

Package ‘trawl’

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Type Package

Title Estimation and Simulation of Trawl Processes

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Description Contains R functions for simulating and estimating integer-valued trawl processes as described in the article Veraart (2019), “Modeling, simulation and inference for multivariate time series of counts using trawl processes”, Journal of Multivariate Analysis, 169, pages 110-129, <[doi:10.1016/j.jmva.2018.08.012](https://doi.org/10.1016/j.jmva.2018.08.012)> and for simulating random vectors from the bivariate negative binomial and the bi- and trivariate logarithmic series distributions.

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 acf_DExp

Autocorrelation function of the double exponential trawl function

Description

This function computes the autocorrelation function associated with the double exponential trawl function.

Usage

```
acf_DExp(x, w, lambda1, lambda2)
```

Arguments

x	The argument (lag) at which the autocorrelation function associated with the double exponential trawl function will be evaluated
w	parameter in the double exponential trawl
lambda1	parameter in the double exponential trawl
lambda2	parameter in the double exponential trawl

Details

The trawl function is parametrised by parameters $0 \leq w \leq 1$ and $\lambda_1, \lambda_2 > 0$ as follows:

$$g(x) = we^{\lambda_1 x} + (1 - w)e^{\lambda_2 x}, \text{ for } x \leq 0.$$

Its autocorrelation function is given by:

$$r(x) = (we^{-\lambda_1 x}/\lambda_1 + (1 - w)e^{-\lambda_2 x}/\lambda_2)/c, \text{ for } x \geq 0,$$

where

$$c = w/\lambda_1 + (1 - w)/\lambda_2.$$

Value

The autocorrelation function of the double exponential trawl function evaluated at x

Examples

```
acf_DExp(1, 0.3, 0.1, 2)
```

acf_Exp

Autocorrelation function of the exponential trawl function

Description

This function computes the autocorrelation function associated with the exponential trawl function.

Usage

```
acf_Exp(x, lambda)
```

Arguments

x	The argument (lag) at which the autocorrelation function associated with the exponential trawl function will be evaluated
lambda	parameter in the exponential trawl

Details

The trawl function is parametrised by the parameter $\lambda > 0$ as follows:

$$g(x) = e^{\lambda x}, \text{ for } x \leq 0.$$

Its autocorrelation function is given by:

$$r(x) = e^{-\lambda x}, \text{ for } x \geq 0.$$

Value

The autocorrelation function of the exponential trawl function evaluated at x

Examples

```
acf_Exp(1, 0.1)
```

```
acf_LM
```

Autocorrelation function of the long memory trawl function

Description

This function computes the autocorrelation function associated with the long memory trawl function.

Usage

```
acf_LM(x, alpha, H)
```

Arguments

x	The argument (lag) at which the autocorrelation function associated with the long memory trawl function will be evaluated
alpha	parameter in the long memory trawl
H	parameter in the long memory trawl

Details

The trawl function is parametrised by the two parameters $H > 1$ and $\alpha > 0$ as follows:

$$g(x) = (1 - x/\alpha)^{-H}, \text{ for } x \leq 0.$$

Its autocorrelation function is given by

$$r(x) = (1 + x/\alpha)^{(1-H)}, \text{ for } x \geq 0.$$

Value

The autocorrelation function of the long memory trawl function evaluated at x

Examples

```
acf_LM(1, 0.3, 1.5)
```

 acf_supIG

Autocorrelation function of the supIG trawl function

Description

This function computes the autocorrelation function associated with the supIG trawl function.

Usage

```
acf_supIG(x, delta, gamma)
```

Arguments

x	The argument (lag) at which the autocorrelation function associated with the supIG trawl function will be evaluated
delta	parameter in the supIG trawl
gamma	parameter in the supIG trawl

Details

The trawl function is parametrised by the two parameters $\delta \geq 0$ and $\gamma \geq 0$ as follows:

$$g(x) = (1 - 2x\gamma^{-2})^{-1/2} \exp(\delta\gamma(1 - (1 - 2x\gamma^{-2})^{1/2})), \text{ for } x \leq 0.$$

It is assumed that δ and γ are not simultaneously equal to zero. Its autocorrelation function is given by:

$$r(x) = \exp(\delta\gamma(1 - \sqrt{1 + 2x/\gamma^2})), \text{ for } x \geq 0.$$

Value

The autocorrelation function of the supIG trawl function evaluated at x

Examples

```
acf_supIG(1, 0.3, 0.1)
```

<code>Bivariate_LSDsim</code>	<i>Simulates from the bivariate logarithmic series distribution</i>
-------------------------------	---

Description

Simulates from the bivariate logarithmic series distribution

Usage

`Bivariate_LSDsim(N, p1, p2)`

Arguments

<code>N</code>	number of data points to be simulated
<code>p1</code>	parameter p_1 of the bivariate logarithmic series distribution
<code>p2</code>	parameter p_2 of the bivariate logarithmic series distribution

Details

The probability mass function of a random vector $X = (X_1, X_2)'$ following the bivariate logarithmic series distribution with parameters $0 < p_1, p_2 < 1$ with $p := p_1 + p_2 < 1$ is given by

$$P(X_1 = x_1, X_2 = x_2) = \frac{\Gamma(x_1 + x_2)}{x_1!x_2!} \frac{p_1^{x_1} p_2^{x_2}}{(-\log(1 - p))},$$

for $x_1, x_2 = 0, 1, 2, \dots$ such that $x_1 + x_2 > 0$. The simulation proceeds in two steps: First, X_1 is simulated from the modified logarithmic distribution with parameters $\tilde{p}_1 = p_1/(1 - p_2)$ and $\delta_1 = \log(1 - p_2)/\log(1 - p)$. Then we simulate X_2 conditional on X_1 . We note that $X_2|X_1 = x_1$ follows the logarithmic series distribution with parameter p_2 when $x_1 = 0$, and the negative binomial distribution with parameters (x_1, p_2) when $x_1 > 0$.

Value

An $N \times 2$ matrix with N simulated values from the bivariate logarithmic series distribution

<code>Bivariate_NBsim</code>	<i>Simulates from the bivariate negative binomial distribution</i>
------------------------------	--

Description

Simulates from the bivariate negative binomial distribution

Usage

`Bivariate_NBsim(N, kappa, p1, p2)`

Arguments

N	number of data points to be simulated
kappa	parameter κ of the bivariate negative binomial distribution
p1	parameter p_1 of the bivariate negative binomial distribution
p2	parameter p_2 of the bivariate negative binomial distribution

Details

A random vector $\mathbf{X} = (X_1, X_2)'$ is said to follow the bivariate negative binomial distribution with parameters κ, p_1, p_2 if its probability mass function is given by

$$P(\mathbf{X} = \mathbf{x}) = \frac{\Gamma(x_1 + x_2 + \kappa)}{x_1!x_2!\Gamma(\kappa)} p_1^{x_1} p_2^{x_2} (1 - p_1 - p_2)^\kappa,$$

where, for $i = 1, 2$, $x_i \in \{0, 1, \dots\}$, $0 < p_i < 1$ such that $p_1 + p_2 < 1$ and $\kappa > 0$.

Value

An $N \times 2$ matrix with N simulated values from the bivariate negative binomial distribution

BivLSD_Cor	<i>Computes the correlation of the components of a bivariate vector following the bivariate logarithmic series distribution</i>
------------	---

Description

Computes the correlation of the components of a bivariate vector following the bivariate logarithmic series distribution

Usage

```
BivLSD_Cor(p1, p2)
```

Arguments

p1	parameter p_1 of the bivariate logarithmic series distribution
p2	parameter p_2 of the bivariate logarithmic series distribution

Value

Correlation of the components of a bivariate vector following the bivariate logarithmic series distribution

BivLSD_Cov	<i>Computes the covariance of the components of a bivariate vector following the bivariate logarithmic series distribution</i>
------------	--

Description

Computes the covariance of the components of a bivariate vector following the bivariate logarithmic series distribution

Usage

BivLSD_Cov(p1, p2)

Arguments

p1	parameter p_1 of the bivariate logarithmic series distribution
p2	parameter p_2 of the bivariate logarithmic series distribution

Value

Covariance of the components of a bivariate vector following the bivariate logarithmic series distribution

BivModLSD_Cor	<i>Computes the correlation of the components of a bivariate vector following the bivariate modified logarithmic series distribution</i>
---------------	--

Description

Computes the correlation of the components of a bivariate vector following the bivariate modified logarithmic series distribution

Usage

BivModLSD_Cor(delta, p1, p2)

Arguments

delta	parameter δ of the bivariate modified logarithmic series distribution
p1	parameter p_1 of the bivariate modified logarithmic series distribution
p2	parameter p_2 of the bivariate modified logarithmic series distribution

Value

Covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution

BivModLSD_Cov	<i>Computes the covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution</i>
---------------	---

Description

Computes the covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution

Usage

```
BivModLSD_Cov(delta, p1, p2)
```

Arguments

delta	parameter δ of the bivariate modified logarithmic series distribution
p1	parameter p_1 of the bivariate modified logarithmic series distribution
p2	parameter p_2 of the bivariate modified logarithmic series distribution

Value

Covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution

fit_DExprawl	<i>Fits the trawl function consisting of the weighted sum of two exponential functions</i>
--------------	--

Description

Fits the trawl function consisting of the weighted sum of two exponential functions

Usage

```
fit_DExprawl(x, Delta = 1, GMMlag = 5, plotacf = FALSE, lags = 100)
```

Arguments

x	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
GMMlag	lag length used in the GMM estimation, the default is 5
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted
lags	number of lags to be used in the plot of the autocorrelation function

Details

The trawl function is parametrised by the three parameters $0 \leq w \leq 1$ and $\lambda_1, \lambda_2 > 0$ as follows:

$$g(x) = we^{\lambda_1 x} + (1 - w)e^{\lambda_2 x}, \text{ for } x \leq 0.$$

The Lebesgue measure of the corresponding trawl set is given by $w/\lambda_1 + (1 - w)/\lambda_2$.

Value

w: the weight parameter (restricted to be in [0,0.5] for identifiability reasons)

lambda1: the first memory parameter (denoted by λ_1 above)

lambda2: the second memory parameter (denoted by λ_2 above)

LM: The Lebesgue measure of the trawl set associated with the double exponential trawl

 fit_Exptrawl

Fits an exponential trawl function to equidistant time series data

Description

Fits an exponential trawl function to equidistant time series data

Usage

```
fit_Exptrawl(x, Delta = 1, plotacf = FALSE, lags = 100)
```

Arguments

x	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted
lags	number of lags to be used in the plot of the autocorrelation function

Details

The trawl function is parametrised by the parameter $\lambda > 0$ as follows:

$$g(x) = e^{\lambda x}, \text{ for } x \leq 0.$$

The Lebesgue measure of the corresponding trawl set is given by $1/\lambda$.

Value

lambda: the memory parameter λ in the exponential trawl

LM: the Lebesgue measure of the trawl set associated with the exponential trawl, i.e. $1/\lambda$.

fit_LMtrawl	<i>Fits a long memory trawl function to equidistant univariate time series data</i>
-------------	---

Description

Fits a long memory trawl function to equidistant univariate time series data

Usage

```
fit_LMtrawl(x, Delta = 1, GMMlag = 5, plotacf = FALSE, lags = 100)
```

Arguments

x	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
GMMlag	lag length used in the GMM estimation, the default is 5
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted
lags	number of lags to be used in the plot of the autocorrelation function

Details

The trawl function is parametrised by the two parameters $H > 1$ and $\alpha > 0$ as follows:

$$g(x) = (1 - x/\alpha)^{-H}, \text{ for } x \leq 0.$$

The Lebesgue measure of the corresponding trawl set is given by $\alpha/(1 - H)$.

Value

alpha: parameter in the long memory trawl

H: parameter in the long memory trawl

LM: The Lebesgue measure of the trawl set associated with the long memory trawl

fit_marginalNB *Fits a negative binomial distribution as marginal law*

Description

Fits a negative binomial distribution as marginal law

Usage

```
fit_marginalNB(x, LM, plotdiag = FALSE)
```

Arguments

x vector of equidistant time series data
 LM Lebesgue measure of the estimated trawl
 plotdiag binary variable specifying whether or not diagnostic plots should be provided

Details

The moment estimator for the parameters of the negative binomial distribution are given by

$$\hat{\theta} = 1 - E(X)/\text{Var}(X),$$

and

$$\hat{m} = E(X)(1 - \hat{\theta})/(\widehat{LM}\hat{\theta}).$$

Value

m: parameter in the negative binomial marginal distribution
 theta: parameter in the negative binomial marginal distribution
 a: Here $a = \theta/(1 - \theta)$. This is given for an alternative parametrisation of the negative binomial marginal distribution

fit_marginalPoisson *Fits a Poisson distribution as marginal law*

Description

Fits a Poisson distribution as marginal law

Usage

```
fit_marginalPoisson(x, LM, plotdiag = FALSE)
```

Arguments

x	vector of equidistant time series data
LM	Lebesgue measure of the estimated trawl
plotdiag	binary variable specifying whether or not diagnostic plots should be provided

Details

The moment estimator for the Poisson rate parameter is given by

$$\hat{v} = E(X)/\widehat{LM}.$$

Value

v: the rate parameter in the Poisson marginal distribution

fit_supIGtrawl	<i>Fits a supIG trawl function to equidistant univariate time series data</i>
----------------	---

Description

Fits a supIG trawl function to equidistant univariate time series data

Usage

```
fit_supIGtrawl(x, Delta = 1, GMMlag = 5, plotacf = FALSE, lags = 100)
```

Arguments

x	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
GMMlag	lag length used in the GMM estimation, the default is 5
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted
lags	number of lags to be used in the plot of the autocorrelation function

Details

The trawl function is parametrised by the two parameters $\delta \geq 0$ and $\gamma \geq 0$ as follows:

$$g(x) = (1 - 2x\gamma^{-2})^{-1/2} \exp(\delta\gamma(1 - (1 - 2x\gamma^{-2})^{1/2})), \text{ for } x \leq 0.$$

It is assumed that δ and γ are not simultaneously equal to zero. The Lebesgue measure of the corresponding trawl set is given by γ/δ .

Value

delta: parameter in the supIG trawl

gamma: parameter in the supIG trawl

LM: The Lebesgue measure of the trawl set associated with the supIG trawl

 fit_trawl_intersection

Finds the intersection of two trawl sets

Description

Finds the intersection of two trawl sets

Usage

```
fit_trawl_intersection(
  fct1 = base::c("Exp", "DExp", "supIG", "LM"),
  fct2 = base::c("Exp", "DExp", "supIG", "LM"),
  lambda11 = 0,
  lambda12 = 0,
  w1 = 0,
  delta1 = 0,
  gamma1 = 0,
  alpha1 = 0,
  H1 = 0,
  lambda21 = 0,
  lambda22 = 0,
  w2 = 0,
  delta2 = 0,
  gamma2 = 0,
  alpha2 = 0,
  H2 = 0,
  LM1,
  LM2,
  plotdiag = FALSE
)
```

Arguments

fct1	specifies the type of the first trawl function
fct2	specifies the type of the second trawl function
lambda11, lambda12, w1	parameters of the (double) exponential trawl functions of the first process
delta1, gamma1	parameters of the supIG trawl functions of the first process
alpha1, H1	parameters of the long memory trawl function of the first process
lambda21, lambda22, w2	parameters of the (double) exponential trawl functions of the second process
delta2, gamma2	parameters of the supIG trawl functions of the second process
alpha2, H2	parameters of the long memory trawl function of the second process
LM1	Lebesgue measure of the first trawl

LM2	Lebesgue measure of the second trawl
plotdiag	binary variable specifying whether or not diagnostic plots should be provided

Details

Computes $R_{12}(0) = \text{Leb}(A_1 \cap A_2)$ based on two trawl functions g_1 and g_2 .

Value

The Lebesgue measure of the intersection of the two trawl sets

fit_trawl_intersection_Exp

Finds the intersection of two exponential trawl sets

Description

Finds the intersection of two exponential trawl sets

Usage

```
fit_trawl_intersection_Exp(lambda1, lambda2, LM1, LM2, plotdiag = FALSE)
```

Arguments

lambda1, lambda2	parameters of the two exponential trawls
LM1	Lebesgue measure of the first trawl
LM2	Lebesgue measure of the second trawl
plotdiag	binary variable specifying whether or not diagnostic plots should be provided

Details

Computes $R_{12}(0) = \text{Leb}(A_1 \cap A_2)$ based on two trawl functions g_1 and g_2 .

Value

The Lebesgue measure of the intersection of the two trawl sets

```
fit_trawl_intersection_LM
```

Finds the intersection of two long memory (LM) trawl sets

Description

Finds the intersection of two long memory (LM) trawl sets

Usage

```
fit_trawl_intersection_LM(alpha1, H1, alpha2, H2, LM1, LM2, plotdiag = FALSE)
```

Arguments

alpha1, H1, alpha2, H2	parameters of the two long memory trawls
LM1	Lebesgue measure of the first trawl
LM2	Lebesgue measure of the second trawl
plotdiag	binary variable specifying whether or not diagnostic plots should be provided

Details

Computes $R_{12}(0) = \text{Leb}(A_1 \cap A_2)$ based on two trawl functions g_1 and g_2 .

Value

the Lebesgue measure of the intersection of the two trawl sets

```
LSD_Mean
```

Computes the mean of the logarithmic series distribution

Description

Computes the mean of the logarithmic series distribution

Usage

```
LSD_Mean(p)
```

Arguments

p	parameter of the logarithmic series distribution
---	--

Details

A random variable X has logarithmic series distribution with parameter $0 < p < 1$ if

$$P(X = x) = (-1)/(\log(1 - p))p^x/x, \text{ for } x = 1, 2, \dots$$

Value

Mean of the logarithmic series distribution

LSD_Var	<i>Computes the variance of the logarithmic series distribution</i>
---------	---

Description

Computes the variance of the logarithmic series distribution

Usage

LSD_Var(p)

Arguments

p parameter of the logarithmic series distribution

Details

A random variable X has logarithmic series distribution with parameter $0 < p < 1$ if

$$P(X = x) = (-1)/(\log(1 - p))p^x/x, \text{ for } x = 1, 2, \dots$$

Value

Variance of the logarithmic series distribution

ModLSD_Mean	<i>Computes the mean of the modified logarithmic series distribution</i>
-------------	--

Description

Computes the mean of the modified logarithmic series distribution

Usage

ModLSD_Mean(delta, p)

Arguments

delta	parameter δ of the modified logarithmic series distribution
p	parameter of the modified logarithmic series distribution

Details

A random variable X has modified logarithmic series distribution with parameters $0 \leq \delta < 1$ and $0 < p < 1$ if $P(X = 0) = (1 - \delta)$ and

$$P(X = x) = (1 - \delta)(-1)/(\log(1 - p))p^x/x, \text{ for } x = 1, 2, \dots$$

Value

Mean of the modified logarithmic series distribution

ModLSD_Var	<i>Computes the variance of the modified logarithmic series distribution</i>
------------	--

Description

Computes the variance of the modified logarithmic series distribution

Usage

```
ModLSD_Var(delta, p)
```

Arguments

delta	parameter δ of the modified logarithmic series distribution
p	parameter of the modified logarithmic series distribution

Details

A random variable X has modified logarithmic series distribution with parameters $0 \leq \delta < 1$ and $0 < p < 1$ if $P(X = 0) = (1 - \delta)$ and

$$P(X = x) = (1 - \delta)(-1)/(\log(1 - p))p^x/x, \text{ for } x = 1, 2, \dots$$

Value

Mean of the modified logarithmic series distribution

plot_2and1hist	<i>Plots the bivariate histogram of two time series together with the univariate histograms</i>
----------------	---

Description

Plots the bivariate histogram of two time series together with the univariate histograms

Usage

```
plot_2and1hist(x, y)
```

Arguments

x	vector of equidistant time series data
y	vector of equidistant time series data (of the same length as x)

Details

This function plots the bivariate histogram of two time series together with the univariate histograms

Value

no return value

plot_2and1hist_gg	<i>Plots the bivariate histogram of two time series together with the univariate histograms using ggplot2</i>
-------------------	---

Description

Plots the bivariate histogram of two time series together with the univariate histograms using ggplot2

Usage

```
plot_2and1hist_gg(x, y, bivbins = 50, xbins = 30, ybins = 30)
```

Arguments

x	vector of equidistant time series data
y	vector of equidistant time series data (of the same length as x)
bivbins	number of bins in the bivariate histogram
xbins	number of bins in the histogram of x
ybins	number of bins in the histogram of y

Details

This function plots the bivariate histogram of two time series together with the univariate histograms

Value

no return value

sim_BivariateTrawl *Simulates a bivariate trawl process*

Description

Simulates a bivariate trawl process

Usage

```
sim_BivariateTrawl(  
  t,  
  Delta = 1,  
  burnin = 10,  
  marginal = base::c("Poi", "NegBin"),  
  dependencetype = base::c("fullydep", "dep"),  
  trawl1 = base::c("Exp", "DExp", "supIG", "LM"),  
  trawl2 = base::c("Exp", "DExp", "supIG", "LM"),  
  v1 = 0,  
  v2 = 0,  
  v12 = 0,  
  kappa1 = 0,  
  kappa2 = 0,  
  kappa12 = 0,  
  a1 = 0,  
  a2 = 0,  
  lambda11 = 0,  
  lambda12 = 0,  
  w1 = 0,  
  delta1 = 0,  
  gamma1 = 0,  
  alpha1 = 0,  
  H1 = 0,  
  lambda21 = 0,  
  lambda22 = 0,  
  w2 = 0,  
  delta2 = 0,  
  gamma2 = 0,  
  alpha2 = 0,  
  H2 = 0  
)
```

Arguments

t	parameter which specifying the length of the time interval $[0, t]$ for which a simulation of the trawl process is required.
Delta	parameter Δ specifying the length of the time step, the default is 1
burnin	parameter specifying the length of the burn-in period at the beginning of the simulation
marginal	parameter specifying the marginal distribution of the trawl
dependencetype	Parameter specifying the type of dependence
trawl1	parameter specifying the type of the first trawl function
trawl2	parameter specifying the type of the second trawl function
v1, v2, v12	parameters of the Poisson distribution
kappa1, kappa2, kappa12, a1, a2	parameters of the (possibly bivariate) negative binomial distribution
lambda11, lambda12, w1	parameters of the exponential (or double-exponential) trawl function of the first process
delta1, gamma1	parameters of the supIG trawl function of the first process
alpha1, H1	parameter of the long memory trawl of the first process
lambda21, lambda22, w2	parameters of the exponential (or double-exponential) trawl function of the second process
delta2, gamma2	parameters of the supIG trawl function of the second process
alpha2, H2	parameter of the long memory trawl of the second process

Details

This function simulates a bivariate trawl process with either Poisson or negative binomial marginal law. For the trawl function there are currently four choices: exponential, double-exponential, supIG or long memory. More details on the precise simulation algorithm is available in the vignette.

sim_UnivariateTrawl *Simulates a univariate trawl process*

Description

Simulates a univariate trawl process

Usage

```

sim_UnivariateTrawl(
  t,
  Delta = 1,
  burnin = 10,
  marginal = base::c("Poi", "NegBin"),
  trawl = base::c("Exp", "DExp", "supIG", "LM"),
  v = 0,
  m = 0,
  theta = 0,
  lambda1 = 0,
  lambda2 = 0,
  w = 0,
  delta = 0,
  gamma = 0,
  alpha = 0,
  H = 0
)

```

Arguments

t	parameter which specifying the length of the time interval $[0, t]$ for which a simulation of the trawl process is required.
Delta	parameter Δ specifying the length of the time step, the default is 1
burnin	parameter specifying the length of the burn-in period at the beginning of the simulation
marginal	parameter specifying the marginal distribution of the trawl
trawl	parameter specifying the type of trawl function
v	parameter of the Poisson distribution
m	parameter of the negative binomial distribution
theta	parameter θ of the negative binomial distribution
lambda1	parameter λ_1 of the exponential (or double-exponential) trawl function
lambda2	parameter λ_2 of the double-exponential trawl function
w	parameter of the double-exponential trawl function
delta	parameter δ of the supIG trawl function
gamma	parameter γ of the supIG trawl function
alpha	parameter α of the long memory trawl function
H	parameter of the long memory trawl function

Details

This function simulates a univariate trawl process with either Poisson or negative binomial marginal law. For the trawl function there are currently four choices: exponential, double-exponential, supIG or long memory. More details on the precise simulation algorithm is available in the vignette.

trawl_DExp	<i>Evaluates the double exponential trawl function</i>
------------	--

Description

Evaluates the double exponential trawl function

Usage

```
trawl_DExp(x, w, lambda1, lambda2)
```

Arguments

x	the argument at which the double exponential trawl function will be evaluated
w	parameter in the double exponential trawl
lambda1	the parameter λ_1 in the double exponential trawl
lambda2	the parameter λ_2 in the double exponential trawl

Details

The trawl function is parametrised by parameters $0 \leq w \leq 1$ and $\lambda_1, \lambda_2 > 0$ as follows:

$$g(x) = we^{\lambda_1 x} + (1 - w)e^{\lambda_2 x z}, \text{ for } x \leq 0.$$

Value

The double exponential trawl function evaluated at x

trawl_Exp	<i>Evaluates the exponential trawl function</i>
-----------	---

Description

Evaluates the exponential trawl function

Usage

```
trawl_Exp(x, lambda)
```

Arguments

x	the argument at which the exponential trawl function will be evaluated
lambda	the parameter λ in the exponential trawl

Details

The trawl function is parametrised by parameter $\lambda > 0$ as follows:

$$g(x) = e^{\lambda x}, \text{ for } x \leq 0.$$

Value

The exponential trawl function evaluated at x

trawl_LM	<i>Evaluates the long memory trawl function</i>
----------	---

Description

Evaluates the long memory trawl function

Usage

trawl_LM(x, alpha, H)

Arguments

x	the argument at which the long memory trawl function will be evaluated
alpha	the parameter α in the long memory trawl
H	the parameter H in the long memory trawl

Details

The trawl function is parametrised by the two parameters $H > 1$ and $\alpha > 0$ as follows:

$$g(x) = (1 - x/\alpha)^{-H}, \text{ for } x \leq 0.$$

Value

the long memory trawl function evaluated at x

trawl_supIG	<i>Evaluates the supIG trawl function</i>
-------------	---

Description

Evaluates the supIG trawl function

Usage

```
trawl_supIG(x, delta, gamma)
```

Arguments

x	the argument at which the supIG trawl function will be evaluated
delta	the parameter δ in the supIG trawl
gamma	the parameter γ in the supIG trawl

Details

The trawl function is parametrised by the two parameters $\delta \geq 0$ and $\gamma \geq 0$ as follows:

$$gd(x) = (1 - 2x\gamma^{-2})^{-1/2} \exp(\delta\gamma(1 - (1 - 2x\gamma^{-2})^{1/2})), \text{ for } x \leq 0.$$

It is assumed that δ and γ are not simultaneously equal to zero.

Value

The supIG trawl function evaluated at x

Trivariate_LSDsim	<i>Simulates from the trivariate logarithmic series distribution</i>
-------------------	--

Description

Simulates from the trivariate logarithmic series distribution

Usage

```
Trivariate_LSDsim(N, p1, p2, p3)
```

Arguments

N	number of data points to be simulated
p1	parameter $p1$ of the trivariate logarithmic series distribution
p2	parameter $p2$ of the trivariate logarithmic series distribution
p3	parameter $p3$ of the trivariate logarithmic series distribution

Details

The probability mass function of a random vector $X = (X_1, X_2, X_3)'$ following the trivariate logarithmic series distribution with parameters $0 < p_1, p_2, p_3 < 1$ with $p := p_1 + p_2 + p_3 < 1$ is given by

$$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = \frac{\Gamma(x_1 + x_2 + x_3)}{x_1!x_2!x_3!} \frac{p_1^{x_1} p_2^{x_2} p_3^{x_3}}{(-\log(1-p))},$$

for $x_1, x_2, x_3 = 0, 1, 2, \dots$ such that $x_1 + x_2 + x_3 > 0$.

The simulation proceeds in two steps: First, X_1 is simulated from the modified logarithmic distribution with parameters $\tilde{p}_1 = p_1/(1 - p_2 - p_3)$ and $\delta_1 = \log(1 - p_2 - p_3)/\log(1 - p)$. Then we simulate $(X_2, X_3)'$ conditional on X_1 . We note that $(X_2, X_3)'|X_1 = x_1$ follows the bivariate logarithmic series distribution with parameters (p_2, p_3) when $x_1 = 0$, and the bivariate negative binomial distribution with parameters (x_1, p_2, p_3) when $x_1 > 0$.

Value

An $N \times 3$ matrix with N simulated values from the trivariate logarithmic series distribution

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