

Package ‘nor1mix’

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Title Normal aka Gaussian 1-d Mixture Models

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Description Onedimensional Normal (i.e. Gaussian) Mixture Models (S3) Classes, for, e.g., density estimation or clustering algorithms research and teaching; providing the widely used Marron-Wand densities. Efficient random number generation and graphics. Fitting to data by efficient ML (Maximum Likelihood) or traditional EM estimation.

Imports stats, graphics

Suggests cluster, copula

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nor1mix-package	<i>Normal aka Gaussian 1-d Mixture Models</i>
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Description

Onedimensional Normal (i.e. Gaussian) Mixture Models (S3) Classes, for, e.g., density estimation or clustering algorithms research and teaching; providing the widely used Marron-Wand densities. Efficient random number generation and graphics. Fitting to data by efficient ML (Maximum Likelihood) or traditional EM estimation.

Details

The DESCRIPTION file:

```

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Title:        Normal aka Gaussian 1-d Mixture Models
Version:      1.3-3
Date:         2024-04-05
Authors@R:   c(person("Martin", "Maechler", role = c("aut", "cre"), email = "maechler@stat.math.ethz.ch", comment = c(OR
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Maintainer:   Martin Maechler <maechler@stat.math.ethz.ch>

```

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llnorMix	Likelihood, Parametrization and EM-Steps For 1D Normal Mixtures
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Note that direct Maximum Likelihood ML (via `optim()`) is typically much faster converging (and more reliably detecting convergence correctly), notably thanks to a smart re-parametrization: use `norMixMLE()`.

Author(s)

Martin Maechler [aut, cre] (<<https://orcid.org/0000-0002-8685-9910>>), Friedrich Leisch [ctb] (`norMixEM()`, <<https://orcid.org/0000-0001-7278-1983>>), Erik Jørgensen [ctb] (`pnorMix()`, `qnorMix()`)

Maintainer: Martin Maechler <maechler@stat.math.ethz.ch>

See Also

The Marron-Wand examples of normal (gaussian) mixtures `MarronWand`.

Multivariate distributions from copulas `Mvdc` from the `copula` package can use `norMix` marginals.

Examples

```
example(dnorMix)
```

clus2norMix

Transform Clustering / Grouping to Normal Mixture

Description

Simple transformation of a clustering or grouping to a normal mixture object (class "norMix", see, `norMix`).

Usage

```
clus2norMix(gr, x, name = deparse(sys.call()))
```

Arguments

gr a grouping/clustering vector with values in $\{1, \dots, K\}$; possibly a [factor](#).
x numeric vector of (original) data (of the same length as **gr**).
name name for [norMix\(\)](#) object; constructed from the call by default.

Value

A call to [norMix\(\)](#) with (μ, sig^2, w) set to the empirical values of the groups (as defined by [split\(x, gr\)](#)).

Note

Via this function, any simple clustering algorithm (such [pam](#)) can be used as simple mixture model fitting procedure.

Author(s)

Martin Maechler, Dec. 2007

See Also

[norMix](#); further [pam\(\)](#) (or [clara\(\)](#)) from package **cluster** for sensible clusterings.

Examples

```
x9 <- rnorMix(500, MW.nm9)
require("cluster")
pxc <- pam(x9, k=3)
plot(pxc, which = 2)# silhouette

(nm.p9 <- clus2norMix(pxc$clustering, x9))
plot(nm.p9, p.norm=FALSE)
lines(MW.nm9, col="thistle")
```

dnorMix

Normal Mixture Density

Description

Evaluate the density function of the normal mixture specified as [norMix](#) object.

Usage

```
dnorMix(x, obj, log = FALSE)

dnorMixL(obj, x = NULL, log = FALSE, xlim = NULL, n = 511)
dpnorMix(x, obj, lower.tail = TRUE)
```

Arguments

obj	an object of class <code>norMix</code> .
x	numeric vector with abscissa values where to evaluate the density (and probability, for <code>dpnorMix()</code>). For <code>dnorMixL()</code> by default, when <code>NULL</code> , it is constructed from <code>n</code> (and <code>xlim</code> if that is specified).
log	logical indicating <i>log</i> -density values should be returned.
xlim	range of abscissa values, used if <code>x == NULL</code> . By default, <code>xlim</code> is taken as mean plus/minus 3 standard deviations of the normal mixture.
n	number of abscissa values to generate if <code>x</code> is not specified.
lower.tail	logical; if <code>TRUE</code> (default), probabilities are $P[X \leq x]$, otherwise, $P[X > x]$.

Value

`dnorMix(x)` returns the numeric vector of density values $f(x)$, logged if `log` is `TRUE`.

`dnorMixL()` returns a list with components

x	the abscissa values.
y	the density values $f(x)$ as for <code>dnorMix()</code> .

`dpnorMix()` returns a list with components

d	the density values $f(x)$ as for <code>dnorMix()</code> .
p	the probability values $F(x)$ as for <code>pnorMix()</code> .

See Also

[rnormMix](#) for random number generation, and [normMix](#) for the construction and further methods, particularly [plot.normMix](#) which makes use `dnorMix`.

Examples

```
ff <- dnorMixL(MW.nm7)
str(ff)
plot(ff, type = "h", ylim = c(0,1)) # rather use plot(ff, ...)

x <- seq.int(-4,5, length.out = 501)
dp <- dpnorMix(x, MW.nm7)
lines(x, dp$d, col = "tomato", lwd=3)
lines(x, dp$p, col = 3, lwd=2)# does not fit y-wise
stopifnot(all.equal(dp$d, dnorMix(x, MW.nm7), tolerance=1e-12),
          all.equal(dp$p, pnorMix(x, MW.nm7), tolerance=1e-12))
```

Description

These functions work with an almost unconstrained parametrization of univariate normal mixtures.

`llnorMix(p, *)` computes the log likelihood,

`obj <- par2norMix(p)` maps parameter vector `p` to a `norMix` object `obj`,

`p <- nM2par(obj)` maps from a `norMix` object `obj` to parameter vector `p`,

where `p` is always a parameter vector in our parametrization.

Partly for didactical reasons, the following functions provide the basic ingredients for the EM algorithm (see also `norMixEM`) to parameter estimation:

`estep.nm(x, obj, p)` computes 1 E-step for the data `x`, given *either* a "norMix" object `obj` or parameter vector `p`.

`mstep.nm(x, z)` computes 1 M-step for the data `x` and the probability matrix `z`.

`emstep.nm(x, obj)` computes 1 E- and 1 M-step for the data `x` and the "norMix" object `obj`.

where again, `p` is a parameter vector in our parametrization, `x` is the (univariate) data, and `z` is a $n \times m$ matrix of (posterior) conditional probabilities, and θ is the full parameter vector of the mixture model.

Usage

```
llnorMix(p, x, m = (length(p) + 1)/3, trafo = c("clr1", "logit"))
```

```
par2norMix(p, trafo = c("clr1", "logit"), name = )
```

```
nM2par(obj, trafo = c("clr1", "logit"))
```

```
estep.nm(x, obj, par)
```

```
mstep.nm(x, z)
```

```
emstep.nm(x, obj)
```

Arguments

<code>p, par</code>	numeric vector: our parametrization of a univariate normal mixture, see details.
<code>x</code>	numeric: the data for which the likelihood is to be computed.
<code>m</code>	integer number of mixture components; this is not to be changed for a given <code>p</code> .
<code>trafo</code>	<code>character</code> string specifying the transformation of the component weight w m -vector (mathematical notation in <code>norMix</code> : $\pi_j, j = 1, \dots, m$) to an $m - 1$ -dimensional unconstrained parameter vector in our parametrization. "logit" has been hard-wired upto <code>nor1mix</code> version 1.2-3, and has been replaced <i>as default</i> in 2019 for <code>nor1mix</code> version 1.2-4 by "clr1" which is more symmetric and basically Aitchinson's centered log ratio, see also CRAN package <code>compositions</code> ' <code>clr()</code> .

name	(for par2norMix():) a name for the "norMix" object that is returned, uses a smart default.
obj	a "norMix" object, see norMix .
z	a $n \times m$ matrix of (posterior) conditional probabilities, $z_{ij} = P(x_i \in C_j \theta)$, where C_j denotes the j -th group ("cluster").

Details

We use a parametrization of a (finite) m -component univariate normal mixture which is particularly apt for likelihood maximization, namely, one whose parameter space is *almost* a full \mathbf{R}^M , $M = 3m - 1$.

For an m -component mixture, we map to and from a parameter vector θ ($= \mathbf{p}$ as \mathbf{R} -vector) of length $3m - 1$. For mixture density

$$\sum_{j=1}^m \pi_j \phi((t - \mu_j)/\sigma_j)/\sigma_j,$$

we transform the π_j (for $j \in 1, \dots, m$) via the transform specified by `trafo` (see below), and log-transform the σ_j . Consequently, θ is partitioned into

`p[1:(m-1)]`: For

`trafo = "logit"`: `p[j]= logit(π_{j+1})` and π_1 is given implicitly as $\pi_1 = 1 - \sum_{j=2}^m \pi_j$.

`trafo = "clr1"`: (**centered log ratio, omitting 1st element**): Set $\ell_j := \ln(\pi_j)$ for $j = 1, \dots, m$, and `p[j]= $\ell_{j+1} - 1/m \sum_{j'=1}^m \ell_{j'}$` for $j = 1, \dots, m - 1$.

`p[m:(2m-1)]`: `p[m-1+ j]= μ_j` , for $j=1:m$.

`p[2m:(3m-1)]`: `p[2*m-1+ j] = $\log(\sigma_j)$` , i.e., $\sigma_j^2 = \exp(2 * p[. + j])$.

Value

`llnorMix()` returns a number, namely the log-likelihood.

`par2norMix()` returns "norMix" object, see [norMix](#).

`nM2par()` returns the parameter vector θ of length $3m - 1$.

`estep.nm()` returns `z`, the matrix of (conditional) probabilities.

`mstep.nm()` returns the model parameters as a [list](#) with components `w`, `mu`, and `sigma`, corresponding to the arguments of `norMix()`. (and see the 'Examples' on using `do.call(norMix, *)` with it.)

`emstep.nm()` returns an *updated* "norMix" object.

Author(s)

Martin Maechler

See Also

[norMix](#), [logLik](#). Note that the log likelihood of a "norMix" object is directly given by `sum(dnorMix(x, obj, log=TRUE))`.

To fit, using the EM algorithm, rather use `norMixEM()` than the `e.step`, `m.step`, or `em.step` functions.

Note that direct likelihood maximization, i.e., MLE, is typically considerably more efficient than the EM, and typically converges well with our parametrization, see [norMixMLE](#).

Examples

```
(obj <- MW.nm10) # "the Claw" -- m = 6 components
length(pp <- nM2par(obj)) # 17 == (3*6) - 1
par2norMix(pp)
## really the same as the initial 'obj' above

## Log likelihood (of very artificial data):
llnorMix(pp, x = seq.int(-2, 2, length.out = 1000))
set.seed(47)## of more realistic data:
x <- rnorMix(1000, obj)
llnorMix(pp, x)

## Consistency check : nM2par() and par2norMix() are inverses
all.EQ <- function(x,y, tol = 1e-15, ...) all.equal(x,y, tolerance=tol, ...)
stopifnot(all.EQ(pp, nM2par(par2norMix(pp))),
          all.EQ(obj, par2norMix(nM2par(obj))),
          check.attributes=FALSE),
          ## Direct computation of log-likelihood:
          all.EQ(sum(dnorMix(x, obj, log=TRUE)),
                 llnorMix(pp, x)) )

## E- and M- steps : -----
rE1 <- estep.nm(x, obj)
rE2 <- estep.nm(x, par=pp) # the same as rE1
z <- rE1
str( rM <- mstep.nm(x, z))
    (rEM <- emstep.nm(x, obj))

stopifnot(all.EQ(rE1, rE2),
          all.EQ(rEM, do.call(norMix, c(rM, name=""))))
```

Description

The fifteen density examples used in Marron and Wand (1992)'s simulation study have been used in quite a few subsequent studies, can all be written as normal mixtures and are provided here for convenience and didactical examples of normal mixtures. Number 16 has been added by Jansen et al.

Usage

```

MW.nm1 # Gaussian
MW.nm2 # Skewed
MW.nm2.old # Skewed(old)
MW.nm3 # Str Skew
MW.nm4 # Kurtotic
MW.nm5 # Outlier
MW.nm6 # Bimodal
MW.nm7 # Separated (bimodal)
MW.nm8 # Asymmetric Bimodal
MW.nm9 # Trimodal
MW.nm10 # Claw
MW.nm11 # Double Claw
MW.nm12 # Asymmetric Claw
MW.nm13 # Asymm. Double Claw
MW.nm14 # Smooth Comb
MW.nm15 # Discrete Comb
MW.nm16 # Distant Bimodal

```

Author(s)

Martin Maechler

Source

They have been translated from Steve Marron's Matlab code, now at <https://marronwebfiles.sites.oasis.unc.edu/OldResearch/parameters/nmpar.m>, however for number 2, the Matlab code had MW.nm2.old; and I've defined MW.nm2 as from the Annals paper; see also the last example below.

References

Marron, S. and Wand, M. (1992) Exact Mean Integrated Squared Error; *Annals of Statistics* **20**, 712–736.

For number 16,

P. Janssen, J. S. Marron, N. Veraverbeke and W. Sarle (1995) Scale measures for bandwidth selection; *Journal of Nonparametric Statistics* **5**, 359–380. doi:10.1080/10485259508832654

Examples

```

MW.nm10
plot(MW.nm14)

## These are defined as norMix() calls in ../R/zMarrWand-dens.R
nms <- ls(pattern = "^MW.nm", "package:nor1mix")
nms <- nms[order(as.numeric(substring(nms,6)))] # w/ warning for "2.old"
for(n in nms) {
  cat("\n",n,"\n"); print(get(n, "package:nor1mix"))
}

```

```

## Plot all of them:
op <- par(mfrow=c(4,4), mgp = c(1.2, 0.5, 0), tcl = -0.2,
          mar = .1 + c(2,2,2,1), oma = c(0,0,3,0))
for(n in nms[-17]) plot(get(n, "package:nor1mix"))
mtext("The Marron-Wand Densities", outer= TRUE, font= 2, cex= 1.6)

## and their Q-Q-plots (not really fast):
prob <- ppoints(N <- 100)
for(n in nms[-17]) # qnorMix() using monotone spline inversion ==> warning
  qqnorm(qnorMix(prob, get(n, "package:nor1mix")), main = n)
mtext("QQ-plots of Marron-Wand Densities", outer = TRUE,
      font = 2, cex = 1.6)
par(op)

## "object" overview:
cbind(sapply(nms, function(n) { o <- get(n)
  sprintf("%-18s: K =%2d; rng = [%3.1f, %2.1f]",
          attr(o, "name"), nrow(o),
          min(o[, "mu"] - 3*o[, "sigma"]),
          max(o[, "mu"] + 3*o[, "sigma"]) )
}))

## Note that Marron-Wand (1992), p.720 give #2 as
MW.nm2
## the parameters of which at first look quite different from
MW.nm2.old
## which has been the definition in the above "Source" Matlab code.
## It's easy to see that  $\mu_{\{nm2\}} = -.3 + 1.2 * \mu_{\{paper\}}$ ,
## and correspondingly,  $s2_{\{nm2\}} = 1.2^2 * s2_{\{paper\}}$ 
## such that they are "identical" apart from scale and location:
op. <- par(mfrow=2:1, mgp= c(1.2,0.5,0), tcl= -0.2, mar=.1+c(2,2,2,1))
plot(MW.nm2)
plot(MW.nm2.old)
par(op.)

```

norMix

Mixtures of Univariate Normal Distributions

Description

Objects of class `norMix` represent finite mixtures of (univariate) normal (aka Gaussian) distributions. Methods for construction, printing, plotting, and basic computations are provided.

Usage

```

norMix(mu, sig2 = rep(1,m), sigma = rep(1,m),
       w = NULL, name = NULL, long.name = FALSE)

```

```

is.norMix(obj)
m.norMix(obj)
var.norMix(x, ...)
## S3 method for class 'norMix'
mean(x, ...)
## S3 method for class 'norMix'
print(x, ...)
## S3 method for class 'norMix'
x[i,j, drop=TRUE]

```

Arguments

mu	numeric vector of length K , say, specifying the means μ of the K normal components.
sig2	deprecated! numeric vector of length K , specifying the variances σ^2 of the K normal components. Do specify sigma instead!
sigma	numeric vector of length K , specifying the standard deviations σ of the K normal components.
w	numeric vector of length K , specifying the mixture proportions π_j of the normal components, $j = 1, \dots, K$. Defaults to equal proportions
name	optional name tag of the result (used for printing).
long.name	logical indicating if the name attribute should use punctuation and hence be slightly larger than by default.
obj, x	an object of class norMix.
i, j, drop	for indexing, see the generic <code>[</code> extractor function.
...	further arguments passed to methods.

Details

The (one dimensional) normal mixtures, R objects of class "norMix", are constructed by `norMix` and tested for by `is.norMix`. `m.norMix()` returns the number of mixture components; the `mean()` method for class "norMix" returns the (theoretical / true) mean $E[X]$ and `var.norMix()` the true variance $E[(X - E[X])^2]$ where $X \sim \langle norm.mixt \rangle$.

The subsetting aka "extract" method (`x[i, j]`; for generic `[`)—when called as `x[i,]`—will typically return a "norMix" object unless matrix indexing selects only one row in which case `x[i, , drop=FALSE]` will return the normal mixture (of one component only).

For further methods (density, random number generation, fitting, ...), see below.

Value

`norMix` returns objects of class "norMix" which are currently implemented as 3-column matrix with column names `mu`, `sigma`, and `w`, and further attributes. The user should rarely need to access the underlying structure directly.

Note

For *estimation* of the parameters of such a normal mixture, we provide a smart parametrization and an efficient implementation of the direct MLE or also the EM algorithm, see `norMixMLE()` which includes `norMixEM()`.

Author(s)

Martin Maechler

See Also

`dnorMix` for the density, `pnorMix` for the cumulative distribution and the quantile function (`qnorMix`), and `rnorMix` for random numbers and `plot.norMix`, the plot method.

`MarronWand` has the Marron-Wand densities as normal mixtures.

`norMixMLE()` and `norMixEM()` provide *fitting* of univariate normal mixtures to data.

Examples

```
ex <- norMix(mu = c(1,2,5))# defaults: sigma = 1, equal proportions ('w')
ex
plot(ex, p.comp = TRUE)# looks like a mixture of only 2; 'p.comp' plots components

## The 2nd Marron-Wand example, see also ?MW.nm2
ex2 <- norMix(name = "#2 Skewed",
              mu = c(0, .5, 13/12),
              sigma = c(1, 2/3, 5/9),
              w = c(.2, .2, .6))

m.norMix (ex2)
mean      (ex2)
var.norMix(ex2)
(e23 <- ex2[2:3,]) # (with re-normalized weights)
stopifnot(is.norMix(e23),
          all.equal(var.norMix(ex2), 719/1080, tol=1e-14),
          all.equal(var.norMix(ex ), 35/9, tol=1e-14),
          all.equal(var.norMix(ex[2:3,]), 13/4, tol=1e-14),
          all.equal(var.norMix(e23), 53^2/(12^3*4),tol=1e-14)
)

plot(ex2, log = "y")# maybe "revealing"
```

norMix2call

Transform "norMix" object into Call, Expression or Function

Description

E.g., for taking symbolic derivatives, it may be useful to get an R *call*, *expression*, or *function* in / of x from a specific "`norMix`" object.

Usage

```
norMix2call(obj, oneArg = TRUE)
## S3 method for class 'norMix'
as.expression(x, oneArg = TRUE, ...)
## S3 method for class 'norMix'
as.function(x, oneArg = TRUE, envir = parent.frame(), ...)
```

Arguments

`obj, x` an R object of class "norMix".

`oneArg` **logical** specifying if expressions of the form `dnorm((x - mu)/sig)` should be used, i.e. **one Argument** only, instead of `dnorm(x, mu, sig)`.

`envir` an **environment**; often the default is perfect.

`...` potentially further arguments (not used in any examples yet).

Value

according to the function used, an R 'language' object, i.e., a **call**, **expression**, or **function**, respectively.

Author(s)

Martin Maechler

See Also

`norMix`. Note that `deriv()` currently only works correctly in case of the default `oneArg = TRUE`.

Examples

```
(cMW2 <- norMix2call(MW.nm2))
deriv(cMW2, "x")

(fMW1 <- as.function (MW.nm1))
(eMW3 <- as.expression(MW.nm3))
stopifnot(is.call (cMW2), is.call(norMix2call(MW.nm2, FALSE)),
          is.function (fMW1), is.function (as.function (MW.nm4)),
          is.expression(eMW3), is.expression(as.expression(MW.nm5))
)
```

norMixFit

*EM and MLE Estimation of Univariate Normal Mixtures***Description**

These functions estimate the parameters of a univariate (finite) normal mixture using the EM algorithm or Likelihood Maximimization via `optim(..., method = "BFGS")`.

Usage

```
norMixEM(x, m, name = NULL, sd.min = 1e-07* diff(range(x))/m,
         trafo = c("clr1", "logit"),
         maxiter = 100, tol = sqrt(.Machine$double.eps), trace = 1)

norMixMLE(x, m, name = NULL,
          trafo = c("clr1", "logit"),
          maxiter = 100, tol = sqrt(.Machine$double.eps), trace = 2)
```

Arguments

<code>x</code>	numeric: the data for which the parameters are to be estimated.
<code>m</code>	integer or factor: If <code>m</code> has length 1 it specifies the number of mixture components, otherwise it is taken to be a vector of initial cluster assignments, see details below.
<code>name</code>	character, passed to <code>norMix</code> . The default, <code>NULL</code> , uses <code>match.call()</code> .
<code>sd.min</code>	number: the minimal value that the normal components' standard deviations (<code>sd</code>) are allowed to take. A warning is printed if some of the final <code>sd</code> 's are this boundary.
<code>trafo</code>	character string specifying the transformation of the component weight w m -vector (mathematical notation in <code>norMix</code> : $\pi_j, j = 1, \dots, m$) to an $(m - 1)$ -dimensional unconstrained parameter vector in our parametrization. See <code>nM2par</code> for details.
<code>maxiter</code>	integer: maximum number of EM iterations.
<code>tol</code>	numeric: EM iterations stop if relative changes of the log-likelihood are smaller than <code>tol</code> .
<code>trace</code>	integer (or logical) specifying if the iterations should be traced and how much output should be produced. The default, 1 prints a final one line summary, where <code>trace = 2</code> produces one line of output per iteration.

Details

Estimation of univariate mixtures can be very sensitive to initialization. By default, `norMixEM` and `norMixMLE` `cut` the data into m groups of approximately equal size. See examples below for other initialization possibilities.

The EM algorithm consists in repeated application of E- and M- steps until convergence. Mainly for didactical reasons, we also provide the functions `estep.nm`, `mstep.nm`, and `emstep.nm`.

The MLE, Maximum Likelihood Estimator, maximizes the likelihood using `optim`, using the same advantageous parametrization as `llnorMix`.

Value

An object of class `norMix`.

Author(s)

EM: Friedrich Leisch, originally; Martin Maechler vectorized it in `m`, added `trace` etc.

MLE: M.Maechler

Examples

```
## use (mu, sigma)
ex <- norMix(mu = c(-1,2,5), sigma = c(1, 1/sqrt(2), sqrt(3)))
tools::assertWarning(verbose=TRUE,
  ## *deprecated* (using 'sig2' will *NOT* work in future!)
  ex. <- norMix(mu = c(-1,2,5), sig2 = c(1, 0.5, 3))
)
stopifnot(all.equal(ex, ex.))
plot(ex, col="gray", p.norm=FALSE)

x <- rnorMix(100, ex)
lines(density(x))
rug(x)

## EM estimation may fail depending on random sample
ex1 <- norMixEM(x, 3, trace=2) #-> warning (sometimes)
ex1
plot(ex1)

## initialization by cut() into intervals of equal length:
ex2 <- norMixEM(x, cut(x, 3))
ex2

## initialization by kmeans():
k3 <- kmeans(x, 3)$cluster
ex3 <- norMixEM(x, k3)
ex3

## Now, MLE instead of EM:
exM <- norMixMLE(x, k3, tol = 1e-12, trace=4)
exM

## real data
data(faithful)
plot(density(faithful$waiting, bw = "SJ"), ylim=c(0,0.044))
rug(faithful$waiting)
```

```
(nmF <- norMixEM(faithful$waiting, 2))
lines(nmF, col=2)
## are three components better?
nmF3 <- norMixEM(faithful$waiting, 3, maxiter = 200)
lines(nmF3, col="forestgreen")
```

plot.norMix

Plotting Methods for 'norMix' Objects

Description

The `plot` and `lines` methods for `norMix` objects draw the normal mixture density, optionally additionally with a fitted normal density.

Usage

```
## S3 method for class 'norMix'
plot(x, type = "l", n = 511, xout = NULL, xlim = NULL, ylim,
     xlab = "x", ylab = "f(x)", main = attr(x, "name"), lwd = 1.4,
     p.norm = !p.comp, p.h0 = TRUE, p.comp = FALSE,
     parNorm = list(col = 2, lty = 2, lwd = 0.4),
     parH0 = list(col = 3, lty = 3, lwd = 0.4),
     parComp = list(col= "blue3", lty = 3, lwd = 0.4), ...)

## S3 method for class 'norMix'
lines(x, type = "l", n = 511, xout = NULL,
      lwd = 1.4, p.norm = FALSE, parNorm = list(col = 2, lty = 2, lwd = 0.4),
      ...)
```

Arguments

<code>x</code>	object of class <code>norMix</code> .
<code>type</code>	character denoting type of plot, see, e.g. lines .
<code>n</code>	number of points to generate if <code>xout</code> is unspecified.
<code>xout</code>	numeric or <code>NULL</code> giving the abscissae at which to draw the density.
<code>xlim</code>	range of <code>x</code> values to use; particularly important if <code>xout</code> is not specified where <code>xlim</code> is passed to dnorMix and gets a smart default if unspecified.
<code>ylim</code>	range of <code>y</code> values to use; by default, if not specified (or containing <code>NA</code>), a smart default is used.
<code>xlab, ylab</code>	labels for the <code>x</code> and <code>y</code> axis with defaults.
<code>main</code>	main title of plot, defaulting to the <code>norMix</code> name.
<code>lwd</code>	line width for plotting with a non-standard default.
<code>p.norm</code>	logical indicating if the normal density with the same mean and variance should be drawn as well.

p.h0	logical indicating if the line $y = 0$ should be drawn.
p.comp	logical indicating if the Gaussian components should also be drawn individually.
parNorm	graphical parameters for drawing the normal density if p.norm is true.
parH0	graphical parameters for drawing the line $y = 0$ if p.h0 is true.
parComp	graphical parameters for drawing the single components if p.comp is true.
...	further arguments passed to and from methods.

Author(s)

Martin Maechler

See Also

[norMix](#) for the construction and further methods, particularly [dnorMix](#) which is used here.

Examples

```
plot(norMix(m=c(0,3), sigma = c(2,1))) # -> var = c(2^2, 1) = c(4, 1)

plot(MW.nm4, p.norm=FALSE, p.comp = TRUE)
plot(MW.nm4, p.norm=FALSE, p.comp = TRUE, ylim = c(0, 2))# now works
stopifnot(all.equal(c(0,2), par("yaxp")[1:2], tol= 1e-15))

## Further examples in ?norMix and ?rnormMix
```

pnorMix

Normal Mixture Cumulative Distribution and Quantiles

Description

Compute cumulative probabilities or quantiles (the inverse) for a normal mixture specified as [norMix](#) object.

Usage

```
pnorMix(q, obj, lower.tail = TRUE, log.p = FALSE)

qnorMix(p, obj, lower.tail = TRUE, log.p = FALSE,
  tol = .Machine$double.eps^0.25, maxiter = 1000, traceRootsearch = 0,
  method = c("interpQspline", "interspline", "eachRoot", "root2"),
  l.interp = pmax(1, pmin(20, 1000 / m)), n.mu.interp = 100)
```

Arguments

<code>obj</code>	an object of class <code>norMix</code> .
<code>p</code>	numeric vector of probabilities. Note that for all methods but "eachRoot", <code>qnorMix(p, *)</code> works with the full vector <code>p</code> , typically using (inverse) interpolation approaches; consequently the result is very slightly dependent on <code>p</code> as a whole.
<code>q</code>	numeric vector of quantiles
<code>.</code>	
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$, otherwise, $P[X > x]$.
<code>log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as $\log(p)$.
<code>tol, maxiter</code>	tolerance and maximal number of iterations for the root search algorithm, see method below and uniroot .
<code>traceRootsearch</code>	logical or integer in $\{0, 1, 2, 3\}$, determining the amount of information printed during root search.
<code>method</code>	a string specifying which algorithm is used for the "root search". Originally, the only method was a variation of "eachRoot", which is the default now when only very few quantiles are sought. For large <code>m.norMix()</code> , the default is set to "root2", currently.
<code>l.interp</code>	positive integer for <code>method = "interQpspline"</code> or <code>"interpspline"</code> , determining the number of values in each "mu-interval".
<code>n.mu.interp</code>	positive integer for <code>method = "interQpspline"</code> or <code>"interpspline"</code> , determining the (maximal) number of mu-values to be used as knots for inverse interpolation.

Details

Whereas the distribution function `pnorMix` is the trivial sum of weighted normal probabilities ([pnorm](#)), its inverse, `qnorMix` is computed numerically: For each `p` we search for `q` such that `pnorMix(obj, q) == p`, i.e., $f(q) = 0$ for $f(q) := \text{pnorMix}(\text{obj}, q) - p$. This is a root finding problem which can be solved by [uniroot](#)(`f`, `lower`, `upper`, `*`). If `length(p) <= 2` or `method = "eachRoot"`, this happens one for one for the *sorted* `p`'s. Otherwise, we start by doing this for the outermost non-trivial ($0 < p < 1$) values of `p`.

For `method = "interQpspline"` or `"interpspline"`, we now compute `p. <- pnorMix(q., obj)` for values `q.` which are a grid of length `l.interp` in each interval $[q_j, q_{j+1}]$, where q_j are the "X-extremes" plus (a sub sequence of length `n.mu.interp` of) the ordered `mu[j]`'s. Then, we use *montone* inverse interpolation ([splinefun](#)(`q.`, `p.`, `method="monoH.FC"`)) plus a few (maximally `maxiter`, typically one!) Newton steps. The default, "interQpspline", additionally logit-transforms the `p.` values to make the interpolation more linear. This method is faster, particularly for large `length(p)`.

Value

a numeric vector of the same length as `p` or `q`, respectively.

Author(s)

Very first version (for length-1 p, q) by Erik Jørgensen <Erik.Jorgensen@agrsci.dk>.

See Also

[dnorMix](#) for the density function.

Examples

```
MW.nm3 # the "strange skew" one
plot(MW.nm3)
## now the cumulative :
x <- seq(-4,4, length.out = 1001)
plot(x, pnorMix(x, MW.nm3), type="l", col=2)
## and some of its inverse :
pp <- seq(.1, .9, by=.1)
plot(qnorMix(pp, MW.nm3), pp)

## The "true" median of a normal mixture:
median.norMix <- function(x) qnorMix(1/2, x)
median.norMix(MW.nm3) ## -2.32
```

r.norMix

Ratio of Normal Mixture to Corresponding Normal

Description

Compute $r(x) = f(x)/f_0(x)$ where $f()$ is a normal mixture density and f_0 the normal density with the same mean and variance as f .

Usage

```
r.norMix(obj, x = NULL, xlim = NULL, n = 511, xy.return = TRUE)
```

Arguments

obj	an object of class norMix.
x	numeric vector with abscissa values where to evaluate the density. Default is constructed from n (and xlim if specified).
xlim	range of abscissa values, used if x == NULL. By default, xlim taken as mean plus/minus 3 standard deviations of the normal mixture.
n	number of abscissa values to generate if x is not specified.
xy.return	logical indicating if the result should be a list or just a numeric vector, see below.

Value

It depends on `xy.return`. If it's false, a numeric vector of the same length as `x`, if true (as per default), a list that can be plotted, with components

`x` abscissa values corresponding to argument `x`.
`y` corresponding values $r(x)$.
`f0` values of the moment matching normal density $f_0(x)$.

Note

The ratio function is used in certain semi-parametric density estimation methods (and theory).

Examples

```
d3 <- rnormMix(m = 5*(0:2), w = c(0.6, 0.3, 0.1))
plot(d3)
rd3 <- r.rnormMix(d3)
str(rd3)
stopifnot(rd3 $ y == r.rnormMix(d3, xy.ret = FALSE))
par(new = TRUE)
plot(rd3, type = "l", col = 3, axes = FALSE, xlab = "", ylab="")
axis(4, col.axis=3)
```

rnormMix

Generate 'Normal Mixture' Distributed Random Numbers

Description

Generate `n` random numbers, distributed according to a normal mixture.

Usage

```
rnormMix(n, obj)
```

Arguments

`n` the number of random numbers desired.
`obj` an object of class `rnormMix`.

Details

For a mixture of m , i.e., `m.rnormMix(obj)`, components, generate the number in each component as multinomial, and then use `rnorm` for each.

Note that these integer (multinomial) numbers are generated via `sample()`, which is by `.Random.seed`, notably from `RNGkind(sample.kind = .)` which changed with R version 3.6.0.

Value

numeric vector of length n.

See Also

[dnorMix](#) for the density, and [norMix](#) for the construction and further methods.

Examples

```
x <- rnorMix(5000, MW.nm10)
hist(x)# you don't see the claw
plot(density(x), ylim = c(0,0.6),
     main = "Estim. and true 'MW.nm10' density")
lines(MW.nm10, col = "orange")
```

 sort.norMix

Sort Method for "norMix" Objects

Description

Sorting a "norMix" object (see [norMix](#)), sorts along the mu values; i.e., for the default decreasing = FALSE the resulting `x[, "mu"]` are sorted from left to right.

Usage

```
## S3 method for class 'norMix'
sort(x, decreasing = FALSE, ...)
```

Arguments

`x` an object of class "norMix".
`decreasing` logical indicating if sorting should be up or down.
`...` further arguments passed to `sort(x[, "mu"], *)`.

Value

a "norMix" object like x.

Examples

```
sort(MW.nm9)
stopifnot(identical(MW.nm2, sort(MW.nm2)))
```

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