

# Package ‘mmtdiff’

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**Version** 1.0.0

**Title** Moment-Matching Approximation for t-Distribution Differences

**Description** Implements the moment-matching approximation for differences of non-standardized t-distributed random variables in both univariate and multivariate settings. The package provides density, distribution function, quantile function, and random generation for the approximated distributions of t-differences. The methodology establishes the univariate approximated distributions through the systematic matching of the first, second, and fourth moments, and extends it to multivariate cases, considering both scenarios of independent components and the more general multivariate t-distributions with arbitrary dependence structures. Methods build on the classical moment-matching approximation method (e.g., Casella and Berger (2024) <[doi:10.1201/9781003456285](https://doi.org/10.1201/9781003456285)>).

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mm\_tdiff\_multivariate\_general  
*Moment-Matching Approximation for General Multivariate t-Differences*

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## Description

Approximates the distribution of differences between two independent multivariate t-distributed random vectors with arbitrary covariance structure.

## Usage

```
mm_tdiff_multivariate_general(mu1, Sigma1, nu1, mu2, Sigma2, nu2)
```

## Arguments

mu1	Location vector of first distribution (length p)
Sigma1	Scale matrix of first distribution (p x p, positive definite)
nu1	Degrees of freedom of first distribution (must be > 4)
mu2	Location vector of second distribution (length p)
Sigma2	Scale matrix of second distribution (p x p, positive definite)
nu2	Degrees of freedom of second distribution (must be > 4)

## Details

This function handles the general case where components may be correlated within each multivariate t-distribution. The approximation uses a single scalar degrees of freedom parameter to capture the overall tail behavior.

Note: For high dimensions with heterogeneous component behaviors, consider using [mm\\_tdiff\\_multivariate\\_independent](#) instead.

## Value

An S3 object of class "mm\_tdiff\_multivariate\_general" containing:

mu_diff	Location vector of difference
Sigma_star	Scale matrix
nu_star	Degrees of freedom (scalar)
method	Character string "multivariate_general"

## Examples

```
Sigma1 <- matrix(c(1, 0.3, 0.3, 1), 2, 2)
Sigma2 <- matrix(c(1.5, 0.5, 0.5, 1.2), 2, 2)
result <- mm_tdiff_multivariate_general(
  mu1 = c(0, 1), Sigma1 = Sigma1, nu1 = 10,
  mu2 = c(0, 0), Sigma2 = Sigma2, nu2 = 15
)
print(result)
```

---

mm\_tdiff\_multivariate\_independent

*Moment-Matching Approximation for Multivariate t-Differences (Independent)*

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## Description

Approximates the distribution of differences between two independent  $p$ -dimensional vectors with independent  $t$ -distributed components.

## Usage

```
mm_tdiff_multivariate_independent(mu1, sigma1, nu1, mu2, sigma2, nu2)
```

## Arguments

mu1	Location vector of first distribution (length $p$ )
sigma1	Scale vector of first distribution (length $p$ , all $> 0$ )
nu1	Degrees of freedom vector of first distribution (length $p$ , all $> 4$ )
mu2	Location vector of second distribution (length $p$ )
sigma2	Scale vector of second distribution (length $p$ , all $> 0$ )
nu2	Degrees of freedom vector of second distribution (length $p$ , all $> 4$ )

## Details

This function applies the univariate moment-matching approximation component-wise when all components are mutually independent. Each component difference  $Z_j = X_{1j} - X_{2j}$  is approximated independently using the univariate method.

This approach is optimal for:

- Marginal inference on specific components
- Cases where components have different tail behaviors
- Maintaining computational efficiency in high dimensions

**Value**

An S3 object of class "mm\_tdiff\_multivariate\_independent" containing:

mu_diff	Location vector of difference
sigma_star	Vector of scale parameters
nu_star	Vector of degrees of freedom
p	Dimension of the vectors
method	Character string "multivariate_independent"

**See Also**

[mm\\_tdiff\\_multivariate\\_general](#) for correlated components

**Examples**

```
result <- mm_tdiff_multivariate_independent(
  mu1 = c(0, 1), sigma1 = c(1, 1.5), nu1 = c(10, 12),
  mu2 = c(0, 0), sigma2 = c(1.2, 1), nu2 = c(15, 20)
)
print(result)
```

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mm\_tdiff\_univariate    *Moment-Matching Approximation for Univariate t-Differences*

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**Description**

Approximates the distribution of the difference between two independent non-standardized t-distributed random variables using the moment-matching method.

**Usage**

```
mm_tdiff_univariate(mu1, sigma1, nu1, mu2, sigma2, nu2)
```

**Arguments**

mu1	Location parameter of first distribution
sigma1	Scale parameter of first distribution (must be > 0)
nu1	Degrees of freedom of first distribution (must be > 4)
mu2	Location parameter of second distribution
sigma2	Scale parameter of second distribution (must be > 0)
nu2	Degrees of freedom of second distribution (must be > 4)

## Details

For two independent non-standardized t-distributed random variables:

- $X1 \sim t(\mu1, \sigma1^2, \nu1)$
- $X2 \sim t(\mu2, \sigma2^2, \nu2)$

The difference  $Z = X1 - X2$  is approximated as:  $Z \sim t(\mu1 - \mu2, \sigma\_star^2, \nu\_star)$

where the effective parameters are computed through moment matching:

- $\sigma\_star$  is derived from the second moment matching
- $\nu\_star$  is derived from the fourth moment matching

The method requires  $\nu1 > 4$  and  $\nu2 > 4$  for the existence of fourth moments. The approximation quality improves as degrees of freedom increase and approaches exactness as  $\nu \rightarrow \infty$  (normal limit).

## Value

An S3 object of class "mm\_tdiff\_univariate" containing:

mu_diff	Location parameter of difference ( $\mu1 - \mu2$ )
sigma_star	Scale parameter
nu_star	Degrees of freedom
input_params	List of input parameters for reference
method	Character string "univariate"

## References

Yamaguchi, Y., Homma, G., Maruo, K., & Takeda, K. Moment-Matching Approximation for Difference of Non-Standardized t-Distributed Variables. (unpublished).

## See Also

[dtdiff](#), [ptdiff](#), [qtdiff](#), [rtdiff](#) for density, distribution function, quantile function, and random generation respectively

## Examples

```
# Example 1: Different scale parameters
result <- mm_tdiff_univariate(
  mu1 = 0, sigma1 = 1, nu1 = 10,
  mu2 = 0, sigma2 = 1.5, nu2 = 15
)
print(result)

# Example 2: Equal parameters (special case)
result_equal <- mm_tdiff_univariate(
  mu1 = 5, sigma1 = 2, nu1 = 20,
  mu2 = 3, sigma2 = 2, nu2 = 20
```

```
)
print(result_equal)
```

---

mvtdiff\_distributions *Distribution Functions for Multivariate Approximated t-Difference*

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### Description

Distribution Functions for Multivariate Approximated t-Difference

### Usage

```
dmvtdiff(x, mm_result, log = FALSE)

pmvtdiff(q, mm_result, lower.tail = TRUE)

rmvtdiff(n, mm_result)
```

### Arguments

x	Matrix of quantiles (n x p) or vector for single point
mm_result	Result from mm_tdiff_multivariate_general()
log	Logical; if TRUE, returns log density
q	Vector of quantiles (length p) for cumulative probability
lower.tail	Logical; if TRUE (default), probabilities are $P(X \leq x)$
n	Number of observations

### Details

These functions implement the distribution functions for the approximated multivariate t-difference based on Theorem 3 from the paper.

**\*\*Note on degrees of freedom:\*\***

- dmvtdiff uses the exact (non-integer) nu\_star from the paper
- pmvtdiff rounds nu\_star to the nearest integer due to mvtnorm::pmvt requirements. This introduces minimal approximation error when nu\_star > 10 (the recommended range).
- rmvtdiff uses the exact (non-integer) nu\_star

### Value

For dmvtdiff: Numeric vector of density values. For pmvtdiff: Numeric scalar of cumulative probability. For rmvtdiff: Matrix of random samples (n x p).

## Examples

```
# Setup
Sigma1 <- matrix(c(1, 0.3, 0.3, 1), 2, 2)
Sigma2 <- matrix(c(1.5, 0.5, 0.5, 1.2), 2, 2)
result <- mm_tdiff_multivariate_general(
  mu1 = c(0, 1), Sigma1 = Sigma1, nu1 = 10,
  mu2 = c(0, 0), Sigma2 = Sigma2, nu2 = 15
)

# Density at a point
dmvtdiff(c(0, 1), result)

# Density at multiple points
x_mat <- matrix(c(0, 1, -1, 0.5), nrow = 2, byrow = TRUE)
dmvtdiff(x_mat, result)

# Cumulative probability
pmvtdiff(c(0, 1), result)

# Random samples
samples <- rmvtdiff(100, result)
head(samples)
```

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tdiff\_distributions     *Distribution Functions for Approximated t-Difference*

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## Description

Distribution Functions for Approximated t-Difference

## Usage

```
dtdiff(x, mm_result)
```

```
ptdiff(q, mm_result)
```

```
qtdiff(p, mm_result)
```

```
rtdiff(n, mm_result)
```

## Arguments

x, q	Vector of quantiles
mm_result	Result from mm_tdiff_univariate()
p	Vector of probabilities
n	Number of observations

**Value**

For `dtdiff`: Numeric vector of density values. For `ptdiff`: Numeric vector of cumulative probabilities. For `qtdiff`: Numeric vector of quantiles. For `rtdiff`: Numeric vector of random samples from the approximated t-difference distribution.

**Examples**

```
result <- mm_tdiff_univariate(0, 1, 10, 0, 1.5, 15)
dtdiff(0, result)
ptdiff(0, result)
qtdiff(c(0.025, 0.975), result)
samples <- rtdiff(100, result)
```

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validate\_approximation

*Validate Moment-Matching Approximation*

---

**Description**

Validates the approximation quality by comparing moments of the approximated distribution with the theoretical moments.

**Usage**

```
validate_approximation(mm_result, n_sim = 10000, seed = NULL)
```

**Arguments**

<code>mm_result</code>	Result from any <code>mm_tdiff</code> function
<code>n_sim</code>	Number of simulations for validation (default: 10000)
<code>seed</code>	Random seed for reproducibility

**Value**

A list containing validation metrics

**Examples**

```
result <- mm_tdiff_univariate(0, 1, 10, 0, 1.5, 15)
validation <- validate_approximation(result)
print(validation)
```

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