

# Package ‘ipd’

May 8, 2026

**Title** Inference on Predicted Data

**Version** 0.4.1

## Description

Performs valid statistical inference on predicted data (IPD) using recent methods, where for a subset of the data, the outcomes have been predicted by an algorithm. Provides a wrapper function with specified defaults for the type of model and method to be used for estimation and inference. Further provides methods for tidying and summarizing results. Salerno et al., (2025) <[doi:10.1093/bioinformatics/btaf055](https://doi.org/10.1093/bioinformatics/btaf055)>.

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**BugReports** <https://github.com/ipd-tools/ipd/issues>

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A

*Calculation of the matrix A based on single dataset***Description**

A function for the calculation of the matrix A based on single dataset

**Usage**

```
A(
  X,
  Y,
  quant = NA,
  theta,
  method = c("ols", "quantile", "mean", "logistic", "poisson")
)
```

**Arguments**

X	Array or data.frame containing covariates
Y	Array or data.frame of outcomes
quant	quantile for quantile estimation
theta	parameter theta
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

matrix A based on single dataset

---

augment.ipd	<i>Augment data from an ipd fit</i>
-------------	-------------------------------------

---

**Description**

Augment data from an ipd fit

**Usage**

```
## S3 method for class 'ipd'
augment(x, data = x@data_u, ...)
```

**Arguments**

x	An object of class ipd.
data	A data.frame to augment; defaults to x@data_u.
...	Ignored.

**Value**

The data.frame with columns .fitted and .resid.

**Examples**

```
dat <- simdat()

fit <- ipd(Y ~ f ~ X1, method = "pspa", model = "ols",
          data = dat, label = "set_label")

augmented_df <- augment(fit)

head(augmented_df)
```

---

`calc_lhat_glm`*Estimate PPI++ Power Tuning Parameter*

---

### Description

Calculates the optimal value of `lhat` for the prediction-powered confidence interval for GLMs.

### Usage

```
calc_lhat_glm(  
  grads,  
  grads_hat,  
  grads_hat_unlabeled,  
  inv_hessian,  
  coord = NULL,  
  clip = FALSE  
)
```

### Arguments

<code>grads</code>	(matrix): $n \times p$ matrix gradient of the loss function with respect to the parameter evaluated at the labeled data.
<code>grads_hat</code>	(matrix): $n \times p$ matrix gradient of the loss function with respect to the model parameter evaluated using predictions on the labeled data.
<code>grads_hat_unlabeled</code>	(matrix): $N \times p$ matrix gradient of the loss function with respect to the parameter evaluated using predictions on the unlabeled data.
<code>inv_hessian</code>	(matrix): $p \times p$ matrix inverse of the Hessian of the loss function with respect to the parameter.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat</code> . If <code>None</code> , it optimizes the total variance over all coordinates. Must be in $(1, \dots, d)$ where $d$ is the shape of the estimand.
<code>clip</code>	(boolean, optional): Whether to clip the value of <code>lhat</code> to be non-negative. Defaults to <code>False</code> .

### Value

(float): Optimal value of `lhat` in  $[0,1]$ .

---

`chen_logistic`*Chen & Chen Logistic*

---

### Description

Helper function for Chen & Chen logistic regression estimation.

### Usage

```
chen_logistic(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

### Arguments

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>intercept</code>	(Logical): Do the design matrices include intercept columns? Default is TRUE.

### Details

Another look at statistical inference with machine learning-imputed data (Gronsbell et al., 2026)  
[doi:10.48550/arXiv.2411.19908](https://doi.org/10.48550/arXiv.2411.19908)

### Value

(list): A list containing the following:

**est** (vector): vector of Chen & Chen logistic regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

### Examples

```
dat <- simdat(model = "logistic")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])
```

```
f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

chen_logistic(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

---

chen\_ols

*Chen & Chen OLS*

---

## Description

Helper function for Chen & Chen OLS estimation

## Usage

```
chen_ols(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

## Arguments

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>intercept</code>	(Logical): Do the design matrices include intercept columns? Default is TRUE.

## Details

Another look at statistical inference with machine learning-imputed data (Gronsbell et al., 2026)  
[doi:10.48550/arXiv.2411.19908](https://doi.org/10.48550/arXiv.2411.19908)

## Value

(list): A list containing the following:

**est** (vector): vector of Chen & Chen OLS regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

## Examples

```
dat <- simdat(model = "ols")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

chen_ols(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

---

chen\_poisson

*Chen & Chen Poisson*

---

## Description

Helper function for Chen & Chen Poisson regression estimation.

## Usage

```
chen_poisson(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

## Arguments

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>intercept</code>	(Logical): Do the design matrices include intercept columns? Default is TRUE.

## Details

Another look at statistical inference with machine learning-imputed data (Gronsbell et al., 2026)  
[doi:10.48550/arXiv.2411.19908](https://doi.org/10.48550/arXiv.2411.19908)

**Value**

(list): A list containing the following:

**est** (vector): vector of Chen & Chen Poisson regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**Examples**

```
dat <- simdat(model = "poisson")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

chen_poisson(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

---

 compute\_cdf

*Empirical CDF of the Data*


---

**Description**

Computes the empirical CDF of the data.

**Usage**

```
compute_cdf(Y, grid, w = NULL)
```

**Arguments**

**Y** (matrix): n x 1 matrix of observed data.

**grid** (matrix): Grid of values to compute the CDF at.

**w** (vector, optional): n-vector of sample weights.

**Value**

(list): Empirical CDF and its standard deviation at the specified grid points.

---

compute_cdf_diff	<i>Empirical CDF Difference</i>
------------------	---------------------------------

---

**Description**

Computes the difference between the empirical CDFs of the data and the predictions.

**Usage**

```
compute_cdf_diff(Y, f, grid, w = NULL)
```

**Arguments**

Y	(matrix): n x 1 matrix of observed data.
f	(matrix): n x 1 matrix of predictions.
grid	(matrix): Grid of values to compute the CDF at.
w	(vector, optional): n-vector of sample weights.

**Value**

(list): Difference between the empirical CDFs of the data and the predictions and its standard deviation at the specified grid points.

---

est_ini	<i>Initial estimation</i>
---------	---------------------------

---

**Description**

est\_ini function for initial estimation

**Usage**

```
est_ini(
  X,
  Y,
  quant = NA,
  method = c("ols", "quantile", "mean", "logistic", "poisson")
)
```

**Arguments**

X	Array or data.frame containing covariates
Y	Array or data.frame of outcomes
quant	quantile for quantile estimation
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

initial estimator

---

glance.ipd

*Glance at an ipd fit*

---

**Description**

Glance at an ipd fit

**Usage**

```
## S3 method for class 'ipd'
glance(x, ...)
```

**Arguments**

x                    An object of class ipd.  
...                   Ignored.

**Value**

A one-row [tibble](#) summarizing the fit.

**Examples**

```
dat <- simdat()

fit <- ipd(Y ~ f ~ X1, method = "pspa", model = "ols",
          data = dat, label = "set_label")

glance(fit)
```

---

ipd

*Inference on Predicted Data (ipd)*

---

**Description**

The main wrapper function to conduct ipd using various methods and models, and returns a list of fitted model components.

**Usage**

```

ipd(
  formula,
  method,
  model,
  data,
  label = NULL,
  unlabeled_data = NULL,
  intercept = TRUE,
  alpha = 0.05,
  alternative = "two-sided",
  na_action = "na.fail",
  ...
)

```

**Arguments**

formula	An object of class formula: a symbolic description of the model to be fitted. Must be of the form $Y - f \sim X$ , where $Y$ is the name of the column corresponding to the observed outcome in the labeled data, $f$ is the name of the column corresponding to the predicted outcome in both labeled and unlabeled data, and $X$ corresponds to the features of interest (i.e., $X = X_1 + \dots + X_p$ ). See <b>1. Formula</b> in the <b>Details</b> below for more information.
method	The IPD method to be used for fitting the model. Must be one of "chen", "pdc", "postpi_analytic", "postpi_boot", "ppi", "ppi_a", "ppi_plusplus", or "pspa". See <b>3. Method</b> in the <b>Details</b> below for more information.
model	The type of downstream inferential model to be fitted, or the parameter being estimated. Must be one of "mean", "quantile", "ols", "logistic", or "poisson". See <b>4. Model</b> in the <b>Details</b> below for more information.
data	A data.frame containing the variables in the model, either a stacked data frame with a specific column identifying the labeled versus unlabeled observations (label), or only the labeled data set. Must contain columns for the observed outcomes ( $Y$ ), the predicted outcomes ( $f$ ), and the features ( $X$ ) needed to specify the formula. See <b>2. Data</b> in the <b>Details</b> below for more information.
label	A string, int, or logical specifying the column in the data that distinguishes between the labeled and unlabeled observations. See the <b>Details</b> section for more information. If NULL, unlabeled_data must be specified. See <b>2. Data</b> in the <b>Details</b> below for more information.
unlabeled_data	(optional) A data.frame of unlabeled data. If NULL, label must be specified. Specifying both the label and unlabeled_data arguments will result in an error message. If specified, must contain columns for the predicted outcomes ( $f$ ), and the features ( $X$ ) needed to specify the formula. See <b>2. Data</b> in the <b>Details</b> below for more information.
intercept	Logical. Should an intercept be included in the model? Default is TRUE.
alpha	The significance level for confidence intervals. Default is 0.05.

alternative	A string specifying the alternative hypothesis. Must be one of "two-sided", "less", or "greater".
na_action	(string, optional) How missing covariate data should be handled. Currently "na.fail" and "na.omit" are accommodated. Defaults to "na.fail".
...	Additional arguments to be passed to the fitting function. See the Details section for more information. See <b>5. Auxiliary Arguments</b> and <b>6. Other Arguments</b> in the <b>Details</b> below for more information.

## Details

### 1. Formula:

The `ipd` function uses one formula argument that specifies both the calibrating model (e.g., PostPI "relationship model", PPI "rectifier" model) and the inferential model. These separate models will be created internally based on the specific method called.

### 2. Data:

The data can be specified in two ways:

1. Single data argument (`data`) containing a stacked `data.frame` and a label identifier (`label`).
2. Two data arguments, one for the labeled data (`data`) and one for the unlabeled data (`unlabeled_data`).

For option (1), provide one data argument (`data`) which contains a stacked `data.frame` with both the unlabeled and labeled data and a `label` argument that specifies the column identifying the labeled versus the unlabeled observations in the stacked `data.frame` (e.g., `label = "set_label"` if the column "set\_label" in the stacked data denotes which set an observation belongs to).

NOTE: Labeled data identifiers can be:

**String** "l", "lab", "label", "labeled", "labelled", "tst", "test", "true"

**Logical** TRUE

**Factor** Non-reference category (i.e., binary 1)

Unlabeled data identifiers can be:

**String** "u", "unlab", "unlabeled", "unlabelled", "val", "validation", "false"

**Logical** FALSE

**Factor** Non-reference category (i.e., binary 0)

For option (2), provide separate data arguments for the labeled data set (`data`) and the unlabeled data set (`unlabeled_data`). If the second argument is provided, the function ignores the `label` identifier and assumes the data provided are not stacked.

NOTE: Not all columns in `data` or `unlabeled_data` may be used unless explicitly referenced in the formula argument or in the `label` argument (if the data are passed as one stacked data frame).

### 3. Method:

Use the method argument to specify the fitting method:

**"chen"** Gronsbell et al. (2026) Chen and Chen Correction

"**pd**" Gan et al. (2024) Prediction Decorrelated Inference  
 "**postpi\_analytic**" Wang et al. (2020) Post-Prediction Inference (PostPI) Analytic Correction  
 "**postpi\_boot**" Wang et al. (2020) Post-Prediction Inference (PostPI) Bootstrap Correction  
 "**ppi**" Angelopoulos et al. (2023) Prediction-Powered Inference (PPI)  
 "**ppi\_a**" Gronsbell et al. (2025) PPI "All" Correction  
 "**ppi\_plusplus**" Angelopoulos et al. (2023) PPI++  
 "**pspa**" Miao et al. (2023) Assumption-Lean and Data-Adaptive Post-Prediction Inference (PSPA)

#### 4. Model:

Use the `model` argument to specify the type of downstream inferential model or parameter to be estimated:

"**mean**" Mean value of a continuous outcome  
 "**quantile**"  $q$ th quantile of a continuous outcome  
 "**ols**" Linear regression coefficients for a continuous outcome  
 "**logistic**" Logistic regression coefficients for a binary outcome  
 "**poisson**" Poisson regression coefficients for a count outcome

The `ipd` wrapper function will concatenate the `method` and `model` arguments to identify the required helper function, following the naming convention "`method_model`".

#### 5. Auxiliary Arguments:

The wrapper function will take method-specific auxiliary arguments (e.g.,  $q$  for the quantile estimation models) and pass them to the helper function through the "`...`" with specified defaults for simplicity.

#### 6. Other Arguments:

All other arguments that relate to all methods (e.g., `alpha`, `ci.type`), or other method-specific arguments, will have defaults.

### Value

a summary of model output.

An S4 object of class `IPD` with the following slots:

`coefficients` Named `numeric` vector of estimated parameters.  
`se` Named `numeric` vector of standard errors.  
`ci` A `matrix` of confidence intervals, with columns `lower` and `upper`.  
`coefTable` A `data.frame` summarizing Estimate, Std. Error, z-value, and  $\Pr(>|z|)$  (glm-style).  
`fit` The raw output `list` returned by the method-specific helper function.  
`formula` The `formula` used for fitting the IPD model.  
`data_l` The labeled `data.frame` used in the analysis.  
`data_u` The unlabeled `data.frame` used in the analysis.  
`method` A `character` string indicating which IPD method was applied.  
`model` A `character` string indicating the downstream inferential model.  
`intercept` A `logical` indicating whether an intercept was included.

**Examples**

```
#-- Generate Example Data

dat <- simdat(n = c(300, 300, 300), effect = 1, sigma_Y = 1)

head(dat)

formula <- Y ~ f ~ X1

#-- Chen and Chen Correction (Gronsbell et al., 2026)

ipd(formula,
  method = "chen", model = "ols",
  data = dat, label = "set_label"
)

#-- Prediction Decorrelated Inference (Gan et al., 2024)

ipd(formula,
  method = "chen", model = "ols",
  data = dat, label = "set_label"
)

#-- PostPI Analytic Correction (Wang et al., 2020)

ipd(formula,
  method = "postpi_analytic", model = "ols",
  data = dat, label = "set_label"
)

#-- PostPI Bootstrap Correction (Wang et al., 2020)

nboot <- 200

ipd(formula,
  method = "postpi_boot", model = "ols",
  data = dat, label = "set_label", nboot = nboot
)

#-- PPI (Angelopoulos et al., 2023)

ipd(formula,
  method = "ppi", model = "ols",
  data = dat, label = "set_label"
)

#-- PPI "All" (Gronsbell et al., 2025)

ipd(formula,
  method = "ppi_a", model = "ols",
  data = dat, label = "set_label"
)
```

```

#-- PPI++ (Angelopoulos et al., 2023)

ipd(formula,
    method = "ppi_plusplus", model = "ols",
    data = dat, label = "set_label"
)

#-- PSPA (Miao et al., 2023)

ipd(formula,
    method = "pspa", model = "ols",
    data = dat, label = "set_label"
)

```

---

ipd-class

*ipd: S4 class for inference on predicted data results*


---

## Description

ipd: S4 class for inference on predicted data results

## Slots

coefficients Numeric vector of parameter estimates.

se Numeric vector of standard errors.

ci Numeric matrix of confidence intervals.

coefTable Data frame summarizing Estimate, Std. Error, z value, and  $\Pr(>|z|)$ .

fit The raw list returned by the helper function.

formula The formula used (class "formula").

data\_l The labeled data (data.frame).

data\_u The unlabeled data (data.frame).

method Character; which IPD method was used.

model Character; which downstream model was fitted.

intercept Logical; was an intercept included?

---

link_grad	<i>Gradient of the link function</i>
-----------	--------------------------------------

---

**Description**

link\_grad function for gradient of the link function

**Usage**

```
link_grad(t, method = c("ols", "logistic", "poisson"))
```

**Arguments**

t	t
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

gradient of the link function

---

link_Hessian	<i>Hessians of the link function</i>
--------------	--------------------------------------

---

**Description**

link\_Hessian function for Hessians of the link function

**Usage**

```
link_Hessian(t, method = c("logistic", "poisson"))
```

**Arguments**

t	t
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

Hessians of the link function

log1pexp

*Log1p Exponential*

---

**Description**

Computes the natural logarithm of 1 plus the exponential of the input, to handle large inputs.

**Usage**

```
log1pexp(x)
```

**Arguments**

x (vector): A numeric vector of inputs.

**Value**

(vector): A numeric vector where each element is the result of  $\log(1 + \exp(x))$ .

---

logistic\_get\_stats

*Logistic Regression Gradient and Hessian*

---

**Description**

Computes the statistics needed for the logistic regression-based prediction-powered inference.

**Usage**

```
logistic_get_stats(  
  est,  
  X_l,  
  Y_l,  
  f_l,  
  X_u,  
  f_u,  
  w_l = NULL,  
  w_u = NULL,  
  use_u = TRUE  
)
```

**Arguments**

<code>est</code>	(vector): Point estimates of the coefficients.
<code>X_l</code>	(matrix): Covariates for the labeled data set.
<code>Y_l</code>	(vector): Labels for the labeled data set.
<code>f_l</code>	(vector): Predictions for the labeled data set.
<code>X_u</code>	(matrix): Covariates for the unlabeled data set.
<code>f_u</code>	(vector): Predictions for the unlabeled data set.
<code>w_l</code>	(vector, optional): Sample weights for the labeled data set.
<code>w_u</code>	(vector, optional): Sample weights for the unlabeled data set.
<code>use_u</code>	(bool, optional): Whether to use the unlabeled data set.

**Value**

(list): A list containing the following:

**grads** (matrix):  $n \times p$  matrix gradient of the loss function with respect to the coefficients.

**grads\_hat** (matrix):  $n \times p$  matrix gradient of the loss function with respect to the coefficients, evaluated using the labeled predictions.

**grads\_hat\_unlabeled** (matrix):  $N \times p$  matrix gradient of the loss function with respect to the coefficients, evaluated using the unlabeled predictions.

**inv\_hessian** (matrix):  $p \times p$  matrix inverse Hessian of the loss function with respect to the coefficients.

---

mean_psi	<i>Sample expectation of psi</i>
----------	----------------------------------

---

**Description**

mean\_psi function for sample expectation of psi

**Usage**

```
mean_psi(
  X,
  Y,
  theta,
  quant = NA,
  method = c("ols", "quantile", "mean", "logistic", "poisson")
)
```

**Arguments**

X	Array or data.frame containing covariates
Y	Array or data.frame of outcomes
theta	parameter theta
quant	quantile for quantile estimation
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

sample expectation of psi

---

mean_psi_pop	<i>Sample expectation of PSPA psi</i>
--------------	---------------------------------------

---

**Description**

mean\_psi\_pop function for sample expectation of PSPA psi

**Usage**

```
mean_psi_pop(
  X_l,
  X_u,
  Y_l,
  f_l,
  f_u,
  w,
  theta,
  quant = NA,
  method = c("ols", "quantile", "mean", "logistic", "poisson")
)
```

**Arguments**

X_l	Array or data.frame containing observed covariates in labeled data.
X_u	Array or data.frame containing observed or predicted covariates in unlabeled data.
Y_l	Array or data.frame of observed outcomes in labeled data.
f_l	Array or data.frame of predicted outcomes in labeled data.
f_u	Array or data.frame of predicted outcomes in unlabeled data.
w	weights vector PSPA linear regression (d-dimensional, where d equals the number of covariates).
theta	parameter theta

quant            quantile for quantile estimation

method           indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

sample expectation of PSPA psi

---

ols                            *Ordinary Least Squares*

---

**Description**

Computes the ordinary least squares coefficients.

**Usage**

```
ols(X, Y, return_se = FALSE)
```

**Arguments**

X                    (matrix): n x p matrix of covariates.

Y                    (vector): p-vector of outcome values.

return\_se           (bool, optional): Whether to return the standard errors of the coefficients.

**Value**

(list): A list containing the following:

**theta** (vector): p-vector of ordinary least squares estimates of the coefficients.

**se** (vector): If return\_se == TRUE, return the p-vector of standard errors of the coefficients.

---

ols\_get\_stats                *OLS Gradient and Hessian*

---

**Description**

Computes the statistics needed for the OLS-based prediction-powered inference.

**Usage**

```
ols_get_stats(
  est,
  X_l,
  Y_l,
  f_l,
  X_u,
  f_u,
  w_l = NULL,
  w_u = NULL,
  use_u = TRUE
)
```

**Arguments**

<code>est</code>	(vector): Point estimates of the coefficients.
<code>X_l</code>	(matrix): Covariates for the labeled data set.
<code>Y_l</code>	(vector): Labels for the labeled data set.
<code>f_l</code>	(vector): Predictions for the labeled data set.
<code>X_u</code>	(matrix): Covariates for the unlabeled data set.
<code>f_u</code>	(vector): Predictions for the unlabeled data set.
<code>w_l</code>	(vector, optional): Sample weights for the labeled data set.
<code>w_u</code>	(vector, optional): Sample weights for the unlabeled data set.
<code>use_u</code>	(boolean, optional): Whether to use the unlabeled data set.

**Value**

(list): A list containing the following:

**grads** (matrix):  $n \times p$  matrix gradient of the loss function with respect to the coefficients.

**grads\_hat** (matrix):  $n \times p$  matrix gradient of the loss function with respect to the coefficients, evaluated using the labeled predictions.

**grads\_hat\_unlabeled** (matrix):  $N \times p$  matrix gradient of the loss function with respect to the coefficients, evaluated using the unlabeled predictions.

**inv\_hessian** (matrix):  $p \times p$  matrix inverse Hessian of the loss function with respect to the coefficients.

---

`optim_est`*One-step update for obtaining estimator*

---

**Description**

`optim_est` function for One-step update for obtaining estimator

**Usage**

```
optim_est(  
  X_l,  
  X_u,  
  Y_l,  
  f_l,  
  f_u,  
  w,  
  theta,  
  quant = NA,  
  method = c("ols", "quantile", "mean", "logistic", "poisson")  
)
```

**Arguments**

<code>X_l</code>	Array or data.frame containing observed covariates in labeled data.
<code>X_u</code>	Array or data.frame containing observed or predicted covariates in unlabeled data.
<code>Y_l</code>	Array or data.frame of observed outcomes in labeled data.
<code>f_l</code>	Array or data.frame of predicted outcomes in labeled data.
<code>f_u</code>	Array or data.frame of predicted outcomes in unlabeled data.
<code>w</code>	weights vector PSPA linear regression (d-dimensional, where d equals the number of covariates).
<code>theta</code>	parameter theta
<code>quant</code>	quantile for quantile estimation
<code>method</code>	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

estimator

---

`optim_weights`*One-step update for obtaining the weight vector*

---

**Description**

`optim_weights` function for One-step update for obtaining estimator

**Usage**

```
optim_weights(  
  X_l,  
  X_u,  
  Y_l,  
  f_l,  
  f_u,  
  w,  
  theta,  
  quant = NA,  
  method = c("ols", "quantile", "mean", "logistic", "poisson")  
)
```

**Arguments**

<code>X_l</code>	Array or data.frame containing observed covariates in labeled data.
<code>X_u</code>	Array or data.frame containing observed or predicted covariates in unlabeled data.
<code>Y_l</code>	Array or data.frame of observed outcomes in labeled data.
<code>f_l</code>	Array or data.frame of predicted outcomes in labeled data.
<code>f_u</code>	Array or data.frame of predicted outcomes in unlabeled data.
<code>w</code>	weights vector PSPA linear regression (d-dimensional, where d equals the number of covariates).
<code>theta</code>	parameter theta
<code>quant</code>	quantile for quantile estimation
<code>method</code>	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

weights

---

pdc\_logistic

*PDC Logistic*

---

## Description

Helper function for PDC logistic regression estimation.

## Usage

```
pdc_logistic(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

## Arguments

X_l	(matrix): n x p matrix of covariates in the labeled data.
Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
X_u	(matrix): N x p matrix of covariates in the unlabeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
intercept	(Logical): Do the design matrices include intercept columns? Default is TRUE.

## Details

Prediction de-correlated inference: A safe approach for post-prediction inference (Gan et al., 2024)  
[doi:10.1111/anzs.12429](https://doi.org/10.1111/anzs.12429)

## Value

(list): A list containing the following:

**est** (vector): vector of PDC logistic regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

## Examples

```
dat <- simdat(model = "logistic")

form <- Y - f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])
```

```
f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

pdc_logistic(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

---

pdc\_ols

*PDC OLS*

---

### Description

Helper function for PDC OLS estimation.

### Usage

```
pdc_ols(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

### Arguments

X_l	(matrix): n x p matrix of covariates in the labeled data.
Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
X_u	(matrix): N x p matrix of covariates in the unlabeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
intercept	(Logical): Do the design matrices include intercept columns? Default is TRUE.

### Details

Prediction de-correlated inference: A safe approach for post-prediction inference (Gan et al., 2024)  
[doi:10.1111/anzs.12429](https://doi.org/10.1111/anzs.12429)

### Value

(list): A list containing the following:

**est** (vector): vector of PDC OLS regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

## Examples

```
dat <- simdat(model = "ols")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

pdc_ols(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

---

pdc\_poisson

*PDC Poisson*

---

## Description

Helper function for PDC Poisson regression estimation.

## Usage

```
pdc_poisson(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

## Arguments

X_l	(matrix): n x p matrix of covariates in the labeled data.
Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
X_u	(matrix): N x p matrix of covariates in the unlabeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
intercept	(Logical): Do the design matrices include intercept columns? Default is TRUE.

## Details

Prediction de-correlated inference: A safe approach for post-prediction inference (Gan et al., 2024)  
[doi:10.1111/anzs.12429](https://doi.org/10.1111/anzs.12429)

**Value**

(list): A list containing the following:

**est** (vector): vector of PDC Poisson regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**Examples**

```
dat <- simdat(model = "poisson")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

pdc_poisson(X_l, Y_l, f_l, X_u, f_u, intercept = TRUE)
```

---

postpi\_analytic\_ols    *PostPI OLS (Analytic Correction)*

---

**Description**

Helper function for PostPI OLS estimation (analytic correction)

**Usage**

```
postpi_analytic_ols(X_l, Y_l, f_l, X_u, f_u, original = FALSE)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>original</code>	(boolean): Logical argument to use original method from Wang et al. (2020). Defaults to FALSE; TRUE retained for posterity.

**Details**

Methods for correcting inference based on outcomes predicted by machine learning (Wang et al., 2020) [doi:10.1073/pnas.2001238117](https://doi.org/10.1073/pnas.2001238117)

**Value**

A list of outputs: estimate of the inference model parameters and corresponding standard error estimate.

**Examples**

```
dat <- simdat(model = "ols")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

postpi_analytic_ols(X_l, Y_l, f_l, X_u, f_u)
```

---

postpi\_boot\_logistic *PostPI Logistic Regression (Bootstrap Correction)*

---

**Description**

Helper function for PostPI logistic regression (bootstrap correction)

**Usage**

```
postpi_boot_logistic(X_l, Y_l, f_l, X_u, f_u, nboot = 100, se_type = "par")
```

**Arguments**

X_l	(matrix): n x p matrix of covariates in the labeled data.
Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
X_u	(matrix): N x p matrix of covariates in the unlabeled data.

`f_u` (vector): N-vector of predictions in the unlabeled data.

`nboot` (integer): Number of bootstrap samples. Defaults to 100.

`se_type` (string): Which method to calculate the standard errors. Options include "par" (parametric) or "npar" (nonparametric). Defaults to "par".

### Details

Methods for correcting inference based on outcomes predicted by machine learning (Wang et al., 2020) [doi:10.1073/pnas.2001238117](https://doi.org/10.1073/pnas.2001238117)

### Value

A list of outputs: estimate of inference model parameters and corresponding standard error based on both parametric and non-parametric bootstrap methods.

### Examples

```
dat <- simdat(model = "logistic")

form <- Y - f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

postpi_boot_logistic(X_l, Y_l, f_l, X_u, f_u, nboot = 200)
```

---

postpi\_boot\_ols

*PostPI OLS (Bootstrap Correction)*

---

### Description

Helper function for PostPI OLS estimation (bootstrap correction)

**Usage**

```

postpi_boot_ols(
  X_l,
  Y_l,
  f_l,
  X_u,
  f_u,
  nboot = 100,
  se_type = "par",
  rel_func = "lm",
  scale_se = TRUE,
  n_t = Inf
)

```

**Arguments**

X_l	(matrix): n x p matrix of covariates in the labeled data.
Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
X_u	(matrix): N x p matrix of covariates in the unlabeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
nboot	(integer): Number of bootstrap samples. Defaults to 100.
se_type	(string): Which method to calculate the standard errors. Options include "par" (parametric) or "npar" (nonparametric). Defaults to "par".
rel_func	(string): Method for fitting the relationship model. Options include "lm" (linear model), "rf" (random forest), and "gam" (generalized additive model). Defaults to "lm".
scale_se	(boolean): Logical argument to scale relationship model error variance. Defaults to TRUE; FALSE option is retained for posterity.
n_t	(integer, optional) Size of the dataset used to train the prediction function (necessary if $n_t < nrow(X_l)$ ). Defaults to Inf.

**Details**

Methods for correcting inference based on outcomes predicted by machine learning (Wang et al., 2020) [doi:10.1073/pnas.2001238117](https://doi.org/10.1073/pnas.2001238117)

**Value**

A list of outputs: estimate of inference model parameters and corresponding standard error based on both parametric and non-parametric bootstrap methods.

**Examples**

```

dat <- simdat(model = "ols")

form <- Y - f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

postpi_boot_ols(X_l, Y_l, f_l, X_u, f_u, nboot = 200)

```

---

ppi\_a\_ols

*PPI "All" OLS*


---

**Description**

Helper function for PPI "All" for OLS estimation

**Usage**

```
ppi_a_ols(X_l, Y_l, f_l, X_u, f_u, w_l = NULL, w_u = NULL)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

## Details

Another look at statistical inference with machine learning-imputed data (Gronsbell et al., 2026)  
[doi:10.48550/arXiv.2411.19908](https://doi.org/10.48550/arXiv.2411.19908)

## Value

(list): A list containing the following:

**est** (vector): vector of PPI OLS regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**rectifier\_est** (vector): vector of the rectifier OLS regression coefficient estimates.

## Examples

```
dat <- simdat()

form <- Y - f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

ppi_a_ols(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_logistic

*PPI Logistic Regression*

---

## Description

Helper function for PPI logistic regression

## Usage

```
ppi_logistic(X_l, Y_l, f_l, X_u, f_u, opts = NULL)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>opts</code>	(list, optional): Options to pass to the optimizer. See <code>?optim</code> for details.

**Details**

Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.1126/science.adi6000](https://doi.org/10.1126/science.adi6000)

**Value**

(list): A list containing the following:

**est** (vector): vector of PPI logistic regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**rectifier\_est** (vector): vector of the rectifier logistic regression coefficient estimates.

**var\_u** (matrix): covariance matrix for the gradients in the unlabeled data.

**var\_l** (matrix): covariance matrix for the gradients in the labeled data.

**grads** (matrix): matrix of gradients for the labeled data.

**grads\_hat\_unlabeled** (matrix): matrix of predicted gradients for the unlabeled data.

**grads\_hat** (matrix): matrix of predicted gradients for the labeled data.

**inv\_hessian** (matrix): inverse Hessian matrix.

**Examples**

```
dat <- simdat(model = "logistic")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
```

```
matrix(ncol = 1)
ppi_logistic(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_mean *PPI Mean Estimation*

---

### Description

Helper function for PPI mean estimation

### Usage

```
ppi_mean(Y_l, f_l, f_u, alpha = 0.05, alternative = "two-sided")
```

### Arguments

**Y\_l** (vector): n-vector of labeled outcomes.

**f\_l** (vector): n-vector of predictions in the labeled data.

**f\_u** (vector): N-vector of predictions in the unlabeled data.

**alpha** (scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05.

**alternative** (string): Alternative hypothesis. Must be one of "two-sided", "less", or "greater".

### Details

Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.1126/science.adi6000](https://doi.org/10.1126/science.adi6000)

### Value

tuple: Lower and upper bounds of the prediction-powered confidence interval for the mean.

### Examples

```
dat <- simdat(model = "mean")

form <- Y - f ~ 1

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)
```

```
ppi_mean(Y_l, f_l, f_u)
```

---

```
ppi_ols
```

```
PPI OLS
```

---

### Description

Helper function for prediction-powered inference for OLS estimation

### Usage

```
ppi_ols(X_l, Y_l, f_l, X_u, f_u, w_l = NULL, w_u = NULL)
```

### Arguments

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

### Details

Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.1126/science.adi6000](https://doi.org/10.1126/science.adi6000)

### Value

(list): A list containing the following:

**est** (vector): vector of PPI OLS regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**rectifier\_est** (vector): vector of the rectifier OLS regression coefficient estimates.

**Examples**

```
dat <- simdat()

form <- Y - f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

ppi_ols(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_plusplus\_logistic *PPI++ Logistic Regression*

---

**Description**

Helper function for PPI++ logistic regression

**Usage**

```
ppi_plusplus_logistic(
  X_l,
  Y_l,
  f_l,
  X_u,
  f_u,
  lhat = NULL,
  coord = NULL,
  opts = NULL,
  w_l = NULL,
  w_u = NULL
)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>lhat</code>	(float, optional): Power-tuning parameter (see <a href="https://doi.org/10.48550/arXiv.2311.01453">doi:10.48550/arXiv.2311.01453</a> ). The default value, NULL, will estimate the optimal value from the data. Setting <code>lhat = 1</code> recovers PPI with no power tuning, and setting <code>lhat = 0</code> recovers the classical point estimate.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat = 1</code> . If NULL, it optimizes the total variance over all coordinates. Must be in $(1, \dots, d)$ where $d$ is the dimension of the estimand.
<code>opts</code>	(list, optional): Options to pass to the optimizer. See <code>?optim</code> for details.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

**Details**

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

(list): A list containing the following:

**est** (vector): vector of PPI++ logistic regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**lambda** (float): estimated power-tuning parameter.

**rectifier\_est** (vector): vector of the rectifier logistic regression coefficient estimates.

**var\_u** (matrix): covariance matrix for the gradients in the unlabeled data.

**var\_l** (matrix): covariance matrix for the gradients in the labeled data.

**grads** (matrix): matrix of gradients for the labeled data.

**grads\_hat\_unlabeled** (matrix): matrix of predicted gradients for the unlabeled data.

**grads\_hat** (matrix): matrix of predicted gradients for the labeled data.

**inv\_hessian** (matrix): inverse Hessian matrix.

**Examples**

```
dat <- simdat(model = "logistic")

form <- Y - f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

ppi_plusplus_logistic(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_plusplus\_logistic\_est

*PPI++ Logistic Regression (Point Estimate)*

---

**Description**

Helper function for PPI++ logistic regression (point estimate)

**Usage**

```
ppi_plusplus_logistic_est(
  X_l,
  Y_l,
  f_l,
  X_u,
  f_u,
  lhat = NULL,
  coord = NULL,
  opts = NULL,
  w_l = NULL,
  w_u = NULL
)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>lhat</code>	(float, optional): Power-tuning parameter (see <a href="https://doi.org/10.48550/arXiv.2311.01453">doi:10.48550/arXiv.2311.01453</a> ). The default value, NULL, will estimate the optimal value from the data. Setting <code>lhat = 1</code> recovers PPI with no power tuning, and setting <code>lhat = 0</code> recovers the classical point estimate.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat = 1</code> . If NULL, it optimizes the total variance over all coordinates. Must be in $(1, \dots, d)$ where $d$ is the dimension of the estimand.
<code>opts</code>	(list, optional): Options to pass to the optimizer. See <code>?optim</code> for details.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

**Details**

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

(vector): vector of prediction-powered point estimates of the logistic regression coefficients.

**Examples**

```
dat <- simdat(model = "logistic")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
```

```
matrix(ncol = 1)

ppi_plusplus_logistic_est(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_plusplus\_mean      *PPI++ Mean Estimation*

---

## Description

Helper function for PPI++ mean estimation

## Usage

```
ppi_plusplus_mean(
  Y_l,
  f_l,
  f_u,
  alpha = 0.05,
  alternative = "two-sided",
  lhat = NULL,
  coord = NULL,
  w_l = NULL,
  w_u = NULL
)
```

## Arguments

<code>Y_l</code>	(vector): n-vector of labeled outcomes.
<code>f_l</code>	(vector): n-vector of predictions in the labeled data.
<code>f_u</code>	(vector): N-vector of predictions in the unlabeled data.
<code>alpha</code>	(scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05.
<code>alternative</code>	(string): Alternative hypothesis. Must be one of "two-sided", "less", or "greater".
<code>lhat</code>	(float, optional): Power-tuning parameter (see <a href="https://doi.org/10.48550/arXiv.2311.01453">doi:10.48550/arXiv.2311.01453</a> ). The default value, NULL, will estimate the optimal value from the data. Setting <code>lhat = 1</code> recovers PPI with no power tuning, and setting <code>lhat = 0</code> recovers the classical point estimate.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat = 1</code> . If NULL, it optimizes the total variance over all coordinates. Must be in (1, ..., d) where d is the dimension of the estimand.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

**Details**

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

tuple: Lower and upper bounds of the prediction-powered confidence interval for the mean.

**Examples**

```
dat <- simdat(model = "mean")

form <- Y - f ~ 1

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

ppi_plusplus_mean(Y_l, f_l, f_u)
```

---

ppi\_plusplus\_mean\_est *PPI++ Mean Estimation (Point Estimate)*

---

**Description**

Helper function for PPI++ mean estimation (point estimate)

**Usage**

```
ppi_plusplus_mean_est(
  Y_l,
  f_l,
  f_u,
  lhat = NULL,
  coord = NULL,
  w_l = NULL,
  w_u = NULL
)
```

**Arguments**

<code>Y_l</code>	(vector): n-vector of labeled outcomes.
<code>f_l</code>	(vector): n-vector of predictions in the labeled data.
<code>f_u</code>	(vector): N-vector of predictions in the unlabeled data.
<code>lhat</code>	(float, optional): Power-tuning parameter (see <a href="https://doi.org/10.48550/arXiv.2311.01453">doi:10.48550/arXiv.2311.01453</a> ). The default value, NULL, will estimate the optimal value from the data. Setting <code>lhat = 1</code> recovers PPI with no power tuning, and setting <code>lhat = 0</code> recovers the classical point estimate.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat = 1</code> . If NULL, it optimizes the total variance over all coordinates. Must be in (1, ..., d) where d is the dimension of the estimand.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

**Details**

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

float or ndarray: Prediction-powered point estimate of the mean.

**Examples**

```
dat <- simdat(model = "mean")
form <- Y - f ~ 1
Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)
f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)
f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)
ppi_plusplus_mean_est(Y_l, f_l, f_u)
```

---

ppi\_plusplus\_ols      *PPI++ OLS*

---

### Description

Helper function for PPI++ OLS estimation

### Usage

```
ppi_plusplus_ols(
  X_l,
  Y_l,
  f_l,
  X_u,
  f_u,
  lhat = NULL,
  coord = NULL,
  w_l = NULL,
  w_u = NULL
)
```

### Arguments

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>lhat</code>	(float, optional): Power-tuning parameter (see <a href="https://doi.org/10.48550/arXiv.2311.01453">doi:10.48550/arXiv.2311.01453</a> ). The default value, NULL, will estimate the optimal value from the data. Setting <code>lhat = 1</code> recovers PPI with no power tuning, and setting <code>lhat = 0</code> recovers the classical point estimate.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat = 1</code> . If NULL, it optimizes the total variance over all coordinates. Must be in $(1, \dots, d)$ where $d$ is the dimension of the estimand.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

### Details

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

(list): A list containing the following:

**est** (vector): vector of PPI++ OLS regression coefficient estimates.

**se** (vector): vector of standard errors of the coefficients.

**lambda** (float): estimated power-tuning parameter.

**rectifier\_est** (vector): vector of the rectifier OLS regression coefficient estimates.

**var\_u** (matrix): covariance matrix for the gradients in the unlabeled data.

**var\_l** (matrix): covariance matrix for the gradients in the labeled data.

**grads** (matrix): matrix of gradients for the labeled data.

**grads\_hat\_unlabeled** (matrix): matrix of predicted gradients for the unlabeled data.

**grads\_hat** (matrix): matrix of predicted gradients for the labeled data.

**inv\_hessian** (matrix): inverse Hessian matrix.

**Examples**

```
dat <- simdat(model = "ols")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])
Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])
f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

ppi_plusplus_ols(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_plusplus\_ols\_est *PPI++ OLS (Point Estimate)*

---

**Description**

Helper function for PPI++ OLS estimation (point estimate)

**Usage**

```
ppi_plusplus_ols_est(
  X_l,
  Y_l,
  f_l,
  X_u,
  f_u,
  lhat = NULL,
  coord = NULL,
  w_l = NULL,
  w_u = NULL
)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>lhat</code>	(float, optional): Power-tuning parameter (see <a href="https://doi.org/10.48550/arXiv.2311.01453">doi:10.48550/arXiv.2311.01453</a> ). The default value, NULL, will estimate the optimal value from the data. Setting <code>lhat = 1</code> recovers PPI with no power tuning, and setting <code>lhat = 0</code> recovers the classical point estimate.
<code>coord</code>	(int, optional): Coordinate for which to optimize <code>lhat = 1</code> . If NULL, it optimizes the total variance over all coordinates. Must be in $(1, \dots, d)$ where $d$ is the dimension of the estimand.
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

**Details**

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

(vector): vector of prediction-powered point estimates of the OLS coefficients.

**Examples**

```
dat <- simdat(model = "ols")
form <- Y ~ f ~ X1
X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])
```

```
Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

ppi_plusplus_ols_est(X_l, Y_l, f_l, X_u, f_u)
```

---

ppi\_plusplus\_quantile *PPI++ Quantile Estimation*

---

## Description

Helper function for PPI++ quantile estimation

## Usage

```
ppi_plusplus_quantile(
  Y_l,
  f_l,
  f_u,
  q,
  alpha = 0.05,
  exact_grid = FALSE,
  w_l = NULL,
  w_u = NULL
)
```

## Arguments

Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
q	(float): Quantile to estimate. Must be in the range (0, 1).
alpha	(scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05.

<code>exact_grid</code>	(bool, optional): Whether to compute the exact solution (TRUE) or an approximate solution based on a linearly spaced grid of 5000 values (FALSE).
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

## Details

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

## Value

tuple: Lower and upper bounds of the prediction-powered confidence interval for the quantile.

## Examples

```
dat <- simdat(model = "quantile")  
  
form <- Y - f ~ X1  
  
Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>  
  matrix(ncol = 1)  
  
f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>  
  matrix(ncol = 1)  
  
f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>  
  matrix(ncol = 1)  
  
ppi_plusplus_quantile(Y_l, f_l, f_u, q = 0.5)
```

---

ppi\_plusplus\_quantile\_est

*PPI++ Quantile Estimation (Point Estimate)*

---

## Description

Helper function for PPI++ quantile estimation (point estimate)

**Usage**

```
ppi_plusplus_quantile_est(
  Y_l,
  f_l,
  f_u,
  q,
  exact_grid = FALSE,
  w_l = NULL,
  w_u = NULL
)
```

**Arguments**

<code>Y_l</code>	(vector): n-vector of labeled outcomes.
<code>f_l</code>	(vector): n-vector of predictions in the labeled data.
<code>f_u</code>	(vector): N-vector of predictions in the unlabeled data.
<code>q</code>	(float): Quantile to estimate. Must be in the range (0, 1).
<code>exact_grid</code>	(bool, optional): Whether to compute the exact solution (TRUE) or an approximate solution based on a linearly spaced grid of 5000 values (FALSE).
<code>w_l</code>	(ndarray, optional): Sample weights for the labeled data set. Defaults to a vector of ones.
<code>w_u</code>	(ndarray, optional): Sample weights for the unlabeled data set. Defaults to a vector of ones.

**Details**

PPI++: Efficient Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.48550/arXiv.2311.01453](https://doi.org/10.48550/arXiv.2311.01453)

**Value**

(float): Prediction-powered point estimate of the quantile.

**Examples**

```
dat <- simdat(model = "quantile")

form <- Y - f ~ 1

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
```

```
matrix(ncol = 1)

ppi_plusplus_quantile_est(Y_l, f_l, f_u, q = 0.5)
```

---

ppi\_quantile

*PPI Quantile Estimation*


---

## Description

Helper function for PPI quantile estimation

## Usage

```
ppi_quantile(Y_l, f_l, f_u, q, alpha = 0.05, exact_grid = FALSE)
```

## Arguments

Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
q	(float): Quantile to estimate. Must be in the range (0, 1).
alpha	(scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05.
exact_grid	(bool, optional): Whether to compute the exact solution (TRUE) or an approximate solution based on a linearly spaced grid of 5000 values (FALSE).

## Details

Prediction Powered Inference (Angelopoulos et al., 2023) [doi:10.1126/science.adi6000](https://doi.org/10.1126/science.adi6000)

## Value

tuple: Lower and upper bounds of the prediction-powered confidence interval for the quantile.

## Examples

```
dat <- simdat(model = "quantile")

form <- Y - f ~ X1

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
```

```
matrix(ncol = 1)

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>

matrix(ncol = 1)

ppi_quantile(Y_l, f_l, f_u, q = 0.5)
```

---

print.ipd

*Print ipd fit*

---

### Description

Print ipd fit

### Usage

```
## S3 method for class 'ipd'
print(x, ...)
```

### Arguments

x                    An object of class ipd.  
...                   Ignored.

### Value

Invisibly returns x.

---

print.summary.ipd

*Print summary.ipd*

---

### Description

Print summary.ipd

### Usage

```
## S3 method for class 'summary.ipd'
print(x, ...)
```

### Arguments

x                    An object of class summary.ipd.  
...                   Ignored.

**Value**

Invisibly returns x.

---

psi

*Estimating equation*

---

**Description**

psi function for estimating equation

**Usage**

```
psi(
  X,
  Y,
  theta,
  quant = NA,
  method = c("ols", "quantile", "mean", "logistic", "poisson")
)
```

**Arguments**

X	Array or data.frame containing covariates
Y	Array or data.frame of outcomes
theta	parameter theta
quant	quantile for quantile estimation
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

estimating equation

---

pspa\_logistic

*PSPA Logistic Regression*

---

**Description**

Helper function for PSPA logistic regression

**Usage**

```
pspa_logistic(X_l, Y_l, f_l, X_u, f_u, weights = NA, alpha = 0.05)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of binary labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of binary predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of binary predictions in the unlabeled data.
<code>weights</code>	(array): $p$ -dimensional array of weights vector for variance reduction. PSPA will estimate the weights if not specified.
<code>alpha</code>	(scalar): type I error rate for hypothesis testing - values in $(0, 1)$ ; defaults to 0.05

**Details**

Post-prediction adaptive inference (Miao et al. 2023) [doi:10.48550/arXiv.2311.14220](https://doi.org/10.48550/arXiv.2311.14220)

**Value**

A list of outputs: estimate of inference model parameters and corresponding standard error.

**Examples**

```
dat <- simdat(model = "logistic")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

pspa_logistic(X_l, Y_l, f_l, X_u, f_u)
```

---

pspa\_mean

*PSPA Mean Estimation*

---

**Description**

Helper function for PSPA mean estimation

**Usage**

```
pspa_mean(Y_l, f_l, f_u, weights = NA, alpha = 0.05)
```

**Arguments**

<code>Y_l</code>	(vector): n-vector of labeled outcomes.
<code>f_l</code>	(vector): n-vector of predictions in the labeled data.
<code>f_u</code>	(vector): N-vector of predictions in the unlabeled data.
<code>weights</code>	(array): 1-dimensional array of weights vector for variance reduction. PSPA will estimate the weights if not specified.
<code>alpha</code>	(scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05.

**Details**

Post-prediction adaptive inference (Miao et al., 2023) [doi:10.48550/arXiv.2311.14220](https://doi.org/10.48550/arXiv.2311.14220)

**Value**

A list of outputs: estimate of inference model parameters and corresponding standard error.

**Examples**

```
dat <- simdat(model = "mean")

form <- Y ~ f ~ 1

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

pspa_mean(Y_l = Y_l, f_l = f_l, f_u = f_u)
```

---

pspa\_ols

*PSPA OLS Estimation*

---

**Description**

Helper function for PSPA OLS for linear regression

**Usage**

```
pspa_ols(X_l, Y_l, f_l, X_u, f_u, weights = NA, alpha = 0.05)
```

**Arguments**

<code>X_l</code>	(matrix): $n \times p$ matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): $n$ -vector of labeled outcomes.
<code>f_l</code>	(vector): $n$ -vector of predictions in the labeled data.
<code>X_u</code>	(matrix): $N \times p$ matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): $N$ -vector of predictions in the unlabeled data.
<code>weights</code>	(array): $p$ -dimensional array of weights vector for variance reduction. PSPA will estimate the weights if not specified.
<code>alpha</code>	(scalar): type I error rate for hypothesis testing - values in $(0, 1)$ ; defaults to 0.05.

**Details**

Post-prediction adaptive inference (Miao et al., 2023) [doi:10.48550/arXiv.2311.14220](https://doi.org/10.48550/arXiv.2311.14220)

**Value**

A list of outputs: estimate of inference model parameters and corresponding standard error.

**Examples**

```
dat <- simdat(model = "ols")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

pspa_ols(X_l, Y_l, f_l, X_u, f_u)
```

pspa\_poisson

*PSPA Poisson Regression***Description**

Helper function for PSPA Poisson regression

**Usage**

```
pspa_poisson(X_l, Y_l, f_l, X_u, f_u, weights = NA, alpha = 0.05)
```

**Arguments**

<code>X_l</code>	(matrix): n x p matrix of covariates in the labeled data.
<code>Y_l</code>	(vector): n-vector of count labeled outcomes.
<code>f_l</code>	(vector): n-vector of binary predictions in the labeled data.
<code>X_u</code>	(matrix): N x p matrix of covariates in the unlabeled data.
<code>f_u</code>	(vector): N-vector of binary predictions in the unlabeled data.
<code>weights</code>	(array): p-dimensional array of weights vector for variance reduction. PSPA will estimate the weights if not specified.
<code>alpha</code>	(scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05

**Details**

Post-prediction adaptive inference (Miao et al., 2023) [doi:10.48550/arXiv.2311.14220](https://doi.org/10.48550/arXiv.2311.14220)

**Value**

A list of outputs: estimate of inference model parameters and corresponding standard error.

**Examples**

```
dat <- simdat(model = "poisson")

form <- Y ~ f ~ X1

X_l <- model.matrix(form, data = dat[dat$set_label == "labeled", ])

Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |>
  matrix(ncol = 1)

f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |>
  matrix(ncol = 1)

X_u <- model.matrix(form, data = dat[dat$set_label == "unlabeled", ])

f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
```

```
matrix(ncol = 1)
pspa_poisson(X_l, Y_l, f_l, X_u, f_u)
```

---

pspa\_quantile

*PSPA Quantile Estimation*


---

## Description

Helper function for PSPA quantile estimation

## Usage

```
pspa_quantile(Y_l, f_l, f_u, q, weights = NA, alpha = 0.05)
```

## Arguments

Y_l	(vector): n-vector of labeled outcomes.
f_l	(vector): n-vector of predictions in the labeled data.
f_u	(vector): N-vector of predictions in the unlabeled data.
q	(float): Quantile to estimate. Must be in the range (0, 1).
weights	(array): 1-dimensional array of weights vector for variance reduction. PSPA will estimate the weights if not specified.
alpha	(scalar): type I error rate for hypothesis testing - values in (0, 1); defaults to 0.05.

## Details

Post-prediction adaptive inference (Miao et al., 2023) [doi:10.48550/arXiv.2311.14220](https://doi.org/10.48550/arXiv.2311.14220)

## Value

A list of outputs: estimate of inference model parameters and corresponding standard error.

## Examples

```
dat <- simdat(model = "quantile")
form <- Y - f ~ 1
Y_l <- dat[dat$set_label == "labeled", all.vars(form)[1]] |> matrix(ncol = 1)
f_l <- dat[dat$set_label == "labeled", all.vars(form)[2]] |> matrix(ncol = 1)
f_u <- dat[dat$set_label == "unlabeled", all.vars(form)[2]] |>
  matrix(ncol = 1)
```

```
pspa_quantile(Y_l = Y_l, f_l = f_l, f_u = f_u, q = 0.5)
```

---

pspa\_y

*PSPA M-Estimation for ML-predicted labels*

---

## Description

pspa\_y function conducts post-prediction M-Estimation.

## Usage

```
pspa_y(
  X_l = NA,
  X_u = NA,
  Y_l,
  f_l,
  f_u,
  alpha = 0.05,
  weights = NA,
  quant = NA,
  intercept = FALSE,
  method
)
```

## Arguments

X_l	Array or data.frame containing observed covariates in labeled data.
X_u	Array or data.frame containing observed or predicted covariates in unlabeled data.
Y_l	Array or data.frame of observed outcomes in labeled data.
f_l	Array or data.frame of predicted outcomes in labeled data.
f_u	Array or data.frame of predicted outcomes in unlabeled data.
alpha	Specifies the confidence level as 1 - alpha for confidence intervals.
weights	weights vector PSPA linear regression (d-dimensional, where d equals the number of covariates).
quant	quantile for quantile estimation
intercept	Boolean indicating if the input covariates' data contains the intercept (TRUE if the input data contains)
method	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

## Value

A summary table presenting point estimates, standard error, confidence intervals (1 - alpha), P-values, and weights.

---

rectified_cdf	<i>Rectified CDF</i>
---------------	----------------------

---

**Description**

Computes the rectified CDF of the data.

**Usage**

```
rectified_cdf(Y_l, f_l, f_u, grid, w_l = NULL, w_u = NULL)
```

**Arguments**

<code>Y_l</code>	(vector): Gold-standard labels.
<code>f_l</code>	(vector): Predictions corresponding to the gold-standard labels.
<code>f_u</code>	(vector): Predictions corresponding to the unlabeled data.
<code>grid</code>	(vector): Grid of values to compute the CDF at.
<code>w_l</code>	(vector, optional): Sample weights for the labeled data set.
<code>w_u</code>	(vector, optional): Sample weights for the unlabeled data set.

**Value**

(vector): Rectified CDF of the data at the specified grid points.

---

rectified_p_value	<i>Rectified P-Value</i>
-------------------	--------------------------

---

**Description**

Computes a rectified p-value.

**Usage**

```
rectified_p_value(
  rectifier,
  rectifier_std,
  imputed_mean,
  imputed_std,
  null = 0,
  alternative = "two-sided"
)
```

**Arguments**

rectifier	(float or vector): Rectifier value.
rectifier_std	(float or vector): Rectifier standard deviation.
imputed_mean	(float or vector): Imputed mean.
imputed_std	(float or vector): Imputed standard deviation.
null	(float, optional): Value of the null hypothesis to be tested. Defaults to $\emptyset$ .
alternative	(str, optional): Alternative hypothesis, either 'two-sided', 'larger' or 'smaller'.

**Value**

(float or vector): The rectified p-value.

---

show, ipd-method	<i>Show an ipd object</i>
------------------	---------------------------

---

**Description**

Display a concise summary of an ipd S4 object, including method, model, formula, and a glm-style coefficient table.

**Usage**

```
## S4 method for signature 'ipd'  
show(object)
```

**Arguments**

object	An object of S4 class ipd.
--------	----------------------------

**Value**

Invisibly returns object after printing.

---

`Sigma_cal`*Variance-covariance matrix of the estimation equation*

---

**Description**

`Sigma_cal` function for variance-covariance matrix of the estimation equation

**Usage**

```
Sigma_cal(  
  X_l,  
  X_u,  
  Y_l,  
  f_l,  
  f_u,  
  w,  
  theta,  
  quant = NA,  
  method = c("ols", "quantile", "mean", "logistic", "poisson")  
)
```

**Arguments**

<code>X_l</code>	Array or data.frame containing observed covariates in labeled data.
<code>X_u</code>	Array or data.frame containing observed or predicted covariates in unlabeled data.
<code>Y_l</code>	Array or data.frame of observed outcomes in labeled data.
<code>f_l</code>	Array or data.frame of predicted outcomes in labeled data.
<code>f_u</code>	Array or data.frame of predicted outcomes in unlabeled data.
<code>w</code>	weights vector PSPA linear regression (d-dimensional, where d equals the number of covariates).
<code>theta</code>	parameter theta
<code>quant</code>	quantile for quantile estimation
<code>method</code>	indicates the method to be used for M-estimation. Options include "mean", "quantile", "ols", "logistic", and "poisson".

**Value**

variance-covariance matrix of the estimation equation

simdat

*Data generation function for various underlying models***Description**

Data generation function for various underlying models

**Usage**

```
simdat(
  n = c(300, 300, 300),
  effect = 1,
  sigma_Y = 1,
  model = "ols",
  shift = 0,
  scale = 1
)
```

**Arguments**

n	Integer vector of size 3 indicating the sample sizes in the training, labeled, and unlabeled data sets, respectively
effect	Regression coefficient for the first variable of interest for inference. Defaults is 1.
sigma_Y	Residual variance for the generated outcome. Defaults is 1.
model	The type of model to be generated. Must be one of "mean", "quantile", "ols", "logistic", or "poisson". Default is "ols".
shift	Scalar shift of the predictions for continuous outcomes (i.e., "mean", "quantile", and "ols"). Defaults to 0.
scale	Scaling factor for the predictions for continuous outcomes (i.e., "mean", "quantile", and "ols"). Defaults to 1.

**Details**

The `simdat` function generates three datasets consisting of independent realizations of  $Y$  (for `model = "mean"` or `"quantile"`), or  $\{Y, \mathbf{X}\}$  (for `model = "ols"`, `"logistic"`, or `"poisson"`): a *training* dataset of size  $n_t$ , a *labeled* dataset of size  $n_l$ , and an *unlabeled* dataset of size  $n_u$ . These sizes are specified by the argument `n`.

NOTE: In the *unlabeled* data subset, outcome data are still generated to facilitate a benchmark for comparison with an "oracle" model that uses the true  $Y^u$  values for estimation and inference.

**Generating Data**

For "mean" and "quantile", we simulate a continuous outcome,  $Y \in \mathbb{R}$ , with mean given by the `effect` argument and error variance given by the `sigma_y` argument.

For "ols", "logistic", or "poisson" models, predictor data,  $\mathbf{X} \in \mathbb{R}^4$  are simulated such that the  $i$ th observation follows a standard multivariate normal distribution with a zero mean vector and identity covariance matrix:

$$\mathbf{X}_i = (X_{i1}, X_{i2}, X_{i3}, X_{i4}) \sim \mathcal{N}_4(\mathbf{0}, \mathbf{I}).$$

For "ols", a continuous outcome  $Y \in \mathbb{R}$  is simulated to depend on  $X_1$  through a linear term with the effect size specified by the effect argument, while the other predictors,  $\mathbf{X} \setminus X_1$ , have nonlinear effects:

$$Y_i = effect \times Z_{i1} + \frac{1}{2}Z_{i2}^2 + \frac{1}{3}Z_{i3}^3 + \frac{1}{4}Z_{i4}^2 + \varepsilon_y,$$

and  $\varepsilon_y \sim \mathcal{N}(0, sigma\_y)$ , where the `sigma_y` argument specifies the error variance.

For "logistic", we simulate:

$$\Pr(Y_i = 1 \mid \mathbf{X}) = \text{logit}^{-1}(effect \times Z_{i1} + \frac{1}{2}Z_{i2}^2 + \frac{1}{3}Z_{i3}^3 + \frac{1}{4}Z_{i4}^2 + \varepsilon_y)$$

and generate:

$$Y_i \sim \text{Bern}[1, \Pr(Y_i = 1 \mid \mathbf{X})]$$

where  $\varepsilon_y \sim \mathcal{N}(0, sigma\_y)$ .

For "poisson", we simulate:

$$\lambda_Y = \exp(effect \times Z_{i1} + \frac{1}{2}Z_{i2}^2 + \frac{1}{3}Z_{i3}^3 + \frac{1}{4}Z_{i4}^2 + \varepsilon_y)$$

and generate:

$$Y_i \sim \text{Poisson}(\lambda_Y)$$

### Generating Predictions

To generate predicted outcomes for "mean" and "quantile", we simulate a continuous variable with mean given by the empirical mean of the training data and error variance given by the `sigma_y` argument.

For "ols", we fit a generalized additive model (GAM) on the simulated *training* dataset and calculate predictions for the *labeled* and *unlabeled* datasets as deterministic functions of  $\mathbf{X}$ . Specifically, we fit the following GAM:

$$Y^{\mathcal{T}} = s_0 + s_1(X_1^{\mathcal{T}}) + s_2(X_2^{\mathcal{T}}) + s_3(X_3^{\mathcal{T}}) + s_4(X_4^{\mathcal{T}}) + \varepsilon_p,$$

where  $\mathcal{T}$  denotes the *training* dataset,  $s_0$  is an intercept term, and  $s_1(\cdot)$ ,  $s_2(\cdot)$ ,  $s_3(\cdot)$ , and  $s_4(\cdot)$  are smoothing spline functions for  $X_1$ ,  $X_2$ ,  $X_3$ , and  $X_4$ , respectively, with three target equivalent degrees of freedom. Residual error is modeled as  $\varepsilon_p$ .

Predictions for *labeled* and *unlabeled* datasets are calculated as:

$$f(\mathbf{X}^{\mathcal{LUU}}) = \hat{s}_0 + \hat{s}_1(X_1^{\mathcal{LUU}}) + \hat{s}_2(X_2^{\mathcal{LUU}}) + \hat{s}_3(X_3^{\mathcal{LUU}}) + \hat{s}_4(X_4^{\mathcal{LUU}}),$$

where  $\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3,$  and  $\hat{s}_4$  are estimates of  $s_0, s_1, s_2, s_3,$  and  $s_4,$  respectively.

NOTE: For continuous outcomes, we provide optional arguments `shift` and `scale` to further apply a location shift and scaling factor, respectively, to the predicted outcomes. These default to `shift = 0` and `scale = 1`, i.e., no location shift or scaling.

For "logistic", we train k-nearest neighbors (k-NN) classifiers on the simulated *training* dataset for values of  $k$  ranging from 1 to 10. The optimal  $k$  is chosen via cross-validation, minimizing the misclassification error on the validation folds. Predictions for the *labeled* and *unlabeled* datasets are obtained by applying the k-NN classifier with the optimal  $k$  to  $\mathbf{X}$ .

Specifically, for each observation in the *labeled* and *unlabeled* datasets:

$$\hat{Y} = \operatorname{argmax}_c \sum_{i \in \mathcal{N}_k} I(Y_i = c),$$

where  $\mathcal{N}_k$  represents the set of  $k$  nearest neighbors in the training dataset,  $c$  indexes the possible classes (0 or 1), and  $I(\cdot)$  is an indicator function.

For "poisson", we fit a generalized linear model (GLM) with a log link function to the simulated *training* dataset. The model is of the form:

$$\log(\mu^{\mathcal{T}}) = \gamma_0 + \gamma_1 X_1^{\mathcal{T}} + \gamma_2 X_2^{\mathcal{T}} + \gamma_3 X_3^{\mathcal{T}} + \gamma_4 X_4^{\mathcal{T}},$$

where  $\mu^{\mathcal{T}}$  is the expected count for the response variable in the *training* dataset,  $\gamma_0$  is the intercept, and  $\gamma_1, \gamma_2, \gamma_3,$  and  $\gamma_4$  are the regression coefficients for the predictors  $X_1, X_2, X_3,$  and  $X_4,$  respectively.

Predictions for the *labeled* and *unlabeled* datasets are calculated as:

$$\hat{\mu}^{\mathcal{LUU}} = \exp(\hat{\gamma}_0 + \hat{\gamma}_1 X_1^{\mathcal{LUU}} + \hat{\gamma}_2 X_2^{\mathcal{LUU}} + \hat{\gamma}_3 X_3^{\mathcal{LUU}} + \hat{\gamma}_4 X_4^{\mathcal{LUU}}),$$

where  $\hat{\gamma}_0, \hat{\gamma}_1, \hat{\gamma}_2, \hat{\gamma}_3,$  and  $\hat{\gamma}_4$  are the estimated coefficients.

## Value

A data.frame containing n rows and columns corresponding to the labeled outcome (Y), the predicted outcome (f), a character variable (set\_label) indicating which data set the observation belongs to (training, labeled, or unlabeled), and four independent, normally distributed predictors (X1, X2, X3, and X4), where applicable.

## Examples

```
#-- Mean

dat_mean <- simdat(c(100, 100, 100),
  effect = 1, sigma_Y = 1,
  model = "mean"
)
```

```
head(dat_mean)

#-- Linear Regression

dat_ols <- simdat(c(100, 100, 100),
  effect = 1, sigma_Y = 1,
  model = "ols"
)

head(dat_ols)
```

---

sim_data_y	<i>Simulate the data for testing the functions</i>
------------	--

---

**Description**

sim\_data\_y for simulation with ML-predicted Y

**Usage**

```
sim_data_y(r = 0.9, binary = FALSE)
```

**Arguments**

r	imputation correlation
binary	simulate binary outcome or not

**Value**

simulated data

---

summary.ipd	<i>Summarize ipd fit</i>
-------------	--------------------------

---

**Description**

Summarize ipd fit

**Usage**

```
## S3 method for class 'ipd'
summary(object, ...)
```

**Arguments**

`object` An object of class `ipd`.  
`...` Ignored.

**Value**

An object of class `summary.ipd` containing:

**call** The model formula.

**coefficients** A glm-style table of estimates, SE, z, p.

**method** Which IPD method was used.

**model** Which downstream model was fitted.

**intercept** Logical; whether an intercept was included.

---

`tidy.ipd`

*Tidy an ipd fit*

---

**Description**

Tidy an ipd fit

**Usage**

```
## S3 method for class 'ipd'
tidy(x, ...)
```

**Arguments**

`x` An object of class `ipd`.  
`...` Ignored.

**Value**

A [tibble](#) with columns `term`, `estimate`, `std.error`, `conf.low`, `conf.high`.

**Examples**

```
dat <- simdat()

fit <- ipd(Y ~ f ~ X1, method = "pspa", model = "ols",
  data = dat, label = "set_label")

tidy(fit)
```

---

wls	<i>Weighted Least Squares</i>
-----	-------------------------------

---

**Description**

Computes the weighted least squares estimate of the coefficients.

**Usage**

```
wls(X, Y, w = NULL, return_se = FALSE)
```

**Arguments**

X	(matrix): n x p matrix of covariates.
Y	(vector): p-vector of outcome values.
w	(vector, optional): n-vector of sample weights.
return_se	(bool, optional): Whether to return the standard errors of the coefficients.

**Value**

(list): A list containing the following:

**theta** (vector): p-vector of weighted least squares estimates of the coefficients.  
**se** (vector): If return\_se == TRUE, return the p-vector of standard errors of the coefficients.

---

zconfint_generic	<i>Normal Confidence Intervals</i>
------------------	------------------------------------

---

**Description**

Calculates normal confidence intervals for a given alternative at a given significance level.

**Usage**

```
zconfint_generic(mean, std_mean, alpha, alternative)
```

**Arguments**

mean	(float): Estimated normal mean.
std_mean	(float): Estimated standard error of the mean.
alpha	(float): Significance level in [0,1]
alternative	(string): Alternative hypothesis, either 'two-sided', 'larger' or 'smaller'.

**Value**

(vector): Lower and upper (1 - alpha) \* 100% confidence limits.

---

`zstat_generic`*Compute Z-Statistic and P-Value*

---

**Description**

Computes the z-statistic and the corresponding p-value for a given test.

**Usage**

```
zstat_generic(value1, value2, std_diff, alternative, diff = 0)
```

**Arguments**

<code>value1</code>	(numeric): The first value or sample mean.
<code>value2</code>	(numeric): The second value or sample mean.
<code>std_diff</code>	(numeric): The standard error of the difference between the two values.
<code>alternative</code>	(character): The alternative hypothesis. Can be one of "two-sided" (or "2-sided", "2s"), "larger" (or "l"), or "smaller" (or "s").
<code>diff</code>	(numeric, optional): The hypothesized difference between the two values. Default is 0.

**Value**

(list): A list containing the following:

**zstat** (numeric): The computed z-statistic.

**pvalue** (numeric): The corresponding p-value for the test.

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