

# Package ‘kza’

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**Description**

Time Series Analysis including break detection, spectral analysis, KZ Fourier Transforms.

**Suggests** polynom

**SystemRequirements** fftw (>= 3.2.2)

**License** GPL-3

**NeedsCompilation** yes

**Repository** CRAN

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 kz *Kolmogorov-Zurbenko filter*


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**Description**

Kolmogorov-Zurbenko low-pass linear filter.

**Usage**

```
kz(x, m, k = 3)
```

**Arguments**

x	The raw data that will be smoothed. The data can have as many as 3 dimensions. KZ will also handle a time series.
m	Window size for the filter. This can be up to 3 dimensions, but not more than the dimensionality of the input data x.
k	Number of iterations.

**Details**

KZ is an iterated moving average. The filter can be used with missing values. One iteration is equivalent to a simple moving average. Three iterations is an approximately Gaussian shaped filter.

**References**

Zurbenko, I. G., 1986: The spectral Analysis of Time Series. North-Holland, 248 pp.

**Examples**

```
## 2 dimensions
set.seed(2)
a <- matrix(rep(0,100*100),nrow=100)
a[35:70,35:70]<-1
a <- a + matrix(rnorm(100*100,0,1),nrow=100)

z<-kz(a,m=c(20,5),k=3)
x <- seq(1,100)
y <- x
op <- par(bg = "white")

c="lightblue"
m="Unsmoothed"
persp(x, y, a, zlab="a", ticktype="detailed", theta = 60, phi = 45, col = c, main=m)

m="KZ(a,m=c(20,5),k=3)"
persp(x, y, z, zlab="z", ticktype="detailed", theta = 60, phi = 45, col = c, main=m)
```

```
#example
t <- seq(0,20,length=20*365)
set.seed(6); e <- rnorm(n = length(t), sd = 2.0)
y <- sin(3*pi*t) + e
z <- kz(y,30)

par(mfrow=c(2,1))
plot(y,ylim=c(-5,5),type="l",main="y = sin(3*pi*t) + noise")
plot(z,ylim=c(-5,5), type="l",main="KZ filter")
lines(sin(3*pi*t), col="blue")
par(mfrow=c(1,1))
```

---

kza

*Kolmogorov-Zurbenko Adaptive*


---

## Description

KZA will recover 2-dimensional or 3-dimensional image or signal buried in noise.

## Usage

```
kza(x, m, y = NULL, k = 3, min_size = round(0.05*m), tol = 1.0e-5, impute_tails = FALSE)
## S3 method for class 'kza'
plot(x, ...)
```

## Arguments

x	A vector of the time series or a matrix (2d) or an array (3d) of an image.
m	The window for the filter.
y	The filtered output from kz.
k	The number of iterations.
min_size	Minimum size of window q.
tol	The smallest value to accept as nonzero.
impute_tails	The default is to drop the tails.
...	Other parameters.

## Details

The selection of parameters of KZA depend on the nature of the data. This function may take a long time to run, depending on the number of dimensions and the size of the dimensions.

## Author(s)

Brian Close <brian.close@gmail.com> and Igor Zurbenko <IZurbenko@albany.edu>

## References

I. Zurbenko, P.S. Porter, S.T. Rao, J.Y. Ku, R. Gui, R.E. Eskridge Detecting Discontinuities in Time Series of Upper-air Data: Development and Demonstration of an Adaptive Filter Technique. Journal of Climate: (1996) Vol. 9, No. 12, pp. 3548-3560. <http://journals.ametsoc.org/action/doSearch?AllField=zurbenko&filter=AllField>

Kevin L. Civerolo, Elvira Brankov, S. T. Rao, Igor Zurbenko Assessing the impact of the acid deposition control program. Atmospheric Environment 35 (2001) 4135-4148 <http://www.elsevier.com/locate/atmosenv>

J.Chen, I.Zurbenko, Nonparametric Boundary detection, Communications in Statistics, Theory and Methods, Vol.26, 12, 2999-3014, 1997.

## Examples

```
#####
# this is an example of detection of a break point in a time series
#####
yrs <- 20
t <- seq(0,yrs,length=yrs*365)
m <- 365

#noise
e <- rnorm(n = length(t),0,1)
trend <- seq(0,-1,length=length(t))

#signal
bkpt <- 3452
brk <- c(rep(0,bkpt),rep(0.5,length(t)-bkpt))
signal <- trend + brk

# y = seasonal + trend + break point + noise
y <- sin(2*pi*t) + signal + e

k.kz <- kz(y,m)

# kza reconstruction of the signal
k.kza <- kza(y,m,y=k.kz,min_size=10)

par(mfrow=c(2,1))
plot(y,type="l", ylim=c(-3,3))
plot(signal,type="l",ylim=c(-3,3),
      main="Signal and KZA Reconstruction")
lines(k.kza$kza, col=4)

#####
# image detection (2d)
#####
set.seed(2)
a <- matrix(rep(0,100*100),nrow=100)
a[35:70,35:70]<-1
a <- a + matrix(rnorm(100*100,0,1),nrow=100)
y<-kz(a,m=15,k=3)
```

```

v <- kza(a,m=15,y=y,k=3,impute_tails=TRUE)

x <- seq(1,100)
y <- x
op <- par(bg = "white")

###
#noise
###
c="lightblue"
persp(x, y, a, zlab="z", zlim=c(-5,5), ticktype="detailed", theta=30, phi=30, col=c)

###
#kza filtered
###
persp(x,y,v$kza,zlab="z",zlim=c(-5,5),ticktype="detailed",theta=30,phi=30,col=c)

###
# another view
###
par(mfrow=c(1,2))
image(a,col=gray(seq(0,1,1/255)))
image(v$kza,col=gray(seq(0,1,1/255)))
par(mfrow=c(1,1))

```

---

kzft

*Kolmogorov-Zurbenko Fourier Transform*


---

## Description

Kolmogorov-Zurbenko Fourier Transform is an iterated Fourier transform.

## Usage

```

kzft(x, f=0, m=1, k=1)
coeff(m, k)
transfer_function(m, k, lamda = seq(-0.5,0.5,by=0.01), omega = 0 )

```

## Arguments

x	The raw data
f	The frequency that KZFT is applied at.
m	The window size for transform
k	The number of iterations for applying the KZFT
lamda	The frequencies used for the calculating the transfer function.
omega	The frequency that KZFT is applied at.

## Details

Kolmogorov-Zurbenko Fourier Transform (KZFT) is the Fourier transform applied over every segment of length  $m$  iterated  $k$  times. The argument `alg="F"` will use Fast Fourier Transforms written in C (fftw library). The `alg="C"` is a slow Fourier Transform but has the advantage of being able to handle missing values. It currently works in one dimension. The `alg="R"` is an R version of KZFT for experimental purposes. The `coeff` function generates the coefficients for the KZFT function.

You will introduce a phase shift and decrease the fidelity of the signal if the product of  $f*m$  is not an integer.

## References

- I. G. Zurbenko, The spectral Analysis of Time Series. North-Holland, 1986.
- I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998.
- R. Neagu, I. G. Zurbenko, Tracking and separating non-stationary multi-component chirp signals, J. Franklin Inst., 499-520, 2002.
- R. H. Shumway, D. S. Stoffer, Time Series Analysis and Its Applications: With R Examples, Springer, 2006.
- Wei Yang and Igor Zurbenko, `kzft`: Kolmogorov-Zurbenko Fourier Transform and Applications, R-Project 2007.
- Igor G. Zurbenko, Amy L. Potrzeba, Tidal Waves in Atmosphere and Their Effects, Acta Geophysica Volume 58, Number 2, 356-373

## See Also

[kzp](#), [kztp](#),

## Examples

```
# example taken from Wei Yang's KZFT package
# coefficients of kzft(201,5)

# function to calculate polynomial coefficients for kzft
## Not run:
a<-coeff(201,5);
t<-seq(1:1001)-501;
z<-cos(2*pi*0.025*t);
plot(z*a,type="l",xlab="Time", ylab="Coefficient", main="Coefficients of the kzft");
lines(a);
lines(-1*a);

## End(Not run)

# example taken from Wei Yang's KZFT package
# transfer function of the kzft(201,5) at frequency 0.025
lamda<-seq(-0.1,0.1,by=0.001)
tf1<-transfer_function(201,1,lamda,0.025)
tf2<-transfer_function(201,5,lamda,0.025)
```

```

matplot(lamda,cbind(log(tf1),log(tf2)),type="l",ylim=c(-15,0),
ylab="Natural log transformation of the coefficients",
xlab="Frequency (cycles/time unit)",
  main="Transfer function of kzft(201,5) at frequency 0.025")

# example with missing values
set.seed(2)
period=101
f<-1/period
t<-1:2000
s<-1*sin(2*pi*f*t)
x<-s
noise<-3*rnorm(length(t))
x<-s+noise
m=101

rand_idx <- sample(t,100,replace=FALSE)
x[rand_idx]<-NA
x<-as.vector(na.omit(x))

system.time(z1<-kzft(x, m=m, k=1, f=f))
system.time(z2<-kzft(x, m=m, k=2, f=f))
system.time(z3<-kzft(x, m=m, k=3, f=f))

par(mfrow=c(2,2))
plot(x,type="l",main="Original time series",xlab="t", ylab="y")
lines(s,col="blue")
plot(2*Re(z1),type="l",main="kzft(101,1)",xlab="t", ylab="y", ylim=c(-6,6))
lines(s,col="blue")
plot(2*Re(z2),type="l",main="kzft(101,2)",xlab="t", ylab="y", ylim=c(-6,6))
lines(s,col="blue")
plot(2*Re(z3),type="l",main="kzft(101,3)",xlab="t", ylab="y", ylim=c(-6,6))
lines(s,col="blue")
par(mfrow=c(1,1))

```

---

kzp

*Kolmogorov-Zurbenko Periodogram*


---

## Description

Kolmogorov-Zurbenko periodogram and smoothing using DiRienzo-Zurbenko (DZ).

## Usage

```

kzp(y, m=length(y), k=1)
## S3 method for class 'kzp'
smooth(object, log=TRUE, smooth_level=0.05, method = "DZ")
## S3 method for class 'kzp'
nonlinearity(x)
## S3 method for class 'kzp'

```

```

variation(x)
## S3 method for class 'kzp'
summary(object, digits=getOption("digits"), top=1, ...)
## S3 method for class 'kzp'
plot(x, ...)

```

### Arguments

y	The raw data.
m	The width of filtering window
k	The number of iterations for the KZFT
object	Output from kzp function.
log	Use logarithm values for smoothing.
smooth_level	Percentage of smoothness to apply.
method	Method used for smoothing; choices are "DZ" or "NZ".
digits	precision of output.
top	list top values
...	Other parameters.
x	periodogram

### Details

The Kolmogorov-Zurbenko Periodogram is an estimate of the spectral density using the Kolmogorov-Zurbenko Fourier Transform (KZFT).

### References

- I. G. Zurbenko, 1986: The spectral Analysis of Time Series. North-Holland, 248 pp.
- I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998.
- A. G. DiRienzo, I. G. Zurbenko, Semi-adaptive nonparametric spectral estimation, Journal of Computational and Graphical Statistics 8(1): 41-59, 1998.
- R. Neagu, I. G. Zurbenko, Algorithm for adaptively smoothing the log-periodogram, Journal of the Franklin Institute 340: 103-123, 2003.
- Wei Yang and Igor Zurbenko, kzft: Kolmogorov-Zurbenko Fourier Transform and Applications, R-Project 2007.

### See Also

[kzft](#), [kztp](#),

**Examples**

```
## Not run:
t<-1:6000
f1<-0.03
f2<-0.04
noise<-15*rnorm(length(t))
amp=1.5
s<-amp*sin(2*pi*f1*t)+amp*sin(2*pi*f2*t)
system.time(a<-kzp(s+noise,m=500,k=3))
b<-smooth.kzp(a, smooth_level=0.01)
par(mfrow=c(3,1))
plot(periodogram(s+noise),type='l')
plot(a)
plot(b)
par(mfrow=c(1,1))

# signal/noise
signal<-kzft(s+noise,m=500,k=3)
print(paste("signal-to-noise ratio = ", round(sqrt(var(2*Re(signal))/var(s+noise)),4) ))

summary(a, digits=2, top=2)

## End(Not run)
```

kzs

*Kolmogorov-Zurbenko Spline***Description**

Kolmogorov-Zurbenko Spline

**Usage**

kzs(y,m=NULL,k=3,t=NULL)

**Arguments**

y	data
m	smooth
k	The number of iterations for applying the KZFT
t	An indexing set

**Details**

Kolmogorov-Zurbenko Spline is essentially the Kolmogorov-Zurbenko Fourier Transform at the zero frequency.

## References

- I. G. Zurbenko, The spectral Analysis of Time Series. North-Holland, 1986.
- I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998.
- R. H. Shumway, D. S. Stoffer, Time Series Analysis and Its Applications: With R Examples, Springer, 2006.
- Derek Cyr and Igor Zurbenko, kzs: Kolmogorov-Zurbenko Spatial Smoothing and Applications, R-Project 2008.
- Derek Cyr and Igor Zurbenko, A Spatial Spline Algorithm and an Application to Climate Waves Over the United States, 2008 Joint Statistical Meetings.

## See Also

[kzft](#),

## Examples

```
n <- 1000
x <- (1:n)/n
true<-((exp(2.5*x)+sin(25*x))-1)/3

noise <- rnorm(n)
y <- true + noise

a<-kzs(y,m=60)

par(mfrow=c(2,1))
plot(y,type='l')
lines(true,col="red")

plot(a,type='l', ylim=c(-2,4))
lines(true,col="red")
par(mfrow=c(1,1))

#####
# second example
#####
t <- seq(from = -round(400*pi), to = round(400*pi), by = .25)
ts <- 0.5*sin(sqrt((2*pi*abs(t))/200))
signal <- ifelse(t < 0, -ts, ts)
et <- rnorm(length(t), mean = 0, sd = 1)
yt <- et + signal

b<-kzs(yt,m=400)
par(mfrow=c(2,1))
plot(yt,type='l')
lines(signal,col="red")

plot(b,type='l', ylim=c(-0.5,1))
lines(signal,col="red")
```

```
par(mfrow=c(1,1))
```

---

kzsv

*Kolmogorov-Zurbenko Adaptive filter with Sample Variance.*


---

### Description

Sample variance of a Kolmogorov-Zurbenko adaptive filter. You want a sigma of at least 3.

### Usage

```
kzsv(object)
## S3 method for class 'kzsv'
summary(object, digits=getOption("digits"), ...)
## S3 method for class 'kzsv'
plot(x, ...)
```

### Arguments

object	The resultant object from kza function.
x	The results of the kza function.
digits	Precision of output.
...	Other parameters.

### Examples

```
x <- c(rep(0,4000),rep(0.5,2000),rep(0,4000))
noise <- rnorm(n = 10000, sd = 1.0) # normally-distributed random variates
v <- x + noise
a<-kza(v, m=1000, k=3)
sv<-kzsv(a)
```

---

kztp

*Kolmogorov-Zurbenko Third-Order Periodogram*


---

### Description

Kolmogorov-Zurbenko Third-Order Periodogram for estimating spectrums

### Usage

```
kztp(x,m,k,box=c(0,0.5,0,0.5))
```

**Arguments**

x	The signal.
m	The window size for the kzft filter.
k	The number of iterations.
box	The window for the application of third-order periodgram.

**Details**

The Kolmogorov-Zurbenko Third-Order Periodogram is used to estimate spectral density of a signal. The smoothing methods are adaptive allowing the bandwidth of the spectral window to vary according to the smoothness of the underlying spectral density. For details, please see to DiRienzo and Zurbenko (1998) and Neagu and Zurbenko (2003).

**References**

I. G. Zurbenko, 1986: The spectral Analysis of Time Series. North-Holland, 248 pp. I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998. W. Yang, I. G. Zurbenko, A semi-adaptive smoothing algorithm in bispectrum estimation, Proceedings of the American Statistical Association, Seattle, 2006. Wei Yang and Igor Zurbenko, kzft: Kolmogorov-Zurbenko Fourier Transform and Applications, R-Project 2007.

**See Also**

[kzft](#), [kzp](#),

**Examples**

```
t<-1:10000
y<-2*sin(2*pi*0.1*t)+3*sin(2*pi*0.2*t) + 10*rnorm(length(t))

a<-kztp(y,50,1)
z<-log(Mod(a))
#z<-smooth.kzp(z)

omega<-seq(0,1,length=51)[2:26]
#filled.contour(omega,omega,s,xlab="freq",ylab="freq",main="Smoothed 3rd Order Periodogram")
```

---

periodogram

*Periodogram*

---

**Description**

Raw periodogram.

**Usage**

```
periodogram(y)
```

**Arguments**

y                    The raw data.

**Details**

Periodogram is an estimate of the spectral density using FFT.

**See Also**

[kzp](#),

**Examples**

```
t<-1:1000
f1<-0.3
f2<-0.4
noise<-15*rnorm(length(t))
s<-3*sin(2*pi*f1*t)+3*sin(2*pi*f2*t)
plot(periodogram(s+noise),type='l')
```

---

rlv

*Rolling Variance Function*

---

**Description**

Rolling variance function, (rlv) will detect data variations of 1-dimensional vector, 2-dimensional matrix and/or 3-dimensional array, which in turn display their local discontinuities (boundaries).

**Usage**

```
rlv(inpt, krnl)
```

**Arguments**

inpt                input data.  
krnl                length of the rolling window for detecting.

**Details**

The rlv is calculated based on a dynamic rolling window. Within a given window, directly estimating its variance allows to capture the local variations of the signal underlying analysis. Since this window is moving, when its size is optimal, the significant local variance atlas will be captured. It is neither necessary to make any assumption, nor to model anything, so it is a nonparametric statistic. However, direct application of rlv to data embedded in heavy background noises could fail to identify the significant local variations. In situations when the estimated data are embedded in background noise, necessary smoothness may be expected.

## References

Igor G Zurbenko, Mingzeng Sun. Estimation of spatial boundaries with rolling variance and 2D KZA algorithm. *Biometrics & Biostatistics International Journal*. 7(4):263.270, 2018

## Examples

```
## A. Signal/objects
# to create a cylinder
circleFun <- function(center = c(0,0),diameter = 1, npoints = 200){
  r = diameter / 2
  tt <- seq(0,2*pi,length.out = npoints)
  xx <- center[1] + r * cos(tt)
  yy <- center[2] + r * sin(tt)
  return(data.frame(x = xx, y = yy))
}
rxo1 <- circleFun(center = c(38, 38), diameter = 38, npoints = 200)
rxo2 <- circleFun(center = c(58, 58), diameter = 38, npoints = 200)
rxo3 <- circleFun(center = c(28, 78), diameter = 18, npoints = 200)
rxo1 <- circleFun(center = c(38, 38), diameter = 18, npoints = 200)
rxo2 <- circleFun(center = c(58, 58), diameter = 18, npoints = 200)
rxo3 <- circleFun(center = c(28, 78), diameter = 8, npoints = 200)

set.seed(3)
s2 <- matrix(rep(0,100*100),nrow=100)
for(i in 1:nrow(rxo1)) {
  s2[rxo1[i,2],38:rxo1[i,1]]=0.5
}
for(i in 1:nrow(rxo2)) {
  s2[rxo2[i,2],58:rxo2[i,1]]=0.5
}
for(i in 1:nrow(rxo3)) {
  s2[rxo3[i,2],28:rxo3[i,1]]=0.3
}
for(i in 1:nrow(rxo1)) {
  s2[rxo1[i,2],38:rxo1[i,1]]=1
}
for(i in 1:nrow(rxo2)) {
  s2[rxo2[i,2],58:rxo2[i,1]]=1
}
for(i in 1:nrow(rxo3)) {
  s2[rxo3[i,2],28:rxo3[i,1]]=0.6
}
signal=s2
## Graphing Library plotly
## Not run:
plot_ly(z=signal, type="surface")

## End(Not run)

## B. Rolling variance atlas
```

```
rolv=rlv(s2,3)
## Not run:
plot_ly(z=rolv, type="surface")

## End(Not run)
```

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