1]$ for regularization towards equal covariance matrix estimations of the classes (in the original space): 0 means equal covariances, 1 (default) means complete heteroscedasticity.
<code>reg.gamm</code>	Similar to <code>reg.lambd</code> : parameter for shrinkage towards diagonal covariance matrices of equal variance in all variables where 0 means diagonality. Default is no shrinkage.
<code>initial.loadings</code>	Initial guess of the matrix of loadings. Must be quadratic of size <code>ncol(x)</code> . Default is the identity matrix. By specification of <code>initial.loadings = "random"</code> a random orthonormal matrix will be generated using <code>qr.Q(qr())</code> of a random matrix with uniformly distributed elements.
<code>sig.levs</code>	Vector of significance levels for <code>eqmean.test</code> (position 1) and <code>homog.test</code> (pos. 2) to stop search for an appropriate dimension of the reduced space.
<code>noutit</code>	Number iterations of the outer loop, i.e. iterations of the likelihood. Default is 7.
<code>ninit</code>	Number of iterations of the inner loop, i.e. reiterations of the loadings matrix within one iteration step of the likelihood.
<code>verbose</code>	Logical indicating whether iteration process should be displayed.
<code>...</code>	For <code>hda.formula</code> : Further arguments passed to function <code>hda.default</code> such as <code>newdim</code> . For <code>hda.default</code> : currently not used.

Details

The function returns the transformation that maximizes the likelihood if the classes are normally distributed but differ only in a `newdim` dimensional subspace and have equal distributions in the remaining dimensions (see Kumar and Andreou, 1998). The scores are uncorrelated for all classes. The algorithm is implemented as it is proposed by Burget (2006). Regularization is computed as proposed by Friedman et al. (1989) and Szepannek et al. (2009).

Value

Returns an object of class `hda`.

<code>hda.loadings</code>	Transformation matrix to be post-multiplied to new data.
<code>hda.scores</code>	Input data after hda transformation. Reduced discriminative space are the first <code>newdim</code> dimensions.
<code>grouping</code>	Corresponding class labels for <code>hda.scores</code> data. Identical to input grouping.
<code>class.dist</code>	Estimated class means and covariance matrices in the transformed space.
<code>reduced.dimension</code>	Input parameter: dimension of the reduced space.
<code>naivebayes</code>	Object of class <code>naiveBayes</code> trained on input data in the reduced space for classification of new (transformed) data. Its computation must be specified by input the parameter <code>crule</code> .
<code>comp.acc</code>	Matrix of accuracies per component and class: reports up to which degree each class k can be classified ($P(f_k > f_{\neq k})$) correctly according to the estimated (normal) distribution in any single component in the identified subspace. Meaningful for reasons of interpretability as HDA is invariant to reordering of the components.
<code>vlift</code>	Returns the variable importance in terms of ratio between the accuracy <code>comp.acc</code> and the resulting accuracy that results if single variable loadings are set to 0. The first element describes overall accuracy lift where the second element is an array of dimension (number of classes, number of components in reduced space, number of variables) specifying the lifts for recognition each class separately.
<code>reg.lambd</code>	Input regularization parameter.
<code>reg.gamm</code>	Input regularization parameter.
<code>eqmean.test</code>	Test on equal means of the classes in the remaining dimensions like in manova based on Wilk's lambda.
<code>homog.test</code>	Test on homoscedasticity of the classes in the remaining dimensions (see e.g. Fahrmeir et al., 1984, p.75.)
<code>hda.call</code>	(Matched) function call.
<code>initial.loadings</code>	Initialization of the loadings matrix.
<code>trace.dimensions</code>	Matrix of p values for different subspace dimensions (as specified in <code>newdim</code>).

Author(s)

Gero Szepannek

References

- Burget, L. (2006): *Combination of speech features using smoothed heteroscedastic discriminant analysis*. Proceedings of Interspeech 2004, pp. 2549-2552.
- Fahrmeir, L. and Hamerle, A. (1984): *Multivariate statistische Verfahren*. de Gruyter, Berlin.
- Friedman, J. (1989): *Regularized discriminant analysis*. JASA 84, 165-175.
- Kumar, N. and Andreou, A. (1998): *Heteroscedastic discriminant analysis and reduced rank HMMs for improved speech recognition*. Speech Communication 25, pp.283-297.
- Szepannek G., Harczos, T., Klefenz, F. and Weihs, C. (2009): *Extending features for automatic speech recognition by means of auditory modelling*. In: Proceedings of European Signal Processing Conference (EUSIPCO) 2009, Glasgow, pp.1235-1239.

See Also

[predict.hda](#), [showloadings](#), [plot.hda](#)

Examples

```
library(mvtnorm)
library(MASS)

# simulate data for two classes
n          <- 50
meana      <- meanb <- c(0,0,0,0,0)
cova       <- diag(5)
cova[1,1]  <- 0.2
for(i in 3:4){
  for(j in (i+1):5){
    cova[i,j] <- cova[j,i] <- 0.75^(j-i)}
  }
covb       <- cova
diag(covb)[1:2] <- c(1,0.2)

xa         <- rmvnorm(n, meana, cova)
xb         <- rmvnorm(n, meanb, covb)
x          <- rbind(xa, xb)
classes    <- as.factor(c(rep(1,n), rep(2,n)))
# rotate simulated data
symmat    <- matrix(runif(5^2),5)
symmat    <- symmat + t(symmat)
even      <- eigen(symmat)$vectors
rotatedspace <- x %*% even
plot(as.data.frame(rotatedspace), col = classes)

# apply linear discriminant analysis and plot data on (single) discriminant axis
lda.res   <- lda(rotatedspace, classes)
plot(rotatedspace %*% lda.res$scaling, col = classes,
      ylab = "discriminant axis", xlab = "Observation index")

# apply heteroscedastic discriminant analysis and plot data in discriminant space
hda.res   <- hda(rotatedspace, classes)
```

```

plot(hda.res$hda.scores, col = classes)

# compare with principal component analysis
pca.res <- prcomp(as.data.frame(rotatedspace), retx = TRUE)
plot(as.data.frame(pca.res$x), col=classes)

# Automatically build classification rule
# this requires package e1071
hda.res2 <- hda(rotatedspace, classes, crule = TRUE)

```

plot.hda

Plot transformed data

Description

Visualizes the scores on selected components of the discriminant space of reduced dimension.

Usage

```

## S3 method for class 'hda'
plot(x, comps = 1:x$reduced.dimension, scores = TRUE, col = x$grouping, ...)

```

Arguments

x	An object of class hda.
comps	A vector of component ids for which the data should be displayed.
scores	Logical indicating whether the scores in the projected space should be plotted. If FALSE estimated densities are plotted.
col	Color vector for the data to be displayed. Per default, different colors represent the classes.
...	Further arguments to be passed to the plot function.

Details

Scatterplots of the scores or estimated densities.

Value

No value is returned.

Author(s)

Gero Szepannek

References

Kumar, N. and Andreou, A. (1998): *Heteroscedastic discriminant analysis and reduced rank HMMs for improved speech recognition*. *Speech Communication* 25, pp.283-297.

Szepannek G., Harczos, T., Klefenz, F. and Weihs, C. (2009): *Extending features for automatic speech recognition by means of auditory modelling*. In: *Proceedings of European Signal Processing Conference (EUSIPCO) 2009, Glasgow*, pp.1235-1239.

See Also

[hda](#), [predict.hda](#), [showloadings](#)

Examples

```
library("mvtnorm")
library("MASS")

# simulate data for two classes
n      <- 50
meana  <- meanb <- c(0,0,0,0,0)
cova   <- diag(5)
cova[1,1] <- 0.2
for(i in 3:4){
  for(j in (i+1):5){
    cova[i,j] <- cova[j,i] <- 0.75^(j-i)}
  }
covb   <- cova
diag(covb)[1:2] <- c(1,0.2)

xa     <- rmvnorm(n, meana, cova)
xb     <- rmvnorm(n, meanb, covb)
x      <- rbind(xa,xb)
classes <- as.factor(c(rep(1,n), rep(2,n)))
## rotate simulated data
symmat <- matrix(runif(5^2),5)
symmat <- symmat + t(symmat)
even   <- eigen(symmat)$vectors
rotatedspace <- x %*% even
plot(as.data.frame(rotatedspace), col = classes)

# apply heteroscedastic discriminant analysis and plot data in discriminant space
hda.res <- hda(rotatedspace, classes)

# plot scores
plot(hda.res)
```

Description

Computes linear transformation of new data into lower dimensional discriminative space using some model produced by [hda](#).

Usage

```
## S3 method for class 'hda'  
predict(object, newdata, alldims = FALSE, task = c("dr", "c"), ...)
```

Arguments

object	Model resulting from a call of hda .
newdata	A matrix or data frame to be transformed into lower dimensional space of the same dimension as the data used for building the model.
alldims	Logical flag specifying whether the result should contain only the reduced space (default) or should also include the redundant dimensions and thus be of the same dimension as the input data. In this case the reduced space is given by the first <code>newdim</code> columns.
task	"dr" for standard application of the hda model to <code>newdata</code> . Choose "c" for classification of new data. This is an interface to predict function of naiveBayes . The option can be chosen if <code>crule = TRUE</code> has been specified in the <code>hda()</code> call.
...	Further arguments to be passed to the naiveBayes predict function.

Value

If option `type = "dr"` the transformed data are returned. For `type = "c"` both the transformed data as well as the resulting object of the naive Bayes prediction are returned.

Author(s)

Gero Szepannek

References

Kumar, N. and Andreou, A. (1998): *Heteroscedastic discriminant analysis and reduced rank HMMs for improved speech recognition*. *Speech Communication* 25, pp. 283-297.

Szepannek G., Harczos, T., Klefenz, F. and Weihs, C. (2009): *Extending features for automatic speech recognition by means of auditory modelling*. In: *Proceedings of European Signal Processing Conference (EUSIPCO) 2009, Glasgow*, pp. 1235-1239.

See Also

[hda](#), [showloadings](#), [plot.hda](#)

Examples

```

library(mvtnorm)
library(MASS)

# simulate data for two classes
n <- 50
meana <- meanb <- c(0,0,0,0,0)
cova <- diag(5)
cova[1,1] <- 0.2
for(i in 3:4){
  for(j in (i+1):5){cova[i,j] <- cova[j,i] <- 0.75^(j-i)}
}
covb <- cova
diag(covb)[1:2] <- c(1,0.2)

xa <- rmvnorm(n,meana,cova)
xb <- rmvnorm(n,meanb,covb)
x <- rbind(xa,xb)
classes <- as.factor(c(rep(1,n),rep(2,n)))
# rotate simulated data
symmat <- matrix(runif(5^2),5)
symmat <- symmat + t(symmat)
even <- eigen(symmat)$vectors
rotatedspace <- x %*% even

# apply heteroscedastic discriminant analysis and plot data in discriminant space
hda.res <- hda(rotatedspace, classes)

# simulate new data
xanew <- rmvnorm(n,meana,cova)
xbnew <- rmvnorm(n,meanb,covb)
xnew <- rbind(xanew,xbnew)
classes <- as.factor(c(rep(1,n),rep(2,n)))
newrotateddata <- x %*% even
plot(as.data.frame(newrotateddata), col = classes)

# transform new data
prediction <- predict(hda.res, newrotateddata)
plot(as.data.frame(prediction), col = classes)

# predict classes for new data on automatically computed naive Bayes classification rule
# this requires package e1071
hda.res2 <- hda(rotatedspace, classes, crule = TRUE)
prediction2 <- predict(hda.res2, newrotateddata, task = "c")
prediction2

```

Description

Visualizes the loadings of the original variables on the components of the transformed discriminant space of reduced dimension.

Usage

```
showloadings(object, comps = 1:object$reduced.dimension, loadings = TRUE, ...)
```

Arguments

object	An object of class hda.
comps	A vector of component ids for which the loadings should be displayed.
loadings	Logical indicating whether loadings or variable importance lifts should be plotted.
...	Further arguments to be passed to the plot functions.

Details

Scatterplots of loadings (or lifts) of any variable on any hda component to give an idea of what variables do mainly contribute to the different discriminant components (see corresponding values of object). Note that as opposed to linear discriminant analysis not only location but also scale differences contribute to class discrimination of the hda components.

Value

No value is returned.

Author(s)

Gero Szepannek

References

- Kumar, N. and Andreou, A. (1998): *Heteroscedastic discriminant analysis and reduced rank HMMs for improved speech recognition*. Speech Communication 25, pp.283-297.
- Szepannek G., Harczos, T., Klefenz, F. and Weihs, C. (2009): *Extending features for automatic speech recognition by means of auditory modelling*. In: Proceedings of European Signal Processing Conference (EUSIPCO) 2009, Glasgow, pp.1235-1239.

See Also

[hda](#), [predict.hda](#), [plot.hda](#)

Examples

```
library(mvtnorm)
library(MASS)

# simulate data for two classes
n      <- 50
meana  <- meanb <- c(0,0,0,0,0)
cova   <- diag(5)
cova[1,1] <- 0.2
for(i in 3:4){
  for(j in (i+1):5){
    cova[i,j] <- cova[j,i] <- 0.75^(j-i)}
  }
covb   <- cova
diag(covb)[1:2] <- c(1,0.2)

xa     <- rmvnorm(n, meana, cova)
xb     <- rmvnorm(n, meanb, covb)
x      <- rbind(xa,xb)
classes <- as.factor(c(rep(1,n), rep(2,n)))
# rotate simulated data
symmat <- matrix(runif(5^2),5)
symmat <- symmat + t(symmat)
even   <- eigen(symmat)$vectors
rotatedspace <- x %*% even
plot(as.data.frame(rotatedspace), col = classes)

# apply heteroscedastic discriminant analysis and plot data in discriminant space
hda.res <- hda(rotatedspace, classes)

# visualize loadings
showloadings(hda.res)
```

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