

# Package ‘trunmnt’

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**Title** Moments of Truncated Multivariate Normal Distribution

**Version** 1.0.0

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**Description** Computes the product moments of the truncated multivariate normal distribution, particularly for cases involving patterned variance-covariance matrices. It also has the capability to calculate these moments with arbitrary positive-definite matrices, although performance may degrade for high-dimensional variables.

**License** GPL-2

**Depends** R ( $\geq 4.1.0$ )

**Imports** fastGHQuad, Rcpp, RcppArmadillo

**Suggests** MomTrunc ( $\geq 6.1$ ), truncnorm ( $\geq 1.0.9$ ), tmvtnorm ( $\geq 1.7$ ), testthat, R.rsp

**LinkingTo** Rcpp, RcppArmadillo, fastGHQuad

**NeedsCompilation** yes

**VignetteBuilder** R.rsp

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trunmnt-package	<i>Moments of Truncated Multivariate Normal Distribution.</i>
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## Description

Based on the algorithm given by Lee (2021), it computes the product moment of a truncated multivariate normal distribution using the multivariate Gaussian quadrature.

## Details

Use `mtrunmnt` to generate a S3 objective and then use `prodmnt` to compute arbitrary order of moments. It can also calculate the mean and variance-covariance matrix of a truncated multivariate normal distribution by `meanvar`. See also `probntrun` and `utrunmnt` for computing the probability of multivariate normal distribution for given limits, and the moment of truncated univariate normal distribution.

The 'trunmnt' package computes the product moments of the Truncated multivariate normal distribution by implementing the algorithm proposed by Lee (2020). This approach relies on multivariate Gaussian quadrature for numerical integration.

**\*\*Limitations:\*\*** While the package supports arbitrary positive-definite matrices, the computational complexity scales poorly with the dimension of the vector. For dimensions  $D > 5$ , using the pattern-specific functions or considering approximation methods is highly recommended.

## Author(s)

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## References

Burkardt, J. (2014). The truncated normal distribution, *Online document*, Available from: [https://people.sc.fsu.edu/~jburkardt/presentations/truncated\\_normal.pdf](https://people.sc.fsu.edu/~jburkardt/presentations/truncated_normal.pdf).

Jaeckel, P. (2005). *A note on multivariate Gauss-Hermite quadrature*. London: ABN-Amro. Available from: <http://www.jaeckel.org/ANoteOnMultivariateGaussHermiteQuadrature.pdf>.

Lee, S.-C. (2021). Moments Calculation for Truncated Multivariate Normal in Nonlinear Generalized Mixed Models. *Communications for Statistical Applications and Methods*, 27, 377–383.

## See Also

[MomTrunc-package](#)

**Examples**

```

set.seed(123)
sigma2e <- 1
sigma2a <- 2
n <- 5
mu <- seq(-1,1, length.out = n)
y <- mu + rnorm(1, sd = sqrt(sigma2a)) + rnorm(n, sd = sqrt(sigma2e))
S <- matrix(sigma2a, ncol = n, nrow = n) + diag(sigma2e, n)
a <- rep(-Inf, n)
b <- rep(Inf, n)
a[y >= 0] <- 0
b[y < 0] <- 0
obj1 <- mtrunmnt(mu, lower = a, upper = b, Sigmae = sigma2e, D = sigma2a)
obj2 <- mtrunmnt(mu, lower = a, upper = b, Sigma = S)
probntrun(obj1)
probntrun(obj2)
prodmnt(obj1, c(2,2,0,0,0))
prodmnt(obj2, c(2,2,0,0,0))
meanvar(obj1)
meanvar(obj2)

```

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meanvar

*Mean and variance for a truncated multivariate normal distribution.*


---

**Description**

meanvar is a S3 generic function of the class `mtrunmnt`. Using the `prodmnt`, it compute the mean and variance-covariance matrix for a truncated multivariate normal distribution.

**Usage**

```
meanvar(Obj)
```

**Arguments**

Obj                    An `mtrunmnt` object created by the `mtrunmnt`.

**Value**

A list with the mean and variance-covariance matrix.

**See Also**

`mtrunmnt`, `meanvarTMD`, `mtmvnorm` and `prodmnt`.

## Examples

```
### A simple example ####

set.seed(123)
sigma2e <- 1
sigma2a <- 2
n <- 5
mu <- seq(-1,1, length.out = n)
y <- mu + rnorm(1, sd = sqrt(sigma2a)) + rnorm(n, sd = sqrt(sigma2e))
a <- rep(-Inf, n)
b <- rep(Inf, n)
a[y >= 0] <- 0
b[y < 0] <- 0
obj <- mtrunmnt(mu, lower = a, upper = b, Sigmae = sigma2e, D = sigma2a)
meanvar(obj)
```

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mtrunmnt

*Creating an S3 object for computing the product moment*


---

## Description

mtrunmnt creates an S3 object designed to compute the product moment for a truncated multivariate normal distribution, utilizing the algorithm by Lee (2020). Key attributes of this object are the nodes and weights of the multivariate Gaussian quadrature and the probability of the truncation interval

## Usage

```
mtrunmnt(
  mu,
  lower = -Inf,
  upper = Inf,
  Sigma = 1,
  Sigmae = 1,
  Z = matrix(rep(1, length(mu)), ncol = 1),
  D = matrix(1, ncol = 1, nrow = 1),
  nGH = 35
)
```

## Arguments

mu	<b>**(Required)**</b> Mean vector of the parent multivariate normal distribution.
lower	vector of lower limits. If the lower limits are the same, a scalar value can be given. Defaults to -Inf.
upper	Vector of upper limits. If the upper limits are the same, a scalar value can be given. Defaults to Inf.

Sigma	The variance-covariance matrix of the parent multivariate normal distribution. It must be given a symmetric positive definite matrix, if Sigmae, D and Z are not specified.
Sigmae	Vector of variances of error terms. If the variances are the same, a scalar value can be given. Defaults to 1.
Z	Design matrix for the random components. Defaults to $n \times 1$ matrix of 1's where $n$ is the dimension of mu. It must be specified carefully with the argument D. $\text{ncol}(Z)$ determines the dimension of D.
D	Variance-covariance matrix of $u$ . See Details. If the random components are independent, you can specify either a vector of variances or a scalar value. A scalar value means that the random components have the same variance. Defaults to 1.
nGH	Number of quadrature points. Defaults to 35.

### Details

Assume the parent multivariate normal distribution comes from a mixed-effects linear model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\epsilon}$$

where  $\mathbf{X}$  and  $\mathbf{Z}$  are design matrices corresponding to  $\boldsymbol{\beta}$  and  $\mathbf{u}$  representing fixed and random effects, respectively, and  $\boldsymbol{\epsilon}$  is the vector of errors. It is assumed that the random effects  $\mathbf{u}$  follows a multivariate normal distribution with mean  $\mathbf{0}$ , and symmetric positive definite variance-covariance matrix  $\mathbf{D}$ . As usual, the distribution of  $\boldsymbol{\epsilon}$  is assumed to be a multivariate normal with mean  $\mathbf{0}$  and variance-covariance matrix  $\sigma_{\epsilon}^2\mathbf{I}$ , but for more flexibility, it can be assumed that the error terms are independent, but do not have equal variance. That is, **we assume**  $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \mathbf{E})$  where  $\mathbf{E}$  is a diagonal matrix. Then,

$$\mathbf{Y} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where  $\boldsymbol{\Sigma} = \mathbf{Z}\mathbf{D}\mathbf{Z}' + \mathbf{E}$ . The variance-covariance structure in mtrunmnt can thus be specified either by providing the individual components  $\mathbf{D}$ ,  $\mathbf{Z}$ , and  $\mathbf{E}$ , or by directly supplying the resulting overall variance-covariance matrix *boldsymbol* $\boldsymbol{\Sigma}$ .

### Value

A mtrunmnt object

### References

Lee, S.-C. (2020). Moments calculation for truncated multivariate normal in nonlinear generalized mixed models. *Communications for Statistical Applications and Methods*, Vol. 27, No. 3, 377–383.

### Examples

```
### Create a mtrunmnt objective ###

set.seed(123)
sigma2e <- 1
sigma2a <- 2
n <- 5
```

```

mu <- seq(-1,1, length.out = n)
y <- mu + rnorm(1, sd = sqrt(sigma2a)) + rnorm(n, sd = sqrt(sigma2e))
S <- matrix(sigma2a, ncol = n, nrow = n) + diag(sigma2e, n)
a <- rep(-Inf, n)
b <- rep(Inf, n)
a[y >= 0] <- 0
b[y < 0] <- 0
obj1 <- mtrunmnt(mu, lower = a, upper = b, Sigmae = sigma2e, D = sigma2a)
obj2 <- mtrunmnt(mu, lower = a, upper = b, Sigma = S)
probntrun(obj1)
probntrun(obj2)
prodmnt(obj1, c(2,2,0,0,0))
prodmnt(obj2, c(2,2,0,0,0))
meanvar(obj1)
meanvar(obj2)

```

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probntrun

---

*Compute the probability mass after truncation.*


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### Description

Compute the probability for the truncation interval  $(\mathbf{a}, \mathbf{b})$ .

### Usage

```
probntrun(Obj)
```

### Arguments

Obj                    An mtrunmnt object created by the `mtrunmnt`.

### Details

`probntrun` is a S3 generic function of the class `mtrunmnt`. Using the multivariate Gaussian quadrature, it computes the probability of truncation interval  $(\mathbf{a}, \mathbf{b})$ .

### Value

a numeric value.

### See Also

`mtrunmnt`, `ptmvnorm`.

**Examples**

```
### A simple example ####
set.seed(123)
sigma2e <- 1
sigma2a <- 2
n <- 5
mu <- seq(-1,1, length.out = n)
y <- mu + rnorm(1, sd = sqrt(sigma2a)) + rnorm(n, sd = sqrt(sigma2e))
a <- rep(-Inf, n)
b <- rep(Inf, n)
a[y >= 0] <- 0
b[y < 0] <- 0
obj <- mtrunmnt(mu, lower = a, upper = b, Sigmae = sigma2e, D = sigma2a)
probntrun(obj)
```

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 prodmnt

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*Compute the product moment.*


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**Description**

It computes the kappa-th order product moment for a truncated multivariate normal distribution.

**Usage**

```
prodmnt(Obj, kappa)
```

**Arguments**

Obj                    mtrunmnt object created by the `mtrunmnt`.

kappa                  Vector of orders of length equal to mu.

**Details**

prodmnt is a S3 generic function of the class `mtrunmnt`. Using the multivariate Gaussian quadrature, it computes the product moment  $E(\prod_{i=1}^n Y_i^{k_i} | a_i < Y_i < b_i, i = 1, \dots, n)$ , where  $Y \sim N(\mu, ZDZ' + E)$ .

**Value**

a numeric value.

**See Also**

[mtrunmnt](#), [momentsTMD](#).

**Examples**

```
### A simple example ####

set.seed(123)
sigma2e <- 1
sigma2a <- 2
n <- 5
mu <- seq(-1,1, length.out = n)
y <- mu + rnorm(1, sd = sqrt(sigma2a)) + rnorm(n, sd = sqrt(sigma2e))
a <- rep(-Inf, n)
b <- rep(Inf, n)
a[y >= 0] <- 0
b[y < 0] <- 0
obj <- mtrunmnt(mu, lower = a, upper = b, Sigmae = sigma2e, D = sigma2a)
prodmnt(obj, c(2,2,0,0,0))
```

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utrunmnt	<i>Compute the <math>k</math>-th order moment for an univariate truncated normal distribution.</i>
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**Description**

The `utrunmnt` function uses the moment generating function to compute any order of moment for the truncated normal distribution.

**Usage**

```
utrunmnt(k, mu = 0, lower = -Inf, upper = Inf, sd = 1)
```

**Arguments**

<code>k</code>	Order of moment. It must be a non-negative integer.
<code>mu</code>	Mean of parent normal distribution. Defaults to 0.
<code>lower</code>	Lower limit. Defaults to -Inf.
<code>upper</code>	Upper limit. Defaults to Inf.
<code>sd</code>	Standard deviation of parent normal distribution. Defaults to 1.

**Value**

a numeric value.

**References**

Burkardt, J. (2014). The truncated normal distribution, *Online document*, Available from: [https://people.sc.fsu.edu/~jburkardt/presentations/truncated\\_normal.pdf](https://people.sc.fsu.edu/~jburkardt/presentations/truncated_normal.pdf).

**Examples**

```
utrunmnt(4, mu = 5, upper = 10)  
utrunmnt(1, mu = 5, lower = -3, upper = 4, sd = 2)
```

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